UNIVERSITE DE ROUEN

EA 4353

H∞ Predictive Control Design for Networked Control Systems

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ESIGELEC

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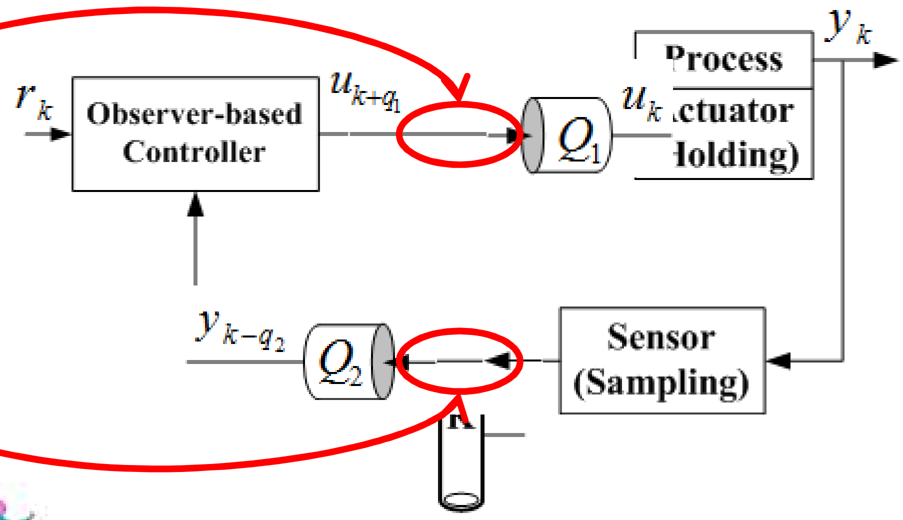
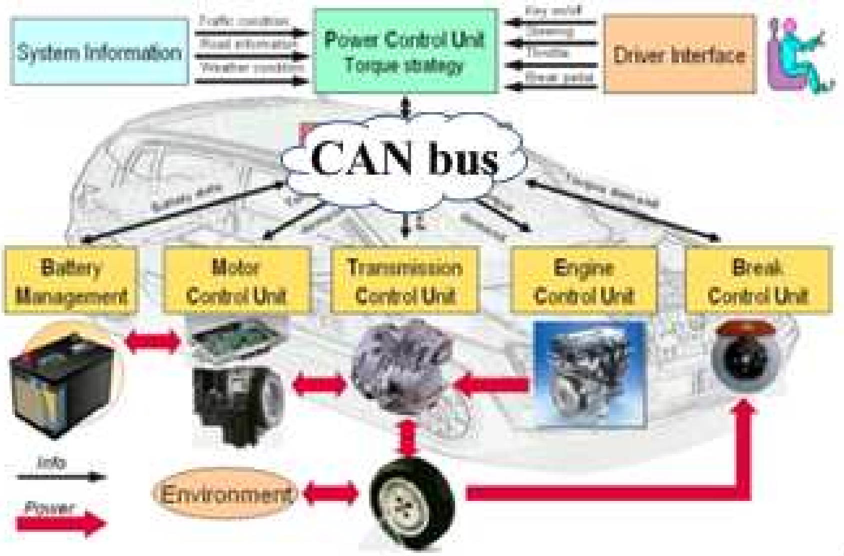
Énoncé du Problème

- Network-induced Delay
- Packet dropout or out-of-order
- Process noise





Combustion Engine for Range Extended Electric Vehicle

- Engine control unit
  （An electromagnetic Valve）
- Battery Management Unit
  （Battery State-of-Charge）

Objective

- 1, Proposed GPC controller
- 2, Extended H∞-GPC algorithm regarding robust issue
- 3, Parameter tuning
- 4, GPC application in an academic model
- 5, Relative study of network-faced state estimation
- 6, Battery state-of-charge estimation problem

Introduction to Generalized Predictive Controller

- CARIMA Model :

$$A(q^{-1})y(t) = q^{-d}B(q^{-1})u(t-1) + \frac{e(t)}{\Delta}$$

- Cost Function :

$$J = \sum_{j=N_1}^{N} \delta(j)[\hat{y}(t+j\,|\,t) - r(t+j)]^2 + \sum_{j=1}^{N_u} \lambda_u(j)[\Delta u(t+j-1)]^2$$

- Control Signal :

