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Fault tolerant control: a Takagi-Sugeno (T-S) -model based predictive approach

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Outline

❖Introduction

❖T-S fuzzy modeling

❖Accommodation: actuator faults

❖ Nonlinear observer

❖Fuzzy model predictive reconfigurable control (Actuator and sensor faults): measurable/ unmeasurable premise variables

❖Conclusion and Outlook

Introduction

The objective of this work is to design a Fault Tolerant Controller (FTC) ensuring trajectory tracking of a desired reference, in the nominal and faulty cases. The FMPC (Fuzzy Model-based Predictive Control) approach is proposed:

- Nonlinear systems subject to faults are described by T-S fuzzy model
- Model-based Predictive Control: minimization of cost function subject to constraints
- The proposed FTC design scheme integrates the state estimation using a nonlinear observer and the dynamic optimizer based on interpolation control to guaranty the stabilization of the faulty plant

T-S fuzzy modeling (1/7):

The modeling choice

- the T-S structure is a universal approximator.
- The T-S fuzzy model is based on rules such as: IF PREMISE THEN CONSEQUENCE.
- Decomposition of the nonlinear system dynamic behavior around several operational areas.
- Each sub-model contributes more or less to approximate the overall system behavior.
- Nonlinear weighting functions are based on interpolation mechanism between these submodels

T-S fuzzy modeling (2/7):
Main methods in the literature

A T-S model can be established using three main principal methods :

1. Identification and parametre estimation from experimental data, [Abonyi, 2002].
2. Nonlinearity sector approach: direct transform of an affine model, [Tanaka&Wang, 2001].

   Advantage: reducing the local model number
3. Linearisation of the nonlinear model: arround different points, [Johansen, 2000].

The number of N local model depends on the desired precision of the modelisation

T-S fuzzy modeling (3/7):

The global non stationary linearization

The nonlinear system state space representation:

$$\begin{cases} \dot{x}(t) = f(x(t), u(t)) \\ y(t) = g(x(t)) \end{cases} \quad (1)$$

Quasi-LPV system with polytopic form, from the Taylor series development around an operating point, [Ben Hamouda, 2013]:

$$\begin{cases} \dot{x}(t) = A(\theta)\, x(t) + B(\theta)\, u(t) \\ y(t) = C(\theta)\, x(t) \end{cases} \quad (2)$$

Interpolation between the local linear models. $\{A_j, B_j\}$ are the sub-models asymtotically stable matrices.

$$\begin{cases} \dot{x}(t) = \sum_{j=1}^{N} \mu_j(\theta)\left(A_j x(t) + B_j u(t)\right) \\ y(t) = \sum_{j=1}^{N} \mu_j(\theta)\, C_j x(t) \end{cases} \quad (3)$$

Polytope is obtained with $N = 2^r$ peaks, where r is the number of premise variables.

Variations of the vector $\theta(x,u)$ is represented by a set of N^{th} peak matrices which define the polytope.

T-S fuzzy modeling (4/7):

Convex polytopique transoformation

Activation function $\mu_j(x(t),u(t))$ (weighting) determines the activation degree of the j^{th} associated local model:

$$\sum_{j=1}^{N} \mu_j(x(t),u(t)) = 1 \text{ and } 0 \leq \mu_j(x(t),u(t)) \leq 1$$

→ depending on various parameters such as the state and the control vectors.

Polytopic convex transformation:

$$\mu_1(x,u) = \frac{\theta(x,u) - \underline{\theta}}{\overline{\theta} - \underline{\theta}} \quad , \quad \mu_2(x,u) = \frac{\overline{\theta} - \theta(x,u)}{\overline{\theta} - \underline{\theta}}$$

T-S fuzzy modeling (5/7):
Choice criteria of premise's variables

The quasi-LPV model is not unique. For each quasi-LPV representation corresponds a particular set of premise's variables.

The choice affects the number of sub-models and the global model structure, [Nagy, 2010].

⇒ This freedom degree is used to facilitate the study of controllability and stability analysis.

1. The control matrix in the quasi-LPV isn't a nul matrix (necessary condition for systems controllability).
2. A minimal set of premise variables is preferred.
3. Choose premise variables which depend on a minimal number of state variables

T-S fuzzy modeling (6/7):

Example: System $\sum$

- The nonlinear system differential equations:

$$\begin{cases} \dot{x}_1(t) = -x_1(t) + u(t) \\ \dot{x}_2(t) = x_1(t) - |x_2(t)|\, x_2(t) - 10 \\ y(t) = x_2(t) \end{cases} \qquad (4)$$

- From the Taylor series development around the operating point $(x_{10}, x_{20}, u_0, y_0)$:

$$\begin{cases} \delta x_1(t) = \begin{bmatrix} -1 & 0 \\ 1 & -2|x_{20}| \end{bmatrix} \delta x(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \delta u(t) \\ \delta y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} \delta x(t) \end{cases} \qquad (5)$$

- The quasi-LPV differential equations:

$$\begin{cases} \dot{x}(t) = A(\theta)\, x(t) + B\, u(t) \\ y(t) = C\, x(t) \end{cases} \qquad (6)$$

with $A(\theta) = \begin{bmatrix} -1 & 0 \\ 1 & -2\theta \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 1 \end{bmatrix}$ and $\theta = |x_{20}|$

The vector $\theta(x,u)$ is supposed to vary arbitrarily in the interval $[0 \quad 10]$.

