Constrained control problems in fault and delay tolerant networked control systems

N. Stanković[1,2] S. Olaru[1] S.-I. Niculescu[2]

[1]Supélec System Sciences (E3S) - Automatic Control Department
{nikola.stankovic,sorin.olaru}@supelec.fr

[2]CNRS - Supélec - Laboratory of Signals and Systems (UMR CNRS 8506)
{nikola.stankovic,silviu.niculescu}@lss.supelec.fr

GT-CPNL Journée du jeudi 17 Octobre 2013, Paris

OUTLINE

SOME OPPORTUNITIES AND CHALLENGES IN CONTROL

BUILDING VERY RELIABLE SYSTEMS FROM UNRELIABLE PARTS

Most engineering systems must continue to operate even when individual components fail. This requires designs that allow the system to automatically reconfigure itself so that its performance degrades gradually rather than abruptly.

CONTROL IN DISTRIBUTED, NETWORKED ENVIRONMENT

Control distributed across multiple computational units, interconnected through packet-based communications. Applications where one cannot ignore computational and communication constraints in performing control operations.

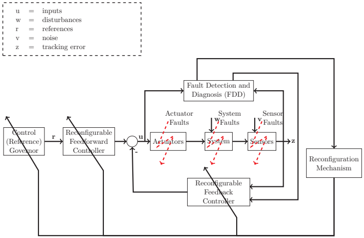
DEALING WITH UNCERTAINTIES AND HARD CONSTRAINTS ON SYSTEM PARAMETERS

In engineering applications the validity of even simple models is often limited by the presence of hard constraints (range of mechanical displacements, limits on acceleration...)

BUILDING VERY RELIABLE SYSTEMS FROM UNRELIABLE PARTS

FAULT IN A DYNAMICAL SYSTEM

- A deviation of the system structure or the system parameters from the nominal characterization.
- Permanent (e.g. damaged components) and temporary (e.g. a temporary change in the work conditions).

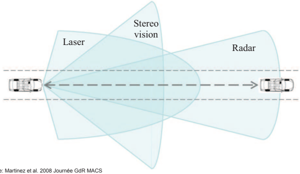


Source: Zhang and Jiang 2008

FAULT TOLERANT CONTROL (FTC)

- Passive FTC: a robust control strategy. It addresses only a limited and predefined range of faults.
- Active FTC: an adaptive control strategy. It includes fault detection and reconfiguration mechanism.

Redundancy



Source: Martinez et al. 2008 Journée GdR MACS

SENSOR FAULTS (E.G. BURGENAIR FLIGHT 301)

- Propagation through the plant can be prevented by the control reconfiguration.

ACTUATOR FAULTS (E.G. EL AL FLIGHT 1862)

- Harder to deal with due to their propagation through the plant.

APPLICATIONS

Safety-critical applications, e.g., in aeronautics, nuclear power plants, chemical industry, collision avoidance systems...

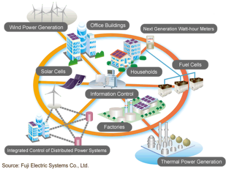
Control in distributed, networked environment

Networked control systems (NCS)

- Shared communication resources.
- Packet-based data transmission.
- Distributed sensing and control.

NCS: Pros and cons

- Pros: increases flexibility, facilitates maintenance, reduces the wiring and implementation cost.
- Cons: bandwidth limitation, network induced delays, packet dropouts.

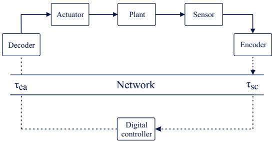


Source: Fuji Electric Systems Co., Ltd.

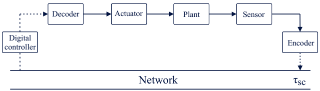
Applications

Smart energy systems, intelligent transportation systems, process industry, automotive industry...

- General single-channel NCS configuration



- Controller collocated with the actuator



NETWORK INDUCED DELAYS

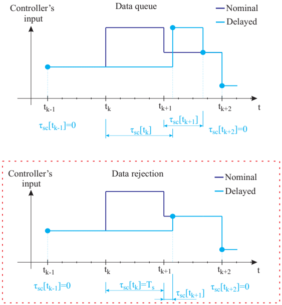
- Appear while transmitting measurements to the controller (τ_{sc}) and/or while transmitting a control signal to the actuator (τ_{ca}).
- Depending on an underlying network protocol, delays can be either constant (e.g. ControlNet) or time-varying (e.g. Ethernet).
- Depend on network access time, network congestion...

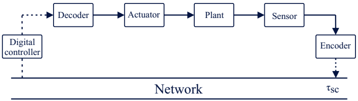
PACKET DROPOUTS

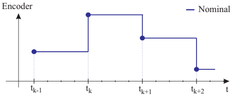
- Due to data loss during the transmission - more common in wireless networks where the links are less reliable.
- Due to rejection of outdated information.

OUTDATED DATA REJECTION VERSUS DATA QUEUE

- With data rejection strategy only the most recent available information is transmitted through the network.
- Data rejection strategy is preferable with respect to network load reduction.
- Data rejection strategy is preferable in real-time control applications.
- Data rejection strategy requires particular protocols.







DELAYS, PACKET DROPOUTS AND FAULTS

- The problem of delays and packet dropouts can be regarded in a general perspective of the FTC design.
- Delays and packet dropouts are considered as a particular mode of performance degradation.

FTC REALIZATION OVER THE NETWORK MAY INDUCE LARGER DELAYS

- FTC requires some degree of redundancy in the installed components (sensors, actuators, controllers).
- Larger number of nodes implies increased usage of communication resources (possible network congestion).

HOW DO WE DEAL WITH TIME-VARYING DELAYS AND PACKET DROPOUTS?

- Passive approach: a robust control strategy.
- Active approach: an adaptive control strategy with delay detection and compensation mechanism which utilizes a model-based predictor.

ATTENTION!

In the presence of exogenous and unmeasured disturbance, prediction-based approach may not respond properly to the control requirements.

OBJECTIVES

FAULT AND DELAY TOLERANT NCS

- To develop an active control method for compensation of network induced sensor-to-controller delays in the presence of a bounded exogenous disturbance.
- To establish a multisensor control scheme with joint abrupt sensor faults and delay tolerance capabilities.