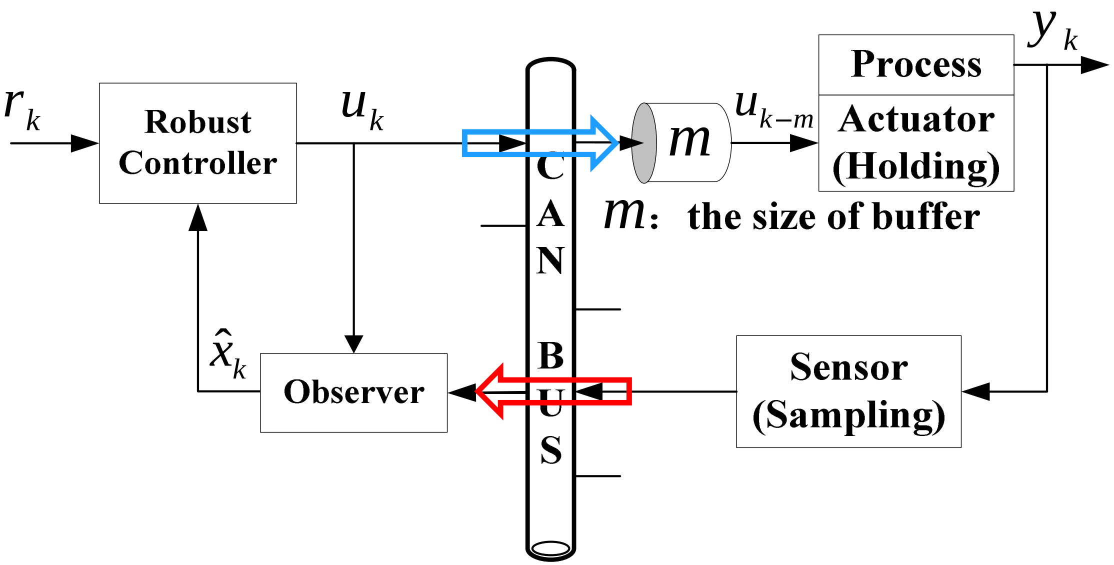
$$u = (G^T G + \lambda I)^{-1} G^T (y_d - f)$$

Main Problems of GPC

- 1, Solution to Network-induced problems
- 2, State-space-model-based system
- 3, Robustness against process noise and model uncertainties
- 4, GPC Parameter computation and tuning
- 5, System stability

Solution to Network-induced problems

The structure of the networked control system :



- Buffering technique for fixing time-delays
- State estimation in NCS
- The maximal value of a consequent packet losses event: S
- The maximal controller-to-actuator delay: T

control channel

feedback channel

State-space-model-based System

Incremental state-space model :

$$\begin{cases} x_{k+1} = \Phi x_k + \Gamma u_{k-m} + E\omega_k \\ z_k = C x_k \\ y_k = z_k + \upsilon_k \end{cases}$$

$$x_k = \Phi x_{k-1} + \Gamma u_{k-m-1} + E\omega_{k-1}$$

$$\Delta\omega_k = \omega_k - \omega_{k-1}$$

$$\Delta u_{k-m} = u_{k-m} - u_{k-m-1}$$

$$\begin{cases} x_{k+1} = (\Phi + I) x_k - \Phi x_{k-1} + \Gamma \Delta u_{k-m} + E\Delta\omega_k \\ z_k = C x_k \\ y_k = z_k + \upsilon_k \end{cases}$$

GPC Design

- The j-step predictive output estimation :

$$\hat{y}_{k+j|k} = \sum_{i=1}^{j-m+1} C\Phi^{i-1} Exp\left[x_{k+m}\right] - \sum_{i=2}^{j-m+1} C\Phi^{i-1} Exp\left[x_{k+m-1}\right] + \sum_{h=2}^{j-m+1} \sum_{i=2}^{h} C\Phi^{h-i}\Gamma\Delta u_{k+j+1-h|k}$$

- Vector form :

$$\hat{Y}_k = G\Delta U_k + \mathrm{T}_k$$

$$\Delta U_k = \left\{\Delta u_{k|k}^T, \Delta u_{k+1|k}^T, \ldots, \Delta u_{k+N_u-1|k}^T\right\}^T$$

$$\mathrm{T}_k = \mathrm{M}_1 \hat{x}_{k+m|k} - \mathrm{M}_2 \hat{x}_{k+m-1|k}$$

$$\hat{Y}_k = \left[\hat{y}_{k+m+1|k}^T,\ \hat{y}_{k+m+2|k}^T,\ \ldots,\ \hat{y}_{k+N|k}^T\right]^T$$

GPC Design

- The GPC cost function :

$$J\left(k,\Delta U_k\right) = Exp\left\{\sum_{j=m+1}^{N}\left(y_{k+j|k}-\gamma_{k+j}\right)^2 + \sum_{j=1}^{N_u}\lambda_j\left(\Delta u_{k+j-1|k}\right)^2\right\}$$

- Vector form :

$$J\left(k,\Delta U_k\right) = \left[\hat{Y}_k - R_k\right]^T\left[\hat{Y}_k - R_k\right] + \Delta U_k^T \lambda \Delta U_k$$

$$\lambda = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \lambda_{N_u} \end{bmatrix}$$

$$R_k = \left[\gamma_{k+m+1}^T,\ \gamma_{k+m+2}^T,\ \dots,\ \gamma_{k+N}^T\right]^T$$

- Calculating control law sequence :

$$\min_{\Delta U_k} J(N)$$

$$\Delta U_k = H\left(I - A_b\right)G^T \bullet$$
$$\bullet\left[A_a\gamma_k - (I - A_b)M_1\hat{x}_{k+m|k} + (I - A_b)M_2\hat{x}_{k+m-1|k}\right]$$