T-S fuzzy modeling (7/7):

Example: System $\sum$

One premise variable $\theta(x,u)$ → two sub-models with polytopic form:

$$M(\theta) = \sum_{i=1}^{2} \mu_i M_i = \mu_1 M_1 + \mu_2 M_2$$

where $\overline{\theta} = 9.5$ and $\underline{\theta} = 0.5$.

The fuzzy model state space representation is written as :

$$\begin{cases} \dot{x}(t) = \sum_{j=1}^{2} \mu_j(\theta)(A_j x(t)) + Bu(t) \\ y(t) = Cx(t) \end{cases} \tag{7}$$

with

$$A_1 = \begin{bmatrix} -1 & 0 \\ 1 & -2\theta_1 \end{bmatrix} \quad , \quad A_2 = \begin{bmatrix} -1 & 0 \\ 1 & -2\theta_2 \end{bmatrix} \; ;$$

$$B = [1 \quad 0]^T \quad , \quad C = [0 \quad 1] \; ;$$

Fuzzy-based Model accommodation (1/8): with stabilized predictions

Repetitive minimization of cost function :

$$J(k) = \sum_{l=1}^{H_p} \left\| x(k+l) - x_d(k+l|k) \right\|_Q^2 + \sum_{l=0}^{H_u-1} \left\| u(k+l|k) \right\|_R^2 \qquad (8)$$

Subject to constraints:

$$u_{\min} \le u(l) \le u_{\max},\ \Delta u_{\min} \le \Delta u(l) \le \Delta u_{\max},\ and\ x_{\min} \le x(l) \le x_{\max}$$

The pre-stabilization is an efficient tool to guarantee nominal closed-loop stability using the MPC controller, [Kale & Chipperfield, 2005].

The proposed strategy is to assume fuzzy state feedback as a baseline controller to which FMPC control signals are added, [Ben Hamouda, 2013].

predicted control

$$u_{FMPC}^{j}(k+i|k) = \begin{cases} -k_{uj}x(k+i|k) + q_{ij}(k), \\ i = 0,\cdots,H_u - 1 \\ -k_{uj}x(k+i|k),\ i \ge H_u \end{cases} \qquad (9)$$

Fuzzy-based Model accommodation (2/8):

Interpolation control

Local controller weighted by the j^{th} activation functions:

$$\begin{cases} \dot{x}(t) = A_j x(t) + B_j u_j(t) \\ y(t) = C_j x(t) \end{cases} \qquad (10)$$

The fuzzy model control law applied to the nonlinear system:

$$u(t) = \sum_{j=1}^{N} \mu_j(\theta) u_j(t) \qquad (11)$$

Control the nonlinear system via the interpolation control laws which are designed from the local controllers around N different operating points.

Fuzzy-based Model accommodation (3/8):

Actuator fault accommodation by perturbations rejection

The j^{th} linear model described by (10) becomes:

$$\begin{cases} \dot{x}(t) = A_j x(t) + B_j u_j(t) + E_a^j f(t) \\ y(t) = C_j x(t) \end{cases} \quad (12)$$

Additive actuator fault

The considered faults accommodation method is based on the following basic equation, [Rodrigues, 2008] :

$$u_j(t) = u_{FMPC}^j(t) + u_F^j(t) \quad (13)$$

Condition

$u_F^j(t)$ must solve the following equality:

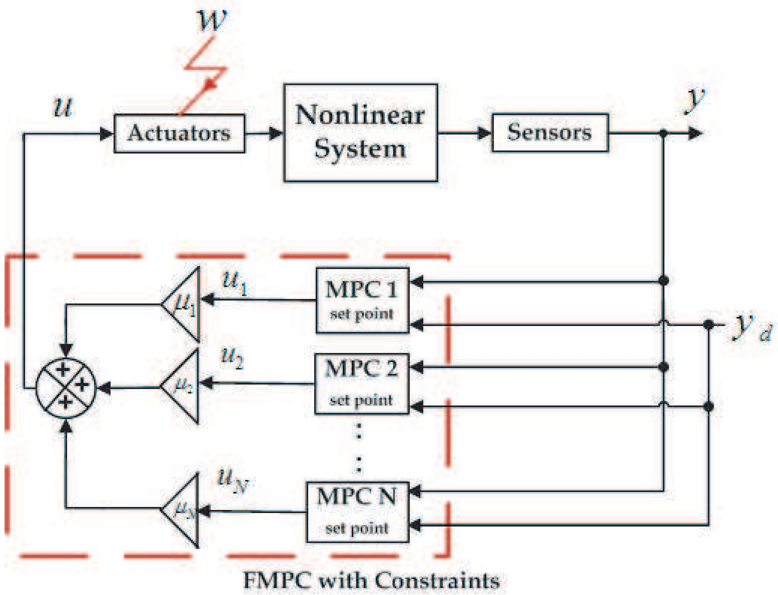
$$B_j u_F^j(t) + E_a^j f(t) = 0, \text{ with } B_j = E_a^j \quad (14)$$

System (12) with the control law (13) under the condition (14) is used to:

→ cancel faults effect in closed loop

Fuzzy-based Model accommodation (4/8):
Interpolation based control using decoupled multiple model

- The MPC optimization is formulated as a Quadratic Programming (QP).
- The LQ-optimal gains of the feed back law are obtained by solving an algebraic Riccati equation.