CONSTRAINED CONTROL PROBLEM AND POSITIVE INVARIANCE

- To characterize positively invariant sets for the discrete-time linear systems with delays.
- To contribute to the establishment of necessary and sufficient conditions for the existence of such sets.

Outline

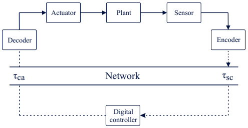
MODELING

PLANT, CONTROL AND NETWORK INDUCED DELAYS

- Linear, time-invariant differential equation with additive disturbance: $\dot{x}(t) = A_c x(t) + B_c u(t) + E_c \omega(t), \ \omega \in \mathcal{W}$.
- Digital control law: $u(t) = u[t_k], \ \forall t \in \mathbb{R}_{[t_k, t_{k+1})}$.
- Piecewise-constant delay: $\tau = \tau_{sc} + \tau_{ca}, \ \tau(t) = \tau[t_k], \ \forall t \in \mathbb{R}_{[t_k, t_{k+1})}, \ \tau[t_k] \in \mathbb{R}_{[0, \tau_{max}]}$.

LINEAR TIME-VARIANT DELAY DIFFERENCE EQUATION

- Time-driven sensor (the output is measured periodically with each pulse of the clock).
- Event-driven controller and actuator (signals are implemented as soon as they are received).

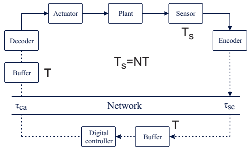


$$x[t_{k+1}] = Ax[t_k] + \sum_{j=0}^{d_m} B_{d_m - j}(\tau_k) u[t_{k+j-d_m}] + E\omega[t_k],$$

where $\tau_k = \begin{bmatrix} \tau[t_k] & \tau[t_{k-1}] & \dots & \tau[t_{k-d_m}] \end{bmatrix}^T, \quad d_m = \left\lceil \frac{\tau_{max}}{T_s} \right\rceil$

TRANSFORMING TIME-VARIANT TO SWITCHING SYSTEM

- Receiving buffers at the controller's and at the actuator's site, read periodically at a higher frequency then the sensor's sampling frequency, i.e., $T_s = NT$.
- Larger N causes faster controller's and actuators reaction. Smaller N provides simpler NCS model with less different configurations.



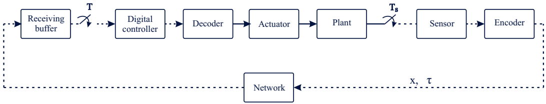
$$x[t_{k+1}] = Ax[t_k] + \sum_{j=0}^{d_m} B_{d_m-j}(\bar{\tau}_k)u[t_{k+j-d_m}] + E\omega[t_k],$$

where $\bar{\tau} = \left(\left\lceil \frac{\tau_{sc} N}{T_s} \right\rceil + \left\lceil \frac{\tau_{ca} N}{T_s} \right\rceil \right) T, \ \bar{\tau}_k = \begin{bmatrix} \bar{\tau}[t_k] & \bar{\tau}[t_{k-1}] & \dots & \bar{\tau}[t_{k-d_m}] \end{bmatrix}^T.$

- A finite number of different configurations - switching dynamics ($\bar{\tau}$ is on a grid of $\mathbb{R}_{[0,\tau_{max}]}$ determined by the inter-sampling period T).
- Simplifies stability analysis.

CONTROLLER COLLOCATED WITH THE ACTUATOR. SENSOR-TO-CONTROLLER DELAYS

- Control input may be updated only once between two consecutive samplings.



RECEIVING BUFFERS AND DELAY REALIZATION

- Delay $\tau \in \mathbb{R}_{(0,\tau max]}$ becomes $\bar{\tau} = (d-1)T_s + \bar{\tau}_{is}T,\ d = \left\lceil \frac{\tau}{T_s} \right\rceil$.
- Inter-sampling delay: $\bar{\tau}_{is} = \left\lceil \frac{\left(\tau - (d-1)T_s\right)N}{T_s} \right\rceil$.
- Number of consecutive dropouts: $d-1$.

MODEL

$$x[t_{k+1}] = Ax[t_k] + B_0(\bar{\tau}_{is})u[t_k] + B_d(\bar{\tau}_{is})u[t_{k-d[t_k]}] + E\omega[t_k],\ \bar{\tau}_{is} \in \mathbb{Z}_{[0,N-1]}$$

$$A = e^{A_c T_s},\quad B_0(\bar{\tau}_{is}) = \int_0^{T_s-\bar{\tau}_{is}T} e^{A_c\zeta}d\zeta B_c,\quad B_d(\bar{\tau}_{is}) = \int_{T_s-\bar{\tau}_{is}T}^{T_s} e^{A_c\zeta}d\zeta B_c \quad \text{and} \quad E = \int_0^{T_s} e^{A_c\zeta}d\zeta E_c.$$

STABILITY ANALYSIS OF NCS

UNPERTURBED MODEL, TIME-VARYING DELAYS

- Linear time-variant delay difference equation: $x[t_{k+1}] = Ax[t_k] + \sum_{j=0}^{d_m} B_{d_m-j}(\tau_k)u[t_{k+j-d_m}]$.
- Equivalent augmented state-space representation:

$$\xi[t_{k+1}] = \mathcal{A}(\tau_k)\xi[t_k] + \mathcal{B}(\tau_k)u[t_k], \quad \xi[t_k] = \begin{bmatrix} x^T[t_k] & u^T[t_{k-1}] & \dots & u^T[t_{k-d_m}] \end{bmatrix}^T$$

$$\mathcal{A}(\tau_k) = \begin{bmatrix} A & B_1(\tau_k) & \dots & B_{d_m-1}(\tau_k) & B_{d_m}(\tau_k) \\ 0 & 0 & \dots & 0 & 0 \\ 0 & I & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & I & 0 \end{bmatrix}, \quad \mathcal{B}(\tau_k) = \begin{bmatrix} B_0(\tau_k) \\ I \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

- For the linear state-feedback control $u = -Kx$:

$$\xi_x[t_{k+1}] = \Lambda(\tau_k)\xi_x[t_k], \quad \xi_x[t_k] = \begin{bmatrix} x^T[t_k] & x^T[t_{k-1}] & \dots & x^T[t_{k-d_m}] \end{bmatrix}^T.$$

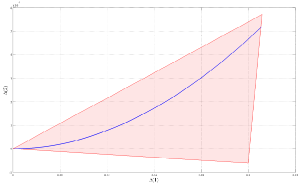
COMMON LYAPUNOV FUNCTION

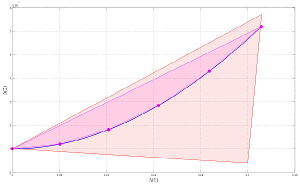
- To guarantee the stability it is sufficient to prove that the system admits a common quadratic Lyapunov function, $V(\xi_x[t_k]) = \xi_x^T[t_k]P\xi_x[t_k]$ such that

$$\Lambda(\tau_k)^T P \Lambda(\tau_k) - P \prec 0, \quad \forall \tau_k \in \mathbb{R}^{d_m+1}_{[0,\tau_{max}]}.$$

How to practically check the stability of a time-varying model?