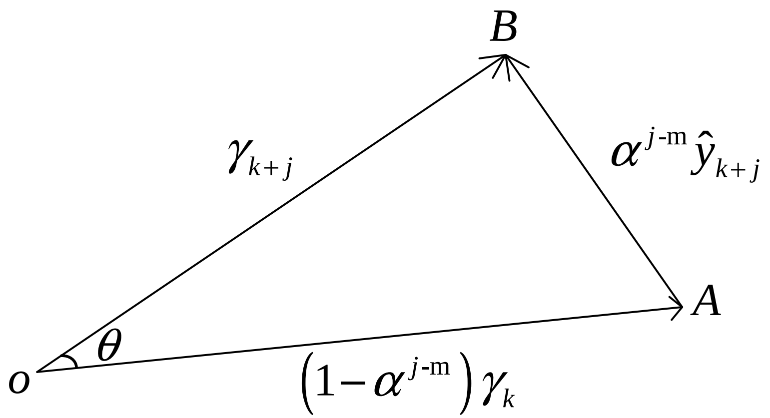
The Future Reference Trajectory

- The Multi-step rule of reference :

$$\gamma_{k+m+1} = \alpha \hat{y}_{k+m+1} + (1-\alpha)\,\gamma_k$$

$$\gamma_{k+j} = \alpha^{j\text{-m}} \hat{y}_{k+j} + \left(1-\alpha^{j\text{-m}}\right)\gamma_k, \quad j \geq m+2$$

- The vector structure diagram of the future reference :



$$R_k = \underbrace{\begin{bmatrix} \alpha & 0 & \cdots & 0 \\ 0 & \alpha^2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \alpha^{N\text{-m}} \end{bmatrix}}_{A_b} \hat{Y}_k + \underbrace{\begin{bmatrix} 1-\alpha \\ 1-\alpha^2 \\ \vdots \\ 1-\alpha^{N\text{-m}} \end{bmatrix}}_{A_a} \gamma_k$$

Robustness against Process Noise and Model Uncertainties

- State space model with incremental control law :

$$x_{k+1} = \underbrace{\left[\Phi + I - \Gamma\beta_1 \mathrm{H}(I-\mathrm{A}_b)G^T(I-\mathrm{A}_b)\mathrm{M}_1\right]}_{\mathrm{M}} x_k - \underbrace{\left[\Phi - \Gamma\beta_1 \mathrm{H}(I-\mathrm{A}_b)G^T(I-\mathrm{A}_b)\mathrm{M}_2\right]}_{\mathrm{N}} x_{k-1} + E\Delta\omega_k$$

- We define H∞ control performance index with a positive scalar r . The system under the incremental control law should satisfy: if (1) is satisfied for $\Delta\omega_k \in \ell_2[0,\infty)$, the system is H∞ norm stable.

$$\left\| Cx_k \right\|_2 \le r \left\| \Delta\omega_k \right\|_2 \qquad (1)$$

↓

$$\Lambda = \sum_{k=0}^{\infty} \left[x_k^T C^T C x_k - r^2 \Delta\omega_k^T \Delta\omega_k \right] \le 0$$

System stability

- Lyapunov function :

$$V_k = x_k^T P x_k + x_{k-1}^T Q x_{k-1} > 0$$

- Proof :

$$\Lambda \le \Lambda + V_k = \sum_{k=0}^{\infty} \left[\Delta V_k + z_k^T z_k - r^2 \Delta\omega_k^T \Delta\omega_k \right] < 0$$

$$= \sum_{k=0}^{\infty} \left\{ \begin{bmatrix} x_k \\ x_{k-1} \\ \Delta\omega_k \end{bmatrix}^T \left(\begin{bmatrix} \mathrm{M}^T \\ \mathrm{N}^T \\ E^T \end{bmatrix} P[\times] + \begin{bmatrix} Q-P+C^TC & 0 & 0 \\ 0 & -Q & 0 \\ 0 & 0 & -r^2 \end{bmatrix} \right) \begin{bmatrix} x_k \\ x_{k-1} \\ \Delta\omega_k \end{bmatrix} \right\} < 0$$

$$\begin{bmatrix} \mathrm{M}^T \\ \mathrm{N}^T \\ E^T \end{bmatrix} P[\times] + \begin{bmatrix} Q-P+C^TC & 0 & 0 \\ 0 & -Q & 0 \\ 0 & 0 & -r^2 \end{bmatrix} < 0 \Rightarrow \begin{bmatrix} Q-P+C^TC & 0 & 0 & * \\ 0 & -Q & 0 & * \\ 0 & 0 & -r^2 & * \\ P\mathrm{M} & P\mathrm{N} & PE & -P \end{bmatrix} < 0$$

Theorem

For a given $r>0$, if there exist symmetric positive definite matrices P and Q such that the LMI is solvable, then the system is H∞ norm stable.