The interpolation based control, [Ben Hamouda, 2013]:

$$u(t) = \sum_{j=1}^{N} \mu_j(\theta)\left[u^j_{FMPC}(t) + u^j_F(t)\right] \quad (15)$$

Fuzzy-based Model accommodation (5/8):

Example: System $\sum$

Controller tuning parameters

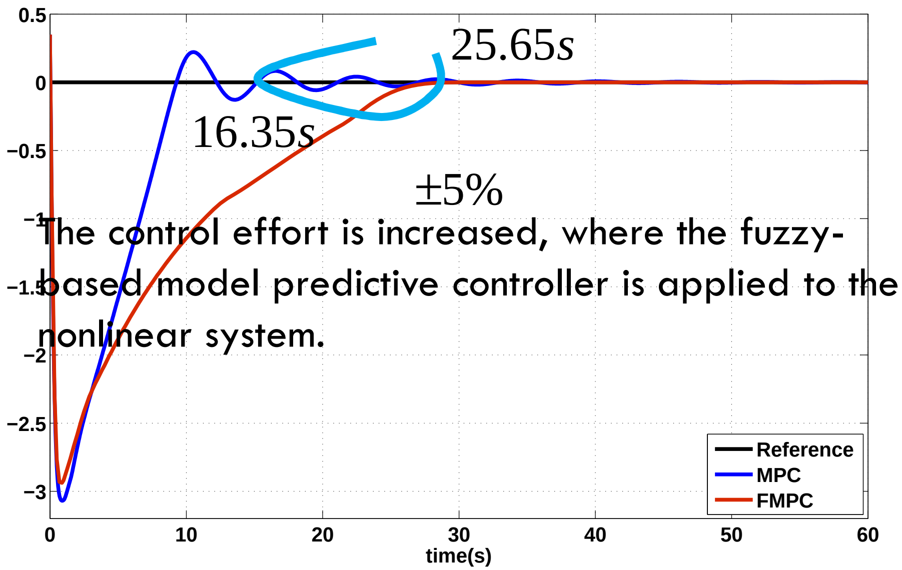
Sample time	$0.5s$
Prediction horizon	$8T_e$
Control horizon	$6T_e$
Input constraints	$-2 \leq u_k \leq 25$
	$-0.358 \leq \Delta u_k \leq 2$
Output constraints	$-3.02 \leq y_k \leq 3.02, \quad \forall k \geq 0$
Input weights	0.1
Output weights	1
Actuator fault matrix	$E_a = [1 \quad 0]^T$

Fuzzy-based Model accommodation (6/8):

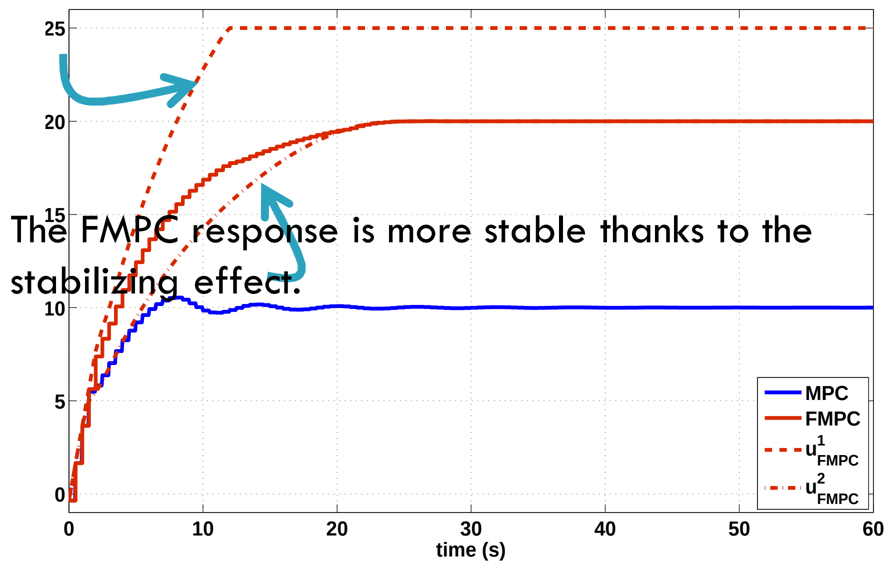
Example: System ∑

Nominal operating:

The outputs responses and control inputs from initial condition $x_0 = (0.32, 0.35)^T$, $y_0 = 0.35$ and $u_0 = 0$



The control effort is increased, where the fuzzy-based model predictive controller is applied to the nonlinear system.