- Over-approximation (convex embedding) of the varying parameter $\Delta(\tau) \rightarrow$ switching dynamics.
- Finite grid of the delay range $\rightarrow$ switching dynamics.





$$\dot{x}(t) = A_c x(t) + B_c u(t-\tau) \quad \rightarrow \quad \begin{bmatrix} x[t_{k+1}] \\ u[t_k] \end{bmatrix} = \begin{bmatrix} e^{A_c T_s} & B - \Delta(\tau) \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x[t_k] \\ u[t_{k-1}] \end{bmatrix} + \begin{bmatrix} \Delta(\tau) \\ 1 \end{bmatrix} u[t_k],$$

$$\text{where} \quad \Delta(\tau) = \int_0^{T_s - \tau[t_k]} e^{A_c \zeta} d\zeta B_c, \quad B = \int_0^{T_s} e^{A_c \zeta} d\zeta B_c.$$

Stability of switching dynamics

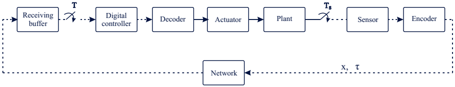
- Sufficient condition: common Lyapunov function obtained by solving a finite number of LMIs.
- Necessary and sufficient condition: joint spectral radius.

OUTLINE

PROBLEM FORMULATION

NOMINAL NCS MODEL

- Nominal (no induced delays) plant dynamics: $x[t_{k+1}] = Ax[t_k] + Bu[t_k] + E\omega[t_k]$.
- Delays: $\bar{\tau}[t_k] = (d[t_k] - 1)T_s + \bar{\tau}_{is}[t_k]T$, $\quad \bar{\tau}_{is} = \left\lceil \frac{\left(\tau - (d-1)T_s\right)N}{T_s} \right\rceil$, $\quad d = \left\lceil \frac{\tau}{T_s} \right\rceil$.
- Reference trajectory: $x_{ref}[t_{k+1}] = Ax_{ref}[t_k] + Bu_{ref}[t_k]$.
- Digital controller: $u[t_k] = u_{ref}[t_k] - K(x[t_k] - x_{ref}[t_k])$. (estimated state-feedback works here as well)



KEY TERMS

- Inter-sampling strategy.
- Model-based predictor.
- Prediction error compensation.

In comparison to sampled-data systems, we'll have one "degree of freedom" more: control action can be updated between two consecutive samplings.

NETWORK INDUCED DELAYS

NOMINAL DYNAMICS WITH RESPECT TO INTER-SAMPLING PERIOD T.

$$x[t_{k+1}] = \tilde{A}^N x[t_k] + \sum_{i=0}^{N-1} \tilde{A}^i \tilde{B} u[t_k] + \sum_{i=0}^{N-1} \tilde{A}^i \tilde{E} \omega[t_k],$$

$$A = \tilde{A}^N, \quad B = \sum_{i=0}^{N-1} \tilde{A}^i \tilde{B}, \quad E = \sum_{i=0}^{N-1} \tilde{A}^i \tilde{E}, \quad T_s = NT.$$

MODEL WHEN DELAYS APPEAR

- Assumption (just for the sake of presentation): delays are smaller than the sampling period T_s.
- $\bar{\tau}[t_k] = \bar{\tau}_{is}[t_k]T, \quad \bar{\tau}_{is} \in \mathbb{Z}_{[0,N]}, \quad \bar{\tau}_{is} = \left\lceil \frac{\tau N}{T_s} \right\rceil.$

$$x[t_{k+1}] = Ax[t_k] + \sum_{i=N-\bar{\tau}_{is}[t_k]}^{N-1} \tilde{A}^i \tilde{B} u[t_{k-1}] + \sum_{i=0}^{N-\bar{\tau}_{is}[t_k]-1} \tilde{A}^i \tilde{B} u[t_k] + E\omega[t_k].$$

ATTENTION

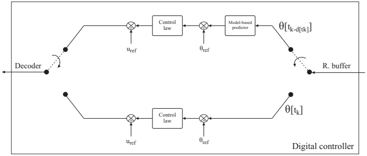
- Allowing $u[t_{k-1}]$ is safe only if the closed-loop system is robust with respect to that delay.
- Delay variation decreases even more a delay margin (maximal allowable delay).

MODEL-BASED PREDICTION

- What is the most relevant control signal we can use until the up-to-date measurements are delivered to the controller?
- Prediction based on the latest received data and mathematical model.
- Accurate model is needed.

ONE STEP PREDICTION

- One step prediction: $\theta_{t_k|t_{k-1}} = Ax[t_{k-1}] + Bu[t_{k-1}]$.
- Instead of using $u[t_{k-1}]$ we employ $u_{t_k|t_{k-1}} = u_{ref}[t_k] - K(\theta_{t_k|t_{k-1}} - x_{ref}[t_k])$.



WHAT ABOUT THE MODEL INACCURACY?

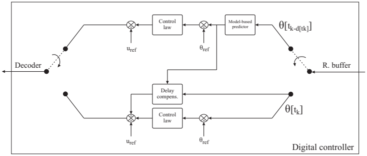
- Prediction does not work well for inaccurate models neither disturbance.
- Prediction error is bounded: $\varepsilon_{t_k|t_{k-1}} = x[t_k] - \theta_{t_k|t_{k-1}} = E\omega[t_{k-1}] \in \mathcal{E}$.