$$\begin{bmatrix} Q-P+C^TC & 0 & 0 & * \\ 0 & -Q & 0 & * \\ 0 & 0 & -r^2I & * \\ P\Phi+P-X(I-A_b)G^T(I-A_b)\mathrm{M}_1 & X(I-A_b)G^T(I-A_b)\mathrm{M}_2-P\Phi & PE & -P \end{bmatrix} < 0$$

where $X=P\Gamma\beta_1\mathrm{H}$

Parameter Computation and Tuning

- Computation of the control-weighting parameter :

$$\lambda_1 = \frac{1}{\Gamma^T K K^T \Gamma}\left[\Gamma^T \Gamma \beta_1 K^T \Gamma - \Gamma^T K (I - \mathrm{A}_b) G^T (I - \mathrm{A}_b) G K^T \Gamma\right]_{K = P^{-1} X}$$

- Parameter tuning (a rule aiming to adapt the control-weighting parameter with respect to the smooth parameter change):

α	α_1	α_2	α_3
λ	λ_1	λ_2	λ_3

❑ a function adapting the control-weighting parameter in real-time when a large reference drop or rise is detected.

❑ a table between the control-weighting parameter and the smooth parameter with respect to any specific maximal controller-to-actuator delay like the above Table.

Academic Model

- System :

$$\Phi = \begin{bmatrix} 1.2 & -0.7 \\ 1 & 0 \end{bmatrix}, \Gamma = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, C = [0.5 \quad 0.1], E = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}$$

$$\omega_{kT} \sim N(0,Q) \quad \text{with variance} \quad Q = I_3$$

$$\upsilon_{kT} \sim N(0,R) \quad \text{with variance} \quad R = 2.5$$

- GPC and H∞ Parameters :

$$r=1$$

$$N_u=3$$

$$N=8$$

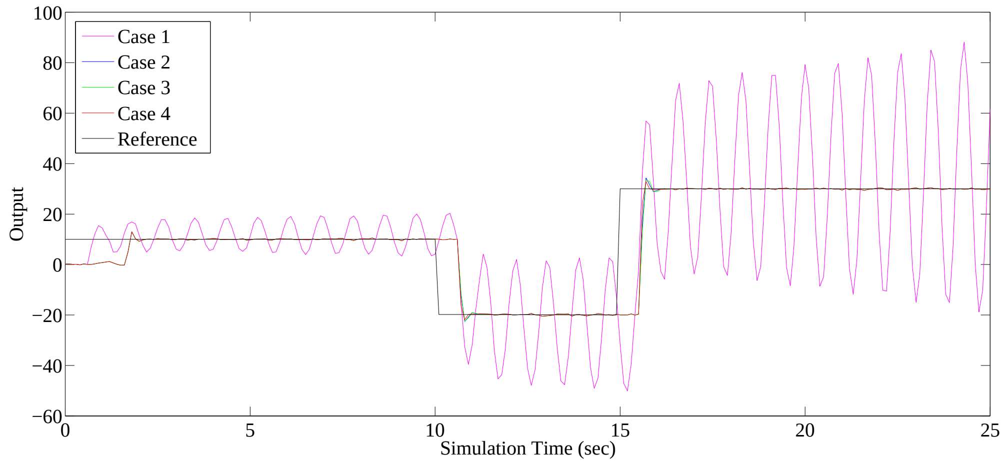
Simulation 1

- case 1 with conventional GPC;
- case 2 with the proposed H∞ predictive control and a constant parameter;
- case 3 with the proposed H∞ predictive control and varying control-weighting parameter;
- case 4 with the proposed H∞ predictive control and varying smooth on control-weighting parameters
- Table of parameter tuning (two sudden jumps at time 10s and 15s):

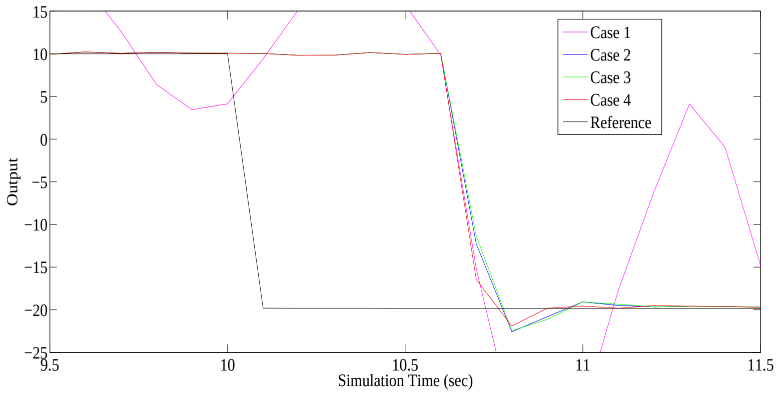
Simulation period/sec	0–10	10–15	15–25
Case 1	α=0.4, λ=0.04	α=0.4, λ=0.04	α=0.4, λ=0.04
Case 2	α=0.73, λ = 0.0024	α=0.73, λ = 0.0024	α=0.73, λ = 0.0024
Case 3	α=0.73, λ = 0.0024	α=0.73, λ = 0.003	α=0.73, λ = 0.0043
Case 4	α=0.73, λ = 0.0024	α=0.4, λ = 0.003	α=0.2, λ = 0.0043