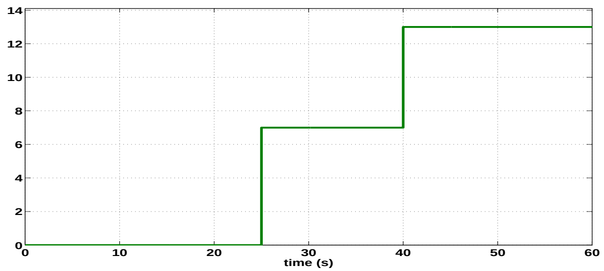


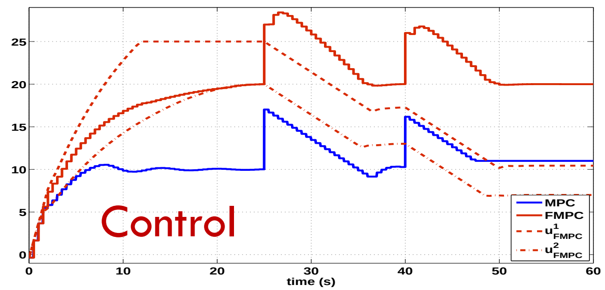
The FMPC response is more stable thanks to the stabilizing effect.

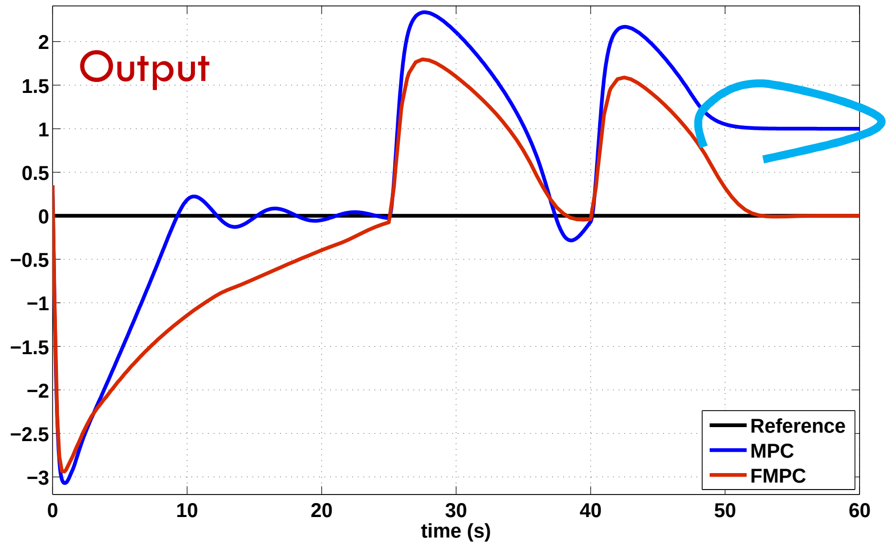
Fuzzy-based Model accommodation (7/8):

Example: System ∑

Actuator scenario fault: Algorithme2 accommodates faults by perturbation rejection.





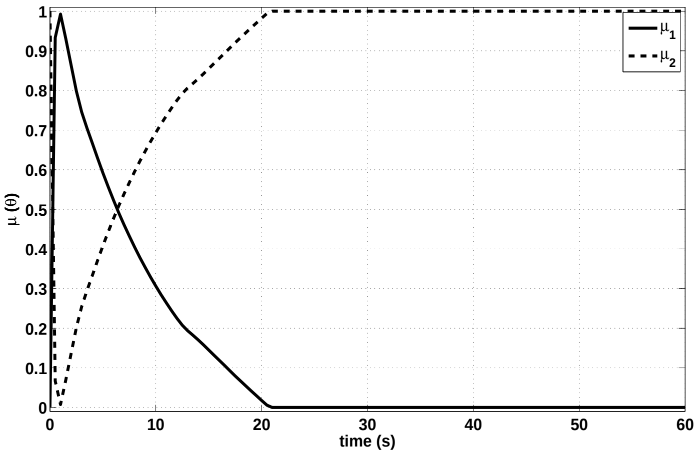


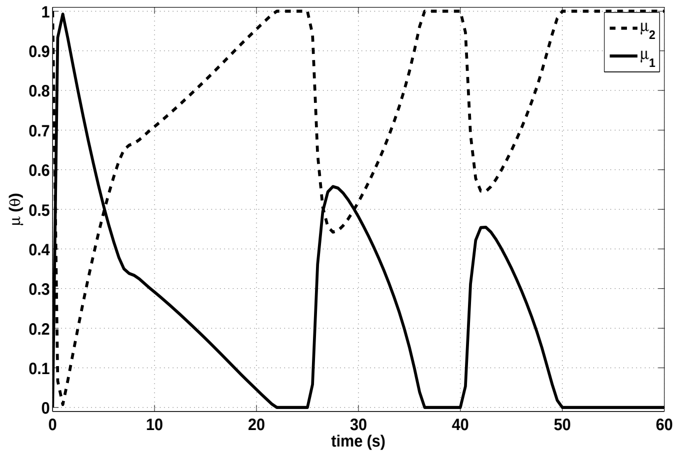
Contrary to the FMPC, the output cannot track the desired trajectory.

Fuzzy-based Model accommodation (8/8):

Example: System ∑

Variation of the activation functions according to the premise variable vs.time in the nominal operating (left) and case linked to the first scenario fault (right)





Nonlinear observer (1/4):
The necessity of the estimation system states

Fuzzy based-model accommodation maintains good tracking performances for nonlinear system subject to actuator faults. Obviously it is not sufficient for the sensor faulty case.