PREDICTION ERROR COMPENSATION

SIMPLE IDEA BEHIND

- If it is not "to late" when up-to-date measurements arrive, we can compensate prediction error with an additional control term.

$$x[t_{k+1}] = Ax[t_k] + \sum_{i=N-\bar{\tau}_{is}[t_k]}^{N-1} \tilde{A}^i \tilde{B} u_{t_k|t_{k-1}} + E\omega[t_k] + \sum_{i=0}^{N-\bar{\tau}_{is}[t_k]-1} \tilde{A}^i \tilde{B}\Big(u[t_k] + \sigma[t_k + (N-1-i)T]\Big).$$



- Using $\theta_{t_k|t_{k-1}} = x[t_k] - \varepsilon_{t_k|t_{k-1}}$:

$$x[t_{k+1}] = Ax[t_k] + Bu[t_k] + E\omega[t_k] + \sum_{i=N-\bar{\tau}_{is}[t_k]}^{N-1} \tilde{A}^i \tilde{B} K \varepsilon_{t_k|t_{k-d[t_k]}} + \sum_{i=0}^{N-\bar{\tau}_{is}[t_k]-1} \tilde{A}^i \tilde{B}\sigma[t_k + (N-1-i)T].$$

- Prediction error $\varepsilon_{t_k|t_{k-1}}$ is "known" to the controller when up-to-date measurements are delivered through the network.

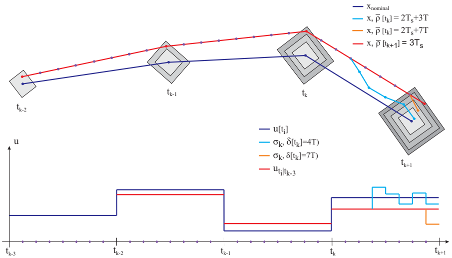
COMPUTING THE COMPENSATION TERM

- Compensation (fro delays smaller than the sampling period) is the solution of the linear equation:

$$\sum_{i=0}^{N-\bar{\tau}_{is}[t_k]-1} \tilde{A}^i \tilde{B} \sigma[t_k + (N-1-i)T] = -\left(\sum_{i=N-\bar{\tau}_{is}[t_k]}^{N-1} \tilde{A}^i \tilde{B} K \varepsilon_{t_k \mid t_{k-d[t_k]}} \right).$$

- Compensation vector exists if and only if $\bar{\tau}_{is}[t_k] \leq N - \mu$, where $\mu \in \mathbb{Z}^+$ is the controllability index of the pair $(\tilde{A}, \tilde{B})$.
- It is assumed that $\bar{\tau}_{is}$ is known (time-sampling or set-based delay detection).

- Larger delays can be treated in a similar way. More “control power” is needed though.



SET-BASED DELAY DETECTION

FAULT DETECTION AND IDENTIFICATION APPROACH

- Data stored in the buffer is denoted by β.
- *Healthy residual* signal is constructed when the stored data is up-to-date.

$$r^H[t_k + iT] = \beta[t_k + iT] - x_{ref}[t_k] = x[t_k] - x_{ref}[t_k] = z[t_k], \quad i \in \mathbb{Z}_{[0,N-1]}.$$

- *Faulty residual* signal is constructed when the stored data is outdated.

$$r^D[t_k + iT] = \beta[t_k + iT] - x_{ref}[t_k] = x[t_{k-1}] - x_{ref}[t_k] = z[t_{k-1}] + x_{ref}[t_{k-1}] - x_{ref}[t_k].$$

- Let $z[t_{k-1}], z[t_k] \in \mathcal{Z}$, where $\mathcal{Z}$ is a tracking error invariant set. Then:

$$\mathcal{R}^H = \mathcal{Z}, \quad \mathcal{R}^D = \mathcal{Z} \oplus \{x_{ref[t_{k-1}]-x_{ref}[t_k]}\}.$$

- Unique delay detection is guaranteed if $\mathcal{R}^H \cap \mathcal{R}^D = \emptyset$.
- Using reference signal in order to separate the residual sets. Less conservative results are obtained for smaller invariant sets.
- Set of admissible reference signals (a non-convex set):

$$\mathcal{D}_{x_{ref}} = \{x_{ref}[t_{k-1}], x_{ref}[t_k] \ : \ x_{ref}[t_{k-1}] - x_{ref}[t_k] \in \mathcal{Z} \oplus \{-\mathcal{Z}\}\}.$$

REFERENCE GOVERNOR DESIGN

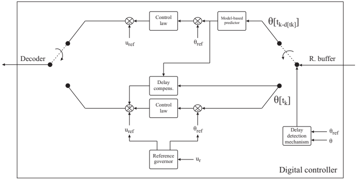
- Reference governor - add-on which adapts the reference signal so the imposed constraints are satisfied.
- Reference governor is implemented in the model predictive control framework through the receding horizon techniques.
- Minimization of the following cost function with respect to hard constraints.

$$u^* = \arg\min_{u_{ref}} \left(\sum_{q=1}^{s} \|(x_r[t_{k+q}] - x_{ref}[t_{k+q}])\|_Q^2 + \sum_{q=0}^{s-1} \|(u_r[t_{k+q}] - u_{ref}[t_{k+q}])\|_P^2\right)$$

subject to:

$$x_{ref}[t_{k+1}] = Ax_{ref}[t_k] + Bu_{ref}[t_k]$$
$$x_{ref}[t_k] \in \mathcal{D}_{x_{ref}}, \quad \forall k \in \mathbb{Z}_+,$$

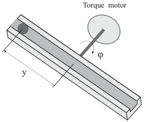
where $x_r[t_{k+1}] = Ax_r[t_k] + Bu_r[t_k]$ is the initial reference dynamics.



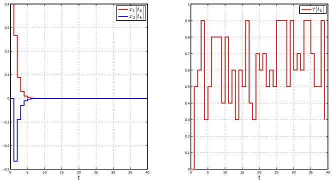
- Non-convex optimization problem. Mixed-integer techniques are required.

EXAMPLE

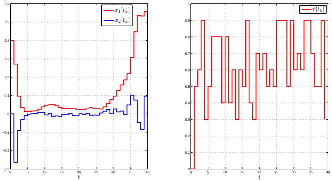
$$\frac{dx(t)}{dt} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 24.525 \end{bmatrix} \varphi(t - \tau(t)),$$
$$\varphi[t_k] = -[0.0271 \quad 0.0543] x[t_k].$$



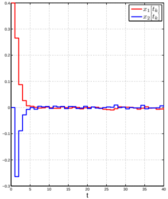
- Model-based predictor and $\|\omega\|_\infty = 0$. Response (left) and delay variation (right).