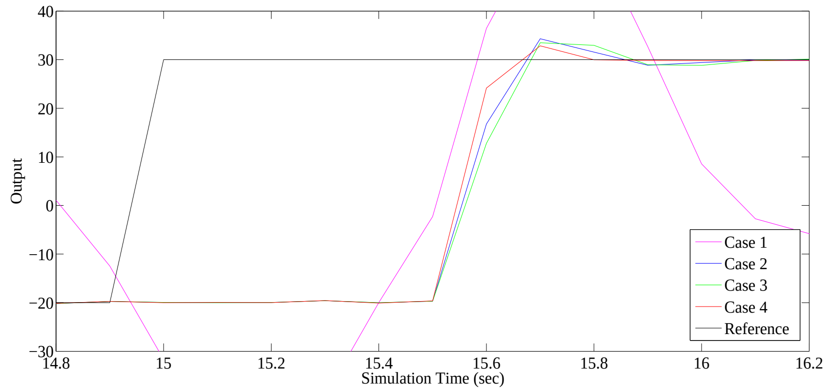
Simulation 2: the adaptive parameter tuning approach

- Outputs and reference of the plant (figure 1):



- Zoom-in (left: figure 2 and right: figure 3) :



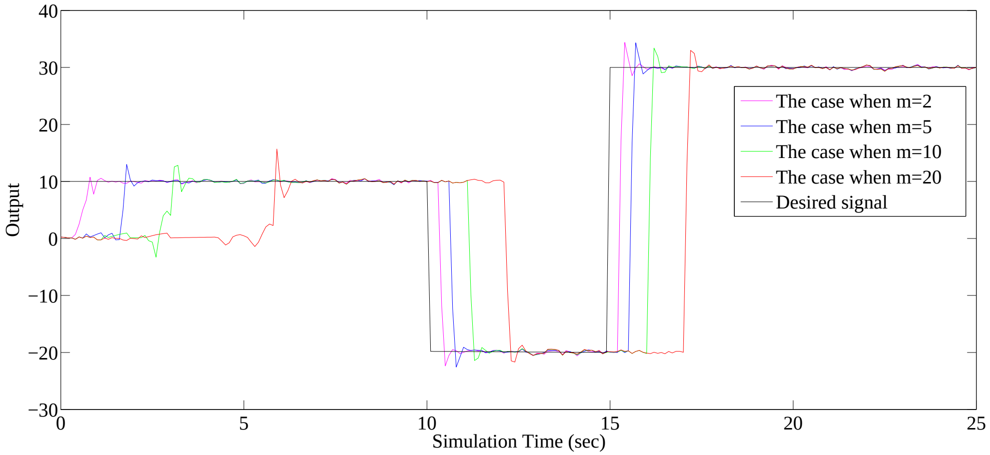


Simulation 3: the influence of the maximal controller-to-actuator delay on the control performance

- Table of parameter tuning :

Case 4	0–10	10–15	15–25
m=2	α=0.73, λ=0.002	α=0.4, λ=0.0024	α=0.2, λ=0.003
m=5	α=0.73, λ = 0.0024	α=0.4, λ = 0.003	α=0.2, λ = 0.0043
m=10	α=0.73, λ = 0.0112	α=0.4, λ = 0.0167	α=0.2, λ = 0.0259
m=20	α=0.73, λ = 0.0174	α=0.4, λ = 0.0224	α=0.2, λ = 0.0321

- Outputs and reference of the plant :



Conclusion

- It achieves the robustness issue of GPC algorithm. The presented H∞ performance index compensates for the performance loss caused by process noise, which is not taken into account in GPC.

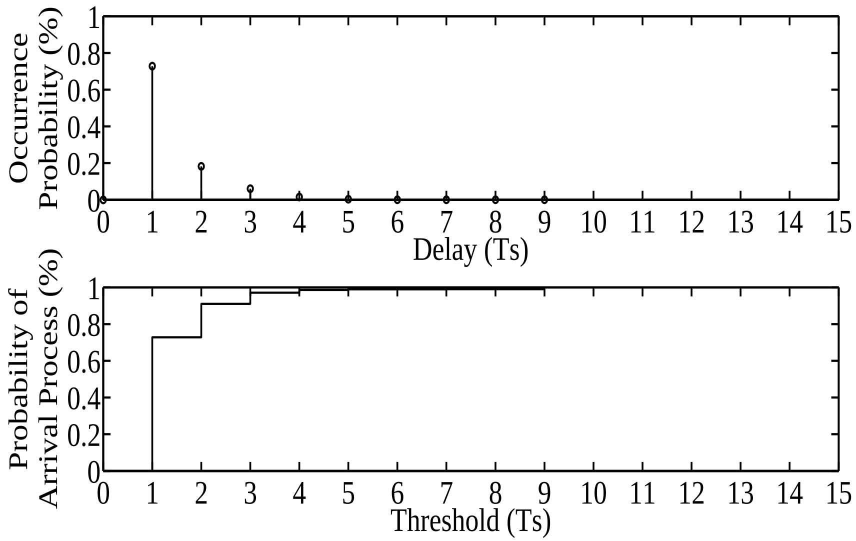
- The proposed approach also allows the adaptation control for GPC with respect to the adjustable smooth parameter for the reference with sudden jumps, which leads to an additional improvement.