In practice, it is assumed that state variables are accessible to control the system.

→ A reliable estimation of unmeasurable variables is necessary.

Nonlinear observer (2/4):
Thau-Luenberger

Transform nonlinear system described by (1) to (16) with: the couple (A,B) is controllable and (A,C) is an observable pair , [Ben Hamouda, 2014].

$$\Rightarrow \quad \begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + f_{NL}(x(t),u(t)) \\ y(t) = Cx(t) \end{cases} \qquad (16)$$

Let the observer be, where L is the oberver gain matrix:

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L(y(t) - \hat{y}(t)) + f_{NL}(\hat{x}(t),u(t)) \\ \hat{y}(t) = C\hat{x}(t) \end{cases} \qquad (17)$$

Assumption: the pair (A,C) is observable, to find an L such that the eigenvalues of A - LC are in the open left half plane.

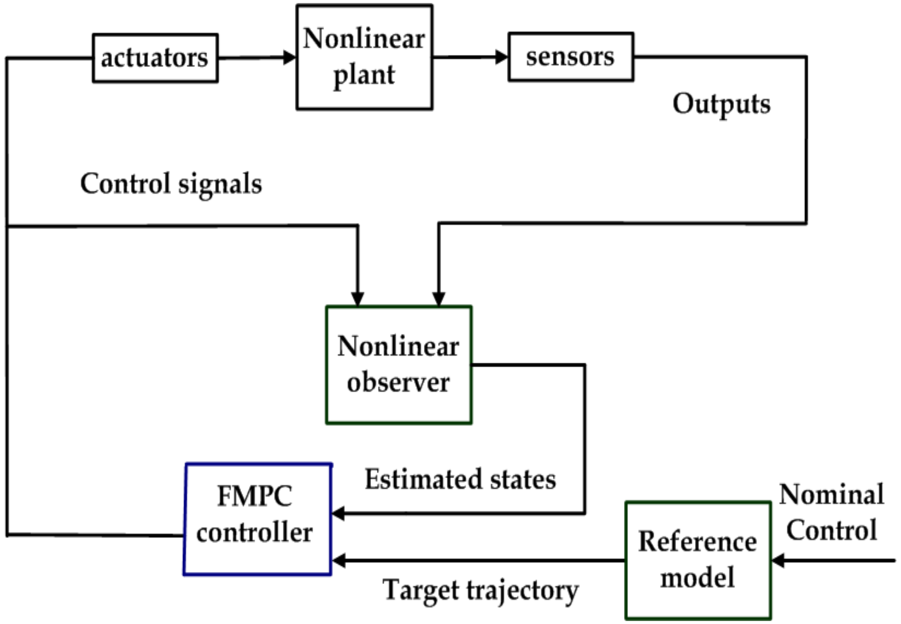
- The LQ-optimal gains of the nonlinear observer is obtained by solving an algebraic Riccati equation.

Nonlinear observer (3/4): Thau-Luenberger

Let e denote that error between the true state and our estimated state, $e = \hat{x} - x$ and wish to determine if it can be made to decay to zero.

The error dynamics are nonlinear:

$$\dot{e} = \underbrace{(A - LC)e}_{Linear\ error} + \underbrace{f_{NL}(e + x, u) - f_{NL}(x, u)}_{Non\ linear\ error} \qquad (18)$$



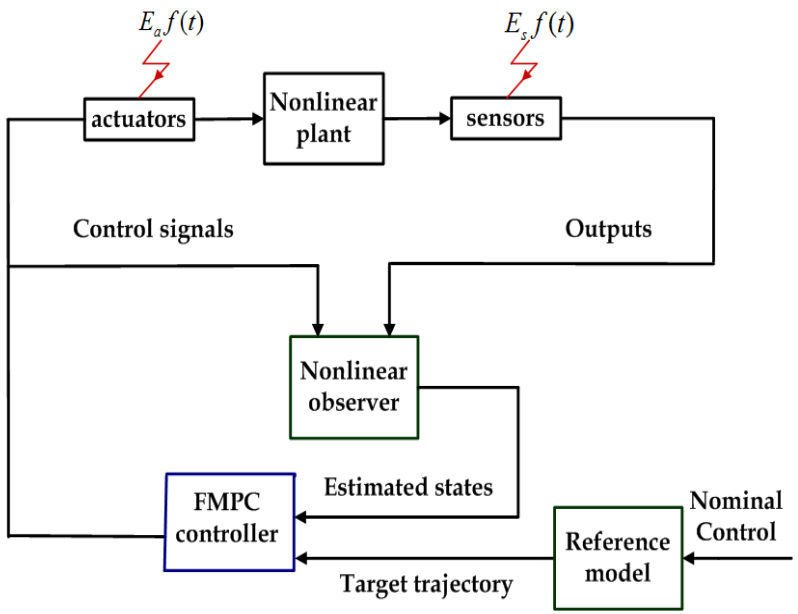
Nonlinear observer (4/4):
In the faulty (actuator/sensor) case

The nonlinear observer:

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + E_a \hat{f}(t) + L(y(t) - \hat{y}(t)) + f_{NL}(\hat{x}(t), u(t)) \\ \hat{y}(t) = C\hat{x}(t) + E_s \hat{f}(t) \end{cases} \quad (19)$$

Let $e_f = f - \hat{f}$ the fault estimation error, the error dynamics, [Ben Hamouda, 2014], are:

$$\dot{e} = \underbrace{(A - LC)e}_{Linear\ error} + \boxed{(E_a - LE_s)e_f(t)} + \underbrace{f_{NL}(x,u) - f_{NL}(x - e, u)}_{Non\ linear\ error} \quad (20)$$



Fuzzy-based Model Predictive Reconfigurable Control (1/10): Proposed MPC based strategy

Repetitive minimization of cost function :

$$J(k) = \sum_{l=1}^{H_p} \left\| \hat{y}(k+l) - y_d(k+l|k) \right\|_Q^2 + \sum_{l=0}^{H_u-1} \left\| \Delta u(k+l|k) \right\|_R^2 \qquad (21)$$

Subject to constraints: $u_{\min} \leq u(l) \leq u_{\max},\ \Delta u_{\min} \leq \Delta u(l) \leq \Delta u_{\max},\ and\ \ x_{\min} \leq x(l) \leq x_{\max}$

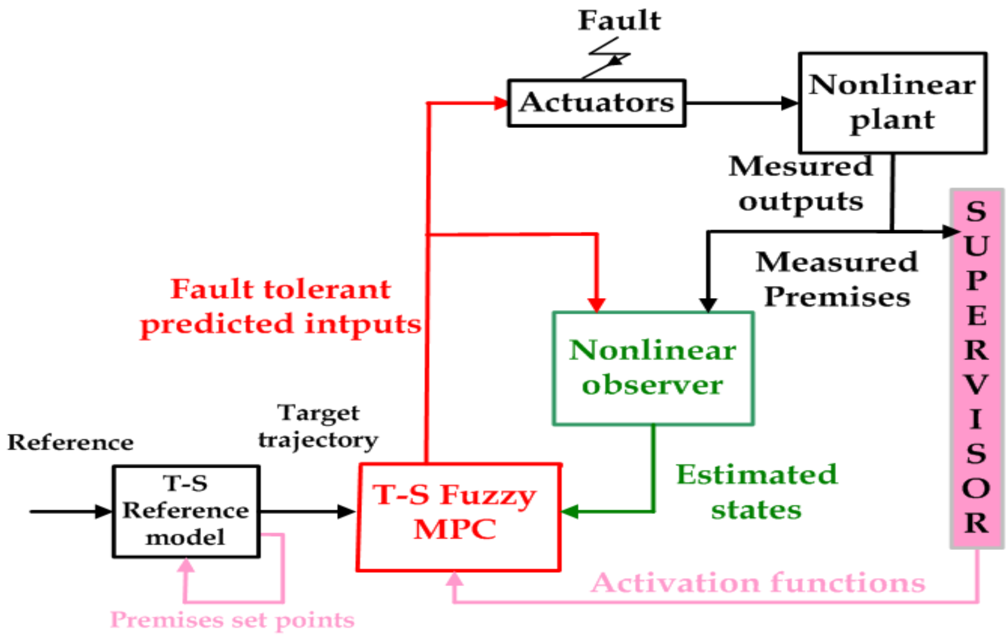
predicted control

The proposed strategy, [Ben Hamouda, 2014]:

$$u_{FMPC}^{j}(k+i|k) = \begin{cases} -k_{uj}\hat{x}(k+i|k) + q_{ij}(k), \\ i = 0, \cdots, H_u - 1 \\ -k_{uj}\hat{x}(k+i|k),\ i \geq H_u \end{cases} \qquad (22)$$

Fuzzy-based Model Predictive Reconfigurable Control (2/10): Measurable premise variables (MPV)

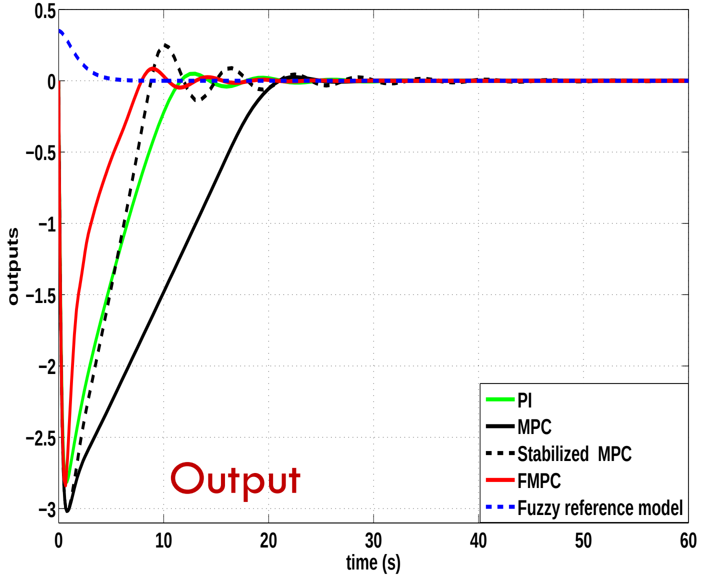
The FTC strategy scheme, [Ben Hamouda, 2014], based on T-S fuzzy model with MPV:

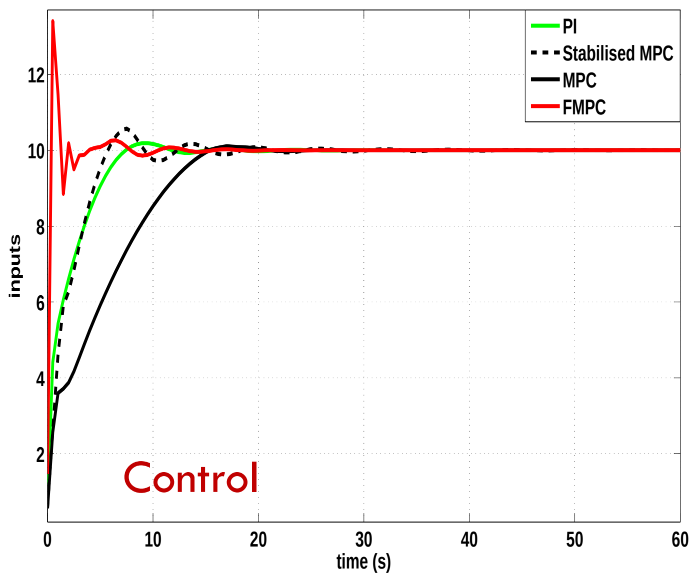


Fuzzy-based Model Predictive Reconfigurable Control (3/10):

Example: System $\sum$

Nominal operating:





The outputs responses and control inputs from initial condition $x_0 = (0.32, 0.35)^T$, $y_0 = 0.35$ and $u_0 = 0$

Fuzzy-based Model Predictive Reconfigurable Control (4/10): MPV

The fuzzy model control law applied to the nonlinear system is:

$$u(t)=\sum_{j=1}^{N}\mu_j(\theta_f)u_j(t) \qquad (23)$$

where:

$$\mu_1(\theta_f)=\frac{\theta_f(x_f)-\underline{\theta}}{\bar{\theta}-\underline{\theta}} \;;\; \mu_2(\theta_f)=1-\mu_1(\theta_f)$$

The T-S reference model:

$$\begin{cases}\dot{x}(t)=\sum_{j=1}^{N}\mu_j(\theta)\big(A_jx(t)+B_ju(t)\big)\\ y(t)=\sum_{j=1}^{N}\mu_j(\theta)\;C_jx(t)\end{cases} \qquad (24)$$

In the presence of the actuator fault, the fuzzy model state space representation is:

$$\begin{cases}\dot{x}_f(t)=\sum_{j=1}^{N}\mu_j(\theta_f)\big(A_jx_f(t)+B_ju(t)+E_a^j f(t)\big)\\ y_f(t)=\sum_{j=1}^{N}\mu_j(\theta_f)\;C_jx_f(t)\end{cases} \qquad (25)$$

Fuzzy-based Model Predictive Reconfigurable Control (5/10):

Unmeasurable premise variables(UPV)

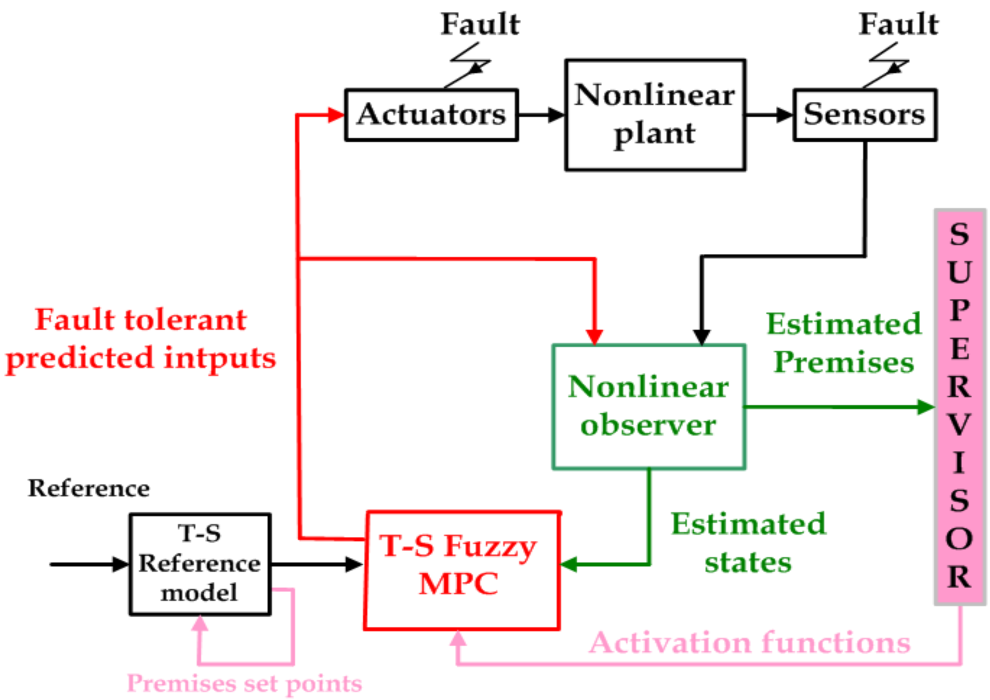
The fuzzy model control law, [Ben Hamouda, 2014], applied to the nonlinear system is:

$$u(t) = \sum_{j=1}^{N} \mu_j(\hat{\theta}_f) u_j(t) \qquad (26)$$

where: $\mu_1(\hat{\theta}_f) = \dfrac{\hat{\theta}_f(x_f) - \underline{\theta}}{\bar{\theta} - \underline{\theta}}$; $\mu_2(\hat{\theta}_f) = 1 - \mu_1(\hat{\theta}_f)$