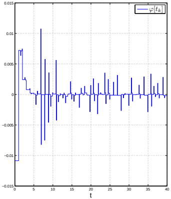


- Model-based predictor and $\|\omega\|_\infty \leq 0.004$. Response (left) and delay variation (right).

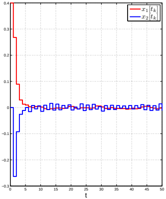


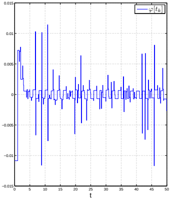
- Model-based predictor and $\|\omega\|_\infty \leq 0.004$. Prediction error compensation. Reference governor disengaged. Response (left) and control signal (right). Same delay variation.





- Model-based predictor and $\|\omega\|_\infty \leq 0.004$. Prediction error compensation. Reference governor engaged. Response (left) and control signal (right). Same delay variation.

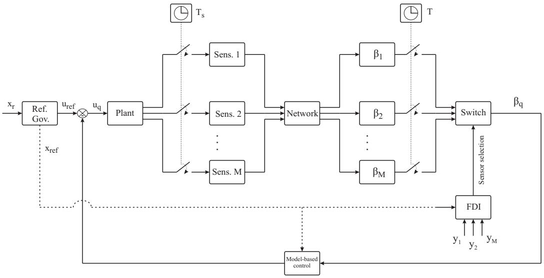




Outline

Nominal multisensor NCS description

- Continuous-time linear plant model: $\dot{x}(t) = A_c x(t) + B_c u(t) + E_c \omega(t), \quad \omega \in \mathcal{W}$.
- Redundant sensors (state feedback transmitted through the network): $y_j[t_k] = x[t_k] + \eta_j[t_k], \quad j \in \mathcal{I}, \quad \mathcal{I} = \mathbb{Z}_{[1,M]}$.
- Control objective: $x_{ref}[t_{k+1}] = Ax_{ref}[t_k] + Bu_{ref}[t_k]$.
- Digital control based on measurements provided by the j^{th} sensor: $u_j[t_k] = u_{ref}[t_k] - K\left(\beta_j[t_k] - x_{ref}[t_k]\right)$.



Nominal closed-loop dynamics

$$x[t_{k+1}] = Ax[t_k] + Bu[t_k] - B\eta_j[t_k] + \sum_{i=0}^{N-1} \tilde{A}^i \tilde{E}\omega[t_k + (N-1-i)T].$$

REGARDING THE STORED DATA, SENSORS ARE CLASSIFIED AS:

- Delayed: $\mathcal{I}^D[t_k + iT] = \left\{ j \in \mathcal{I} \ : \ \beta_j[t_k + iT] = x[t_{k-1}] + \eta_j[t_{k-1}] \right\}$;
- Faulty: $\mathcal{I}^F[t_k + iT] = \left\{ j \in \mathcal{I} \ : \ \beta_j[t_k + iT] = \eta_j^F[t_k] \ \vee \ \beta_j[t_k + iT] = \eta_j^F[t_{k-1}] \right\}$;
- Healthy: $\mathcal{I}^H[t_k + iT] = \left\{ j \in \mathcal{I} \ : \ \beta_j[t_k + iT] = x[t_k] + \eta_j[t_k] \ \wedge \ j \notin \mathcal{I}^F[t_k + sT], \ \forall s \in \mathbb{Z}_{[0,i)} \right\}$;
- Under recovery: $\mathcal{I}^R[t_k + iT] = \left\{ j \in \mathcal{I} \ : \ \beta_j[t_k + iT] = x[t_k] + \eta_j[t_k] \ \wedge \ j \in \mathcal{I}^F[t_k + sT], \ s \in \mathbb{Z}_{[0,i)} \right\}$.

FAULT SCENARIO

- Sensors are prone to abrupt faults:

$$y_j[t_k] = x[t_k] + \eta_j[t_k] \ \underset{RECOVERY}{\overset{FAULT}{\rightleftharpoons}} \ y_j[t_k] = 0 \cdot x[t_k] + \eta_j^F[t_k].$$

- Data transmission from sensors to the controller is subject to random and time-varying delays that are less than a sampling period.
- Sets $\mathcal{I}^D[t_k + iT] \neq \emptyset \ \ \forall i \in \mathbb{Z}_{[0,N-\mu)}$ and $\mathcal{I}^H[t_k + (N - \mu)T] \neq \emptyset$, where $\mu \in \mathbb{Z}^+$ is the controllability index of the pair $(\tilde{A}, \tilde{B})$.

- According to the fault scenario, outdated measurements can be used if the most recent ones are not available. Data provided by a faulty sensor needs to be avoided.

FAULT DETECTION IN THE PRESENCE OF DELAYS

RESIDUAL SIGNALS

- Healthy: when data stored in a buffer are *up-to-date* and provided by the functional sensor $\left(j \in \mathcal{I}^H[t_k + iT]\right)$:

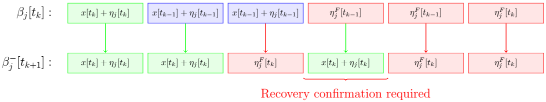
$$r_j^H[t_k + iT] = \beta_j[t_k + iT] - x_{ref}[t_k] = z[t_k] + \eta_j[t_k].$$

- Faulty: when data stored in a buffer are provided by the sensor which is affected by an abrupt fault $\left(j \in \mathcal{I}^F[t_k + iT]\right)$:

$$r_j^F[t_k + iT] = \beta_j[t_k + iT] - x_{ref}[t_k] = \eta_j^F[t_k] - x_{ref}[t_k].$$

- Delayed: when data stored in a buffer are *outdated* and provided by the functional sensor $\left(j \in \mathcal{I}^D[t_k + iT]\right)$:

$$r_j^D[t_k + iT] = \beta_j[t_k + iT] - x_{ref}[t_k] = x[t_{k-1}] + \eta_j[t_{k-1}] - x_{ref}[t_k] = z[t_{k-1}] + \eta_j[t_{k-1}] + x_{ref}[t_{k-1}] - x_{ref}[t_k].$$



RESIDUAL SETS

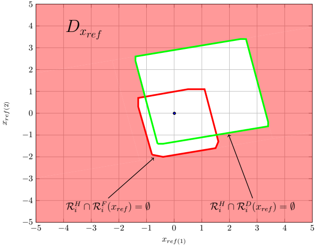
- Healthy: $\mathcal{R}_j^H = \mathcal{Z} \oplus \mathcal{N}_j$.
- Faulty: $\mathcal{R}_j^F(x_{ref}) = \{-x_{ref}[t_k]\} \oplus \mathcal{N}_j^F$.
- Delayed: $\mathcal{R}_j^D(x_{ref}) = \{x_{ref}[t_{k-1}] - x_{ref}[t_k]\} \oplus \mathcal{Z} \oplus \mathcal{N}_j$.