Modeling of Observation Arrival Process

Bernoulli Process:

$$\gamma_k = \begin{cases} 1 \ , \ \text{if packet arrives at time k} \\ 0 \ , \ \text{otherwise} \end{cases}$$

Poisson Distribution:



Problem 0: Given available measurements $Y_k = \{\gamma_1 y_1, \ldots, \gamma_k y_k\}$, seek the state estimate $\hat{x}_k$, which minimizes $J_k = E\left[(x_k - \hat{x}_k)(x_k - \hat{x}_k)^T\right]$.

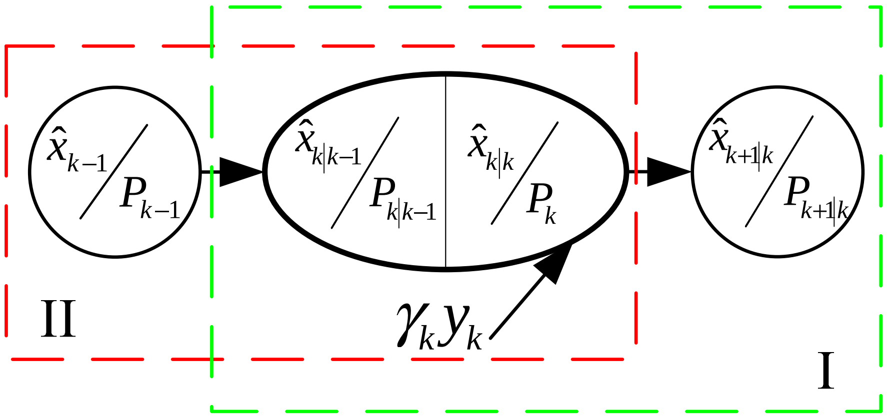
Generalized State Estimation

Problem 1: For observation arrival process $Y_k = \{\gamma_1 y_1, \ldots, \gamma_k y_k\}$, seek a critical value λ_c for observation arrival probability, which enables state estimation with intermittent observations stable for all initial conditions as $\lambda > \lambda_c$.

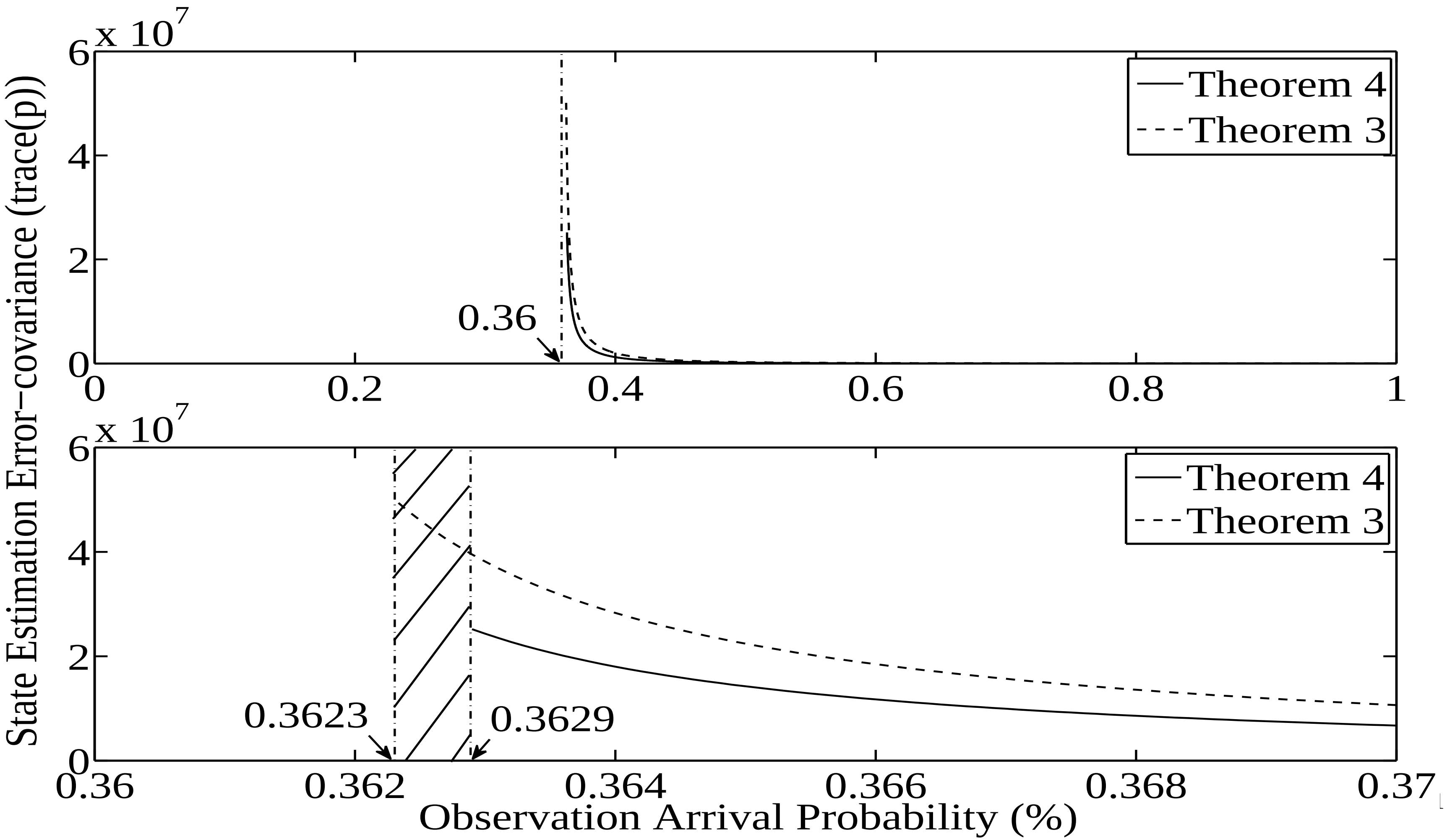
State-space model with noise:

$$\begin{cases} x_{k+1} = \Phi x_k + \Gamma u_k + \omega_k \\ y_k = C x_k + D u_k + \upsilon_k \end{cases}, k \geq 1 \quad \textbf{(6)}$$

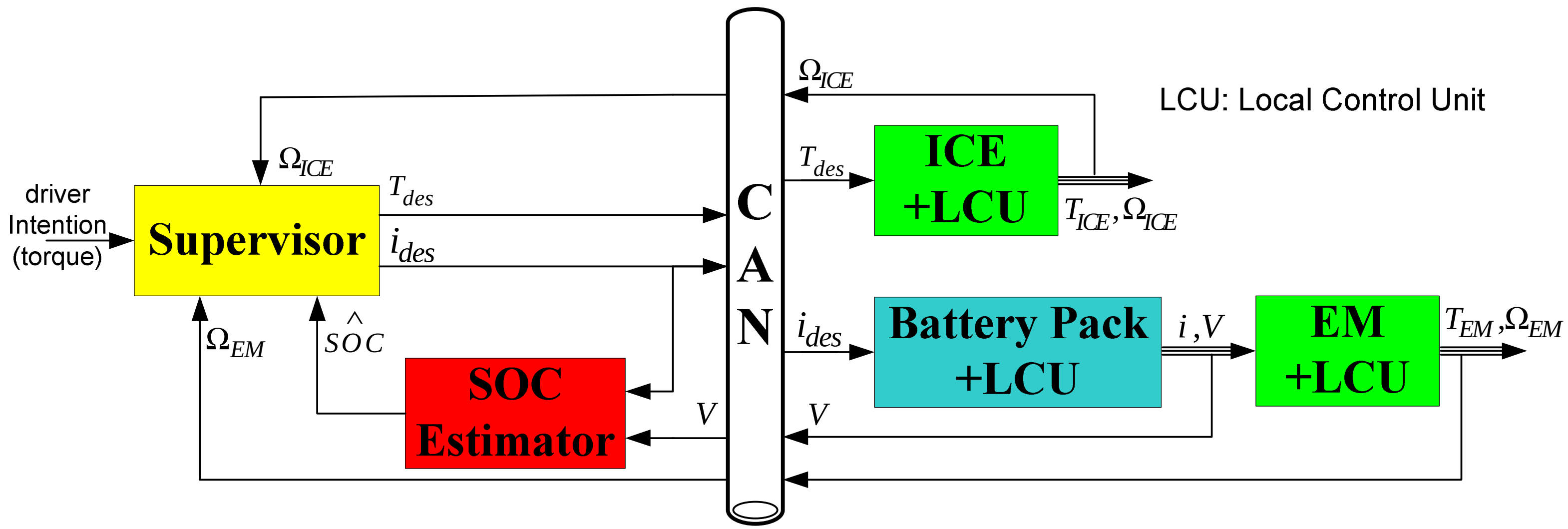
Transition model with respect to observation arrival process:



Generalized Convergence Condition Subject to Observation Arrival Probability



Battery State-of-Charge Estimation



ICE – Internal Combustion Engine;
EM – Electric Motor.