UNEQUIVOCAL FAULT/DELAY DETECTION

$$\mathcal{R}_j^H \cap \mathcal{R}_j^F(x_{ref}) = \emptyset, \quad \mathcal{R}_j^H \cap \mathcal{R}_j^D(x_{ref}) = \emptyset$$

REFERENCE ADMISSIBLE SET

$$\mathcal{D}_{x_{ref}} = \{x_{ref}[t_k], x_{ref}[t_{k-1}] \ : \ \mathcal{R}_j^H \cap \mathcal{R}_j^F(x_{ref}) = \emptyset, \ \mathcal{R}_j^H \cap \mathcal{R}_j^D(x_{ref}) = \emptyset, \ \forall j \in \mathcal{I}\}.$$

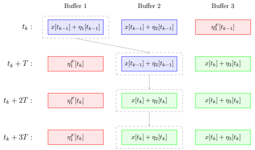


- Estimated state feedback case can be tackled in the same way. However, verification of a sensor recovery is more complex.
- For defined reference admissible set, reference governor is designed as it was already presented.

FTC STRATEGY FOR MULTISENSOR NCS

Functioning of the scheme is shown through an example:

- Three redundant sensors.
- $N = 4$ inter-sampling instants.



- $\mathcal{I}^H[t_k] = \emptyset, \quad \mathcal{I}^D[t_k] = \{1, 2\}$ and $\mathcal{I}^F[t_k] = \{3\}$:

$$x[t_k + T] = \tilde{A}x[t_k] + \tilde{B}\Big(u_{ref}[t_k] - K\left(\theta_1[t_k|t_{k-1}] - x_{ref}[t_k]\right)\Big) + \tilde{E}\omega[t_k].$$

- $\mathcal{I}^H[t_k + T] = \emptyset, \quad \mathcal{I}^D[t_k + T] = \{2\}, \quad \mathcal{I}^F[t_k + T] = \{1\}$ and $\mathcal{I}^R[t_k + T] = \{3\}$:

$$x[t_k + 2T] = \tilde{A}^2 x[t_k] + \sum_{i=0}^{1} \tilde{A}^i \tilde{B}\Big(u_{ref}[t_k] - K\left(\theta_2[t_k|t_{k-1}] - x_{ref}[t_k]\right)\Big) + \sum_{i=0}^{1} \tilde{A}^i \tilde{E}\omega[t_k + (1 - i)T] + \tilde{A}BK\gamma_{21}[t_k],$$

where $\gamma_{21}[t_k] = \theta_2[t_k|t_{k-1}] - \theta_1[t_k|t_{k-1}]$ and $\theta_2[t_k|t_{k-1}] = A\beta_2[t_k] + B\Big(u_{ref}[t_{k-1}] - K\left(\beta_2[t_k] - x_{ref}[t_{k-1}]\right)\Big)$.

- $\mathcal{I}^H[t_k + 2T] = \{2\}$, $\mathcal{I}^D[t_k + 2T] = \emptyset$, $\mathcal{I}^F[t_k + 2T] = \{1\}$ and $\mathcal{I}^R[t_k + 2T] = \{3\}$:

$$x[t_{k+1}] = \underbrace{Ax[t_k] + Bu[t_k] - B\eta_2[t_k] + \sum_{i=0}^{3} \tilde{A}^i \tilde{E}\omega[t_k + (3-i)T]}_{\text{Nominal dynamics}}$$

$$+ \sum_{i=2}^{3} \tilde{A}^i \tilde{B}K\varepsilon_2[t_k] + \tilde{A}^3 BK\gamma_{21}[t_k] + \sum_{i=0}^{1} \tilde{A}^i \tilde{B}\sigma[t_k + (3-i)T],$$

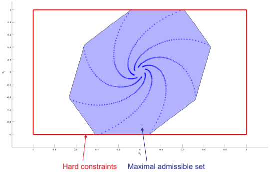
  where $\varepsilon_2[t_k] = \beta_2[t_k + 2T] - \theta_2[t_k | t_{k-1}]$.
- Since $\gamma_{21}[t_k]$ and $\varepsilon_2[t_k]$ are available, one can determine $\sigma_k = \begin{bmatrix} \sigma[t_k + 2T] & \sigma[t_k + 3T] \end{bmatrix}$.

OUTLINE

DEALING WITH UNCERTAINTIES AND HARD CONSTRAINTS ON SYSTEM PARAMETERS

POSITIVELY INVARIANT SET

- Hard physical constraints can be satisfied by ensuring the existence of a positively invariant set inside the admissible region.
- A positively invariant set with respect to all disturbance signals from a bounded region is referred to as robust.



APPLICATION

- Model-predictive control, fault tolerant control, reference governor design, disturbance attenuation...

LINEAR SYSTEMS WITH DELAYS

- Several concepts of positive invariance (Laypunov-Krasovskii, Lyapunov-Razumikhin, set factorization...).

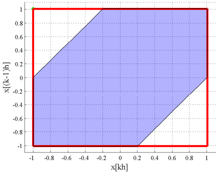
POSITIVELY INVARIANT SETS AND TIME-DELAYS

APPROACH BASED ON LYAPUNOV-KRASOVSKII FUNCTIONS

- Analysis carried out by augmenting the state-space.
- Stability depends on delay parameters $d_i,\, i \in \mathbb{Z}_{[1,m]}$.

$$x[kh] = \sum_{i=1}^{m} A_i x[(k-d_i)h] \quad \rightarrow \quad \begin{bmatrix} x[kh] \\ \vdots \\ x[(k-d_1+1)h] \\ \vdots \\ x[(k-d_m+1)h] \end{bmatrix} = \begin{bmatrix} 0 & \dots & A_1 & \dots & 0 & A_m \\ I & \dots & 0 & \dots & 0 & 0 \\ \vdots & & \vdots & & \vdots & \vdots \\ 0 & \dots & 0 & \dots & I & 0 \end{bmatrix} \begin{bmatrix} x[(k-1)h] \\ \vdots \\ x[(k-d_1)h] \\ \vdots \\ x[(k-d_m)h] \end{bmatrix}.$$

- Example : For the given system $x[kh] = x[(k-1)h] - 0.8x[(k-2)h]$ define the maximal set of initial conditions such that $-1 \leq x[kh] \leq 1, \ \forall k \in \mathbb{Z}_{[0,\infty)}$.

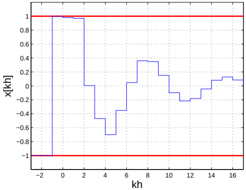


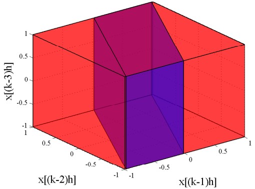
- Advantage :
  - Necessary and sufficient conditions for the existence.
- Disadvantages :
  - Higher dimension - more complex computation.
  - Initial conditions are interdependent - may not correspond to a practical requirement.

POSITIVELY INVARIANT SETS AND TIME-DELAYS

APPROACH BASED ON LYAPUNOV-RAZUMIKHIN FUNCTIONS

- Lyapunov-Razumikhin function : $V(x[0]) \leq \rho \max_{\theta_i \in [-d_m, -1]} \{V(x[\theta_i h])\}, \forall x[\theta_i h] \in \mathbb{X}, 0 \leq \rho \leq 1$ such that $\alpha_1(\|x\|) \leq V(x) \leq \alpha_2(\|x\|)$, where α_1, α_2 are continuous, non-decreasing and radially unbounded scalar functions with $\alpha 1(0) = \alpha_2(0) = 0$.
- Relaxes constraints imposed on initial conditions by Lyapunov-Krasovskii approach.





- Advantage :
  - Positively invariant sets in the original state-space.
- Disadvantages :
  - Does not have a lot to do with delay values.
  - Necessary and sufficient condition for the existence is the open problem.

ASYMPTOTIC & ROBUST ASYMPTOTIC STABILITY

DISCRETE-TIME DELAY-DIFFERENCE EQUATION WITH $h = 1$

- The zero solution is asymptotically stable if and only if $\det\left(I - \sum_{i=1}^{m} A_i z^{d_i}\right) \neq 0, \ \forall z \in \mathbb{D} \cup \partial\mathbb{D}$, where $\mathbb{D}$ and $\partial\mathbb{D}$ are the open unit disc and the unit circle, respectively.

$$x[k] = \sum_{i=1}^{m} A_i x[k - d_i],$$

- Delays, as the components of a vector, define a **ray** in delay-parameter space:

$$\mathcal{T}(\vec{d}) := \{\alpha\vec{d} : \alpha \in \mathbb{R}_{(0,\infty)}\}.$$

- Asymptotic stability of the zero solution is the global property on one ray - **stable ray**.
- The zero solution is not necessarily stable if a stable ray is subject to a perturbation of its direction.
- The discrete-time delay-difference equation is robustly asymptotically stable with respect to delay uncertainty, if and only if its zero solution is asymptotically stable for all $\vec{d} \in \mathbb{Z}^{m}_{(0,\infty)}$.

CONTINUOUS-TIME DELAY-DIFFERENCE EQUATIONS

- General class of delay-difference equations largely treated in the theory of neutral functional differential equations.
- The zero solution is asymptotically stable if and only if $\det\left(I - \sum_{i=1}^{m} A_i e^{-sr_i}\right) \neq 0, \forall s \in \mathbb{C}_+ \cup \partial\mathbb{C}$, where $\mathbb{C}_+$ and $\partial\mathbb{C}$ are open right-hand complex plane and the imaginary axis.

$$x(t) = \sum_{i=1}^{m} A_i x(t - r_i), \quad t \in \mathbb{R}_{[0,\infty)}, \ \vec{r} = [r_1 \dots r_m]^T \in \mathbb{R}^m_{[0,\infty)}.$$

- If the zero solution is asymptotically stable for all $\vec{r} \in \mathbb{R}^m_{(0,\infty)}$, then it is referred to as delay-independent stability.
- Delay-independent stability is a fragile property with respect to a perturbation of the stable ray direction.
- **Delay-interference phenomenon** - stability is disrupted by an arbitrary small perturbation of direction of a stable ray.
- Delay-independent stability hold if and only if:

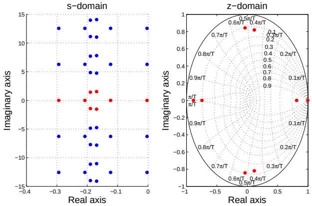
$$\sup\left\{\rho\left(\sum_{i=1}^{m} A_i e^{j\theta_i}\right) \ : \ \vec{\theta} \in [0, 2\pi]^m\right\} < 1.$$

CONTINUOUS-TIME DELAY-DIFFERENCE EQUATIONS

DELAY-INDEPENDENT STABILITY WHEN $m = 2$

- If $\exists \vec{r} \in (\mathbb{R}^+)^2$ such that delay-difference equation is asymptotically stable then it is delay-independent stable if and only if:

$$\sigma(U, V) \cap \partial\mathbb{D} = \emptyset \quad \text{where}$$
$$U = \begin{bmatrix} 0 & I \\ -B_0 & -B_1 \end{bmatrix}, \quad V = \begin{bmatrix} I & 0 \\ 0 & B_2 \end{bmatrix}$$
$$B_0 = A_1 \otimes A_2^T, \; B_1 = A_1 \otimes A_1^T + A_2 \otimes A_2^T - I,$$
$$B_2 = A_2 \otimes A_1^T.$$



- Special case of interest - commensurate delays i.e. $\exists h \in \mathbb{R}_{(0,\infty)}$ and $\vec{d} \in \mathbb{Z}^m_{(0,\infty)}$ such that $\vec{r} = h\vec{d}$.

$$x(t) = \sum_{i=1}^{m} A_i x(t - d_i h).$$

- Poles of discrete-time (z) and continuous-time (s) delay-difference equation are related by $z = e^{sh}$.