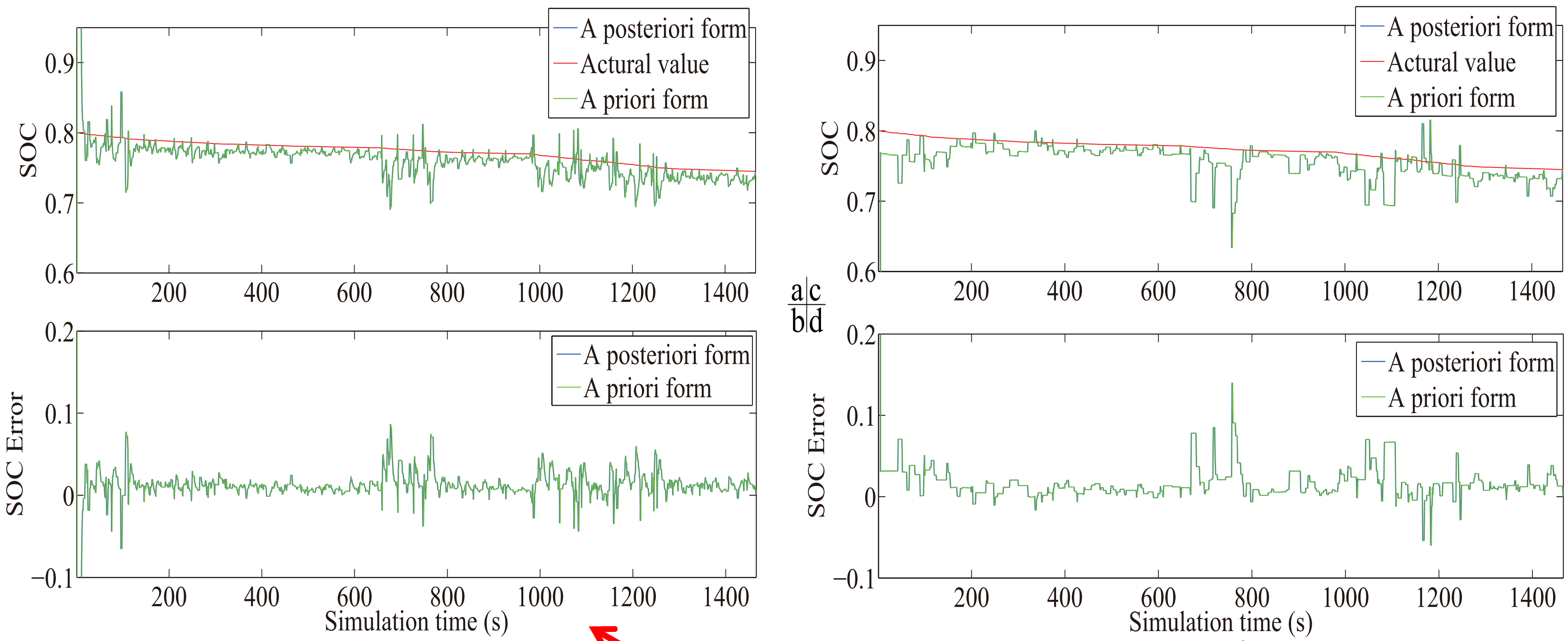


Table1. Comparison of Mean Absolute Estimation Errors

	λ_c=0.6016	λ_c=0.1851
Ave_{pre}	0.0934	0.9761
Ave_{post}	0.0049	0.8876

Perspectives et travaux futurs

(i). The convergence subject to state estimation error-covariance is limited by a critical boundary of observation arrival probability.

(i). if it is possible to directly or non-directly identify the intensity of system noise online, it would develop an adaptive buffering strategy based on this work so as to always keep the filer working with a lower error-covariance.

Publications

- **Y.M. ZHANG, V. SIRCOULOMB, N. LANGLOIS.** "Observer Design for Discrete-time Systems Subject to Long Time-delays". IEEE Chinese Control & Decision Conference 24th, Taiyuan, China, pp. 2949-2954, May 23-25, 2012.
- **Y.M. ZHANG, V. SIRCOULOMB, N. LANGLOIS.** "Stable Observer-based Control for Long Network-induced Delays". IEEE Chinese Control & Decision Conference 24th, Taiyuan, China, pp. 2966-2971, May 23-25, 2012.
- **Y.M. ZHANG, V. SIRCOULOMB, N. LANGLOIS**. "Kalman Filtering for Discrete-time Networked Control Systems with Incomplete Observations: A Dropping Over-delayed Packets Approach". IFAC SSSC joint conference, Grenoble, France, February 4-6, 2013.
- **Y.M. ZHANG, V. SIRCOULOMB, N. LANGLOIS,** "Robust Linear Estimator for Networked Control Systems with Incomplete Observations: A Dropping Over-delayed Packets Approach". IEEE ISIE joint conference, Taipei, Taiwan, May 28-31, 2013.
- **Y.M. ZHANG, V. SIRCOULOMB, N. LANGLOIS.** “Predictive Control Design for Discrete-Time Networked Control Systems”, 11th IFAC International Workshop on Adaptation and Learning in Control and Signal Processing, Caen, France, July 3-5, 2013.
- **ZHANG Y., SIRCOULOMB V., LANGLOIS N.,** Robust Optimal Estimation with Lossy Network: A Probabilistic Approach. Application to Battery State of Charge Estimation, accepted at International Journal of Robust and Nonlinear Control, October 2013.

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