ROBUST ASYMPTOTIC STABILITY & SET INVARIANCE

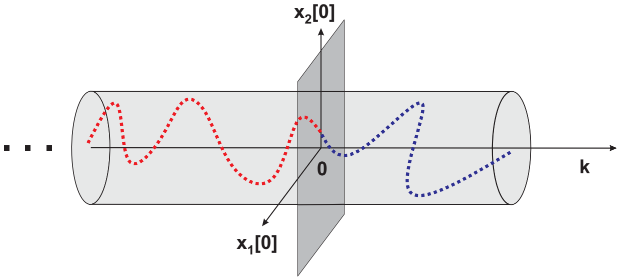
- Since it holds for all real delays from delay-parameter space, delay-independent stability is the sufficient condition for the robust asymptotic stability.
- Delay-independent stability is arbitrary close to be the necessary condition as well.

$$\sup\left\{\rho\left(\sum_{i=1}^{m} A_i e^{j\theta_i}\right) \;:\; \vec{\theta} \in [0, 2\pi]^m\right\} < 1 \Rightarrow \det\left(I - \sum_{i=1}^{m} A_i z^{d_i}\right) \neq 0, \;\; \forall z \in \mathbb{D} \cup \partial\mathbb{D}, \;\; \forall \vec{d} \in \mathbb{Z}^m_{(0,\infty)}.$$

POSITIVELY INVARIANT SET ⇔ LYAPUNOV-RAZUMIKHIN BASED APPROACH

- A set $\mathbb{S}$ is positively invariant if for all $x[-d_i] \in \mathbb{S}$, $1 \leq i \leq m$, $x[k] \in \epsilon\mathbb{S}$, $0 \leq \epsilon \leq 1$, for all k.
- If $0 \leq \epsilon < 1$, then $\mathbb{S}$ is called ϵ-contractive.
- The existence of a positively invariant set depends only on the **linear mapping** of delay-difference equations, not on discrete or continuous-time domain.

COMPUTATIONAL CONDITIONS



- If a delay-difference equation admits an ϵ-contractive set then it is robustly asymptotically (delay-independent) stable.
- Necessary condition : Linear discrete-time delay-difference equation $x[kh] = \sum_{i=1}^{m} A_i x[(k-d_i)h]$ admits an ϵ-contractive set only if $\sigma(U, V) \cap \partial\mathbb{D} = \emptyset$.
- Open question : Is the proposed algebraic condition also sufficient?

ILLUSTRATIVE EXAMPLE - SCALAR CASE

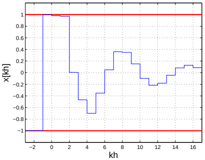
- Robust asymptotic (discrete-time) stability $\Leftrightarrow$ delay-independent (continuous-time) stability:

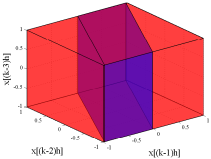
$$\rho\Big(\sum_{i=1}^{m} a_i e^{j\theta_i}\Big) \leq 1 \;\Leftrightarrow\; \sum_{i=1}^{m} |a_i| \leq 1 \;\Leftrightarrow\; \|A\|_\infty = 1,$$

where A is the extended state-space system matrix.

- Robust asymptotic stability $\Leftrightarrow$ the existence of a positively invariant set.
- Positively invariant set - infinity norm hypersphere $\mathbb{B}_\infty^{dm}$ such that $A\mathbb{B}_\infty^{dm} \subseteq \mathbb{B}_\infty^{dm}, \forall \vec{d} \in \mathbb{Z}^m_{(0,\infty)}$.

- Example : $x[kh] = 0.5x[(k-1)h] - 0.48x[(k-3)h]$.





ILLUSTRATIVE EXAMPLE - GENERAL CASE

- Example 1 : $x[k] = \begin{bmatrix} 0.5 & 0.16 \\ -0.21 & 0.45 \end{bmatrix} x[k-d_1] + \begin{bmatrix} 0.51 & -0.01 \\ 0.02 & 0.51 \end{bmatrix} x[k-d_2].$

NECESSARY CONDITIONS PROPOSED IN THE LITERATURE

- $\rho(A) < 1$, where A is extended state-space system matrix. HOLD
- $\rho(A_1) < 1, \rho(A_2), \rho(-A1 + A2) < 1$ and $\rho(A1 - A2) < 1$. HOLD
- $\sigma(U, V) \cap \partial\mathbb{D} = \emptyset$. DOES NOT HOLD

- Example 2 : $x[k] = \begin{bmatrix} 0.35 & 0.13 \\ -0.51 & -0.01 \end{bmatrix} x[k-d_1] + \begin{bmatrix} 0.51 & -0.01 \\ 0.03 & 0.51 \end{bmatrix} x[k-d_2].$

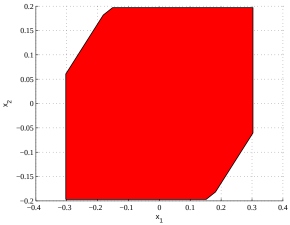
SUFFICIENT CONDITIONS PROPOSED IN THE LITERATURE

- $\|A_1 + A_2\| < 1$. DOES NOT HOLD
- $\sigma(U, V) \cap \partial\mathbb{D} = \emptyset$. HOLD

Polyhedral set factorization. An alternative positive invariance concept - example

Consider the delay difference equation $x[t_{k+1}] = x[t_k] - 0.5x[t_{k-2}]$ which does not allow a $D-$invariant set.

$$\begin{bmatrix} x_{k+1} \\ x_k \\ x_{k-1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -0.5 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x[t_k] \\ x[t_{k-1}] \\ x[t_{k-2}] \end{bmatrix}$$



has a strictly stable transition matrix, i.e., admits invariant sets in $\mathbb{R}^3$ which are not factorisable in $\mathbb{R}$.
There is a *nonminimal* state delay difference equation:

$$\begin{bmatrix} x_{k+1} \\ x_{k+1} - x_k \end{bmatrix} = \begin{bmatrix} 0.5 & 0.5 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_k \\ x_k - x_{k-1} \end{bmatrix} + \begin{bmatrix} 0 & 0.5 \\ -0.5 & 0.5 \end{bmatrix} \begin{bmatrix} x_{k-1} \\ x_{k-1} - x_{k-2} \end{bmatrix}$$

for which a $D-$invariant set $P \subset \mathbb{R}^2$ exists.

Thank you!