In the presence of the faults, the fuzzy model state space representation is:

$$\begin{cases} \dot{x}_f(t) = \sum_{j=1}^{N} \mu_j(\hat{\theta}_f) \left(A_j x_f(t) + B_j u(t) + E_a^j f(t) \right) \\ y_f(t) = \sum_{j=1}^{N} \mu_j(\hat{\theta}_f) \left(C_j x_f(t) + E_s^j f(t) \right) \\ \dot{f}(t) = 0 \end{cases} \qquad (27)$$



Fuzzy-based Model Predictive Reconfigurable Control (6/10):

Example: System ∑

To obtain the representation of the proposed nonlinear observer, the system described by (4) is transformed to (16) with $f_{NL}(x(t)) = \begin{bmatrix} 0 & -|x_2(t)|x_2(t) - 10 \end{bmatrix}^T$.

The ranks of the controllability and observability matrices are equal to the system state matrix one. The conditions of the nonlinear observer are checked where the observer gain matrix is: $L = \begin{bmatrix} 0.2169 & 1.1974 \end{bmatrix}^T$

The linear-quadratic controller gains are: $K_{u1} = \begin{bmatrix} 0.4151 & 0.0013 \end{bmatrix}$ and $K_{u2} = \begin{bmatrix} 0.5425 & 0.1896 \end{bmatrix}$

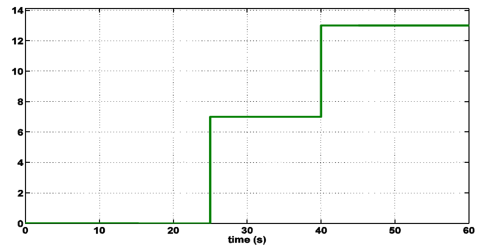
In the presence of the actuator fault, the fuzzy model state space representation is written as:

$$\begin{cases} \dot{x}_f(t) = \sum_{j=1}^{2} \mu_j(\theta_f)\left(A_j x_f(t)\right) + Bu(t) + E_a f(t) \\ y_f(t) = Cx_f(t) \end{cases} \qquad (28)$$

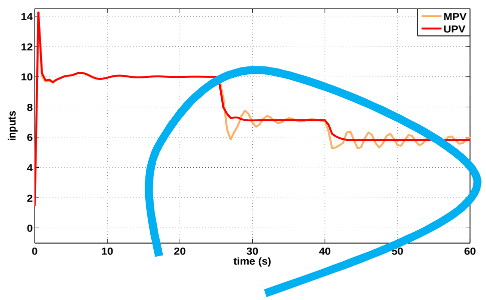
Fuzzy-based Model Predictive Reconfigurable Control (7/10):

Example: System ∑

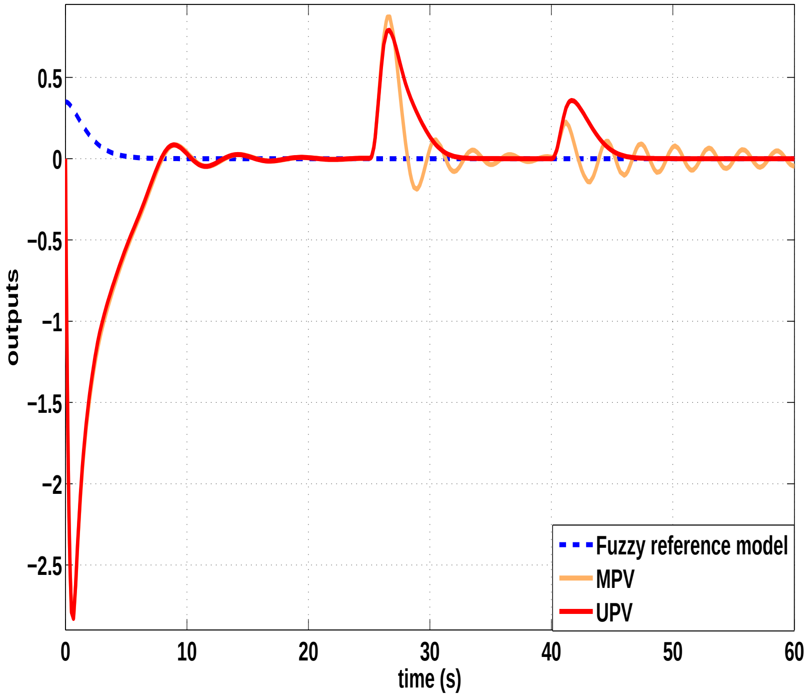
Actuator fault: MPV/UPV



Control signal vs.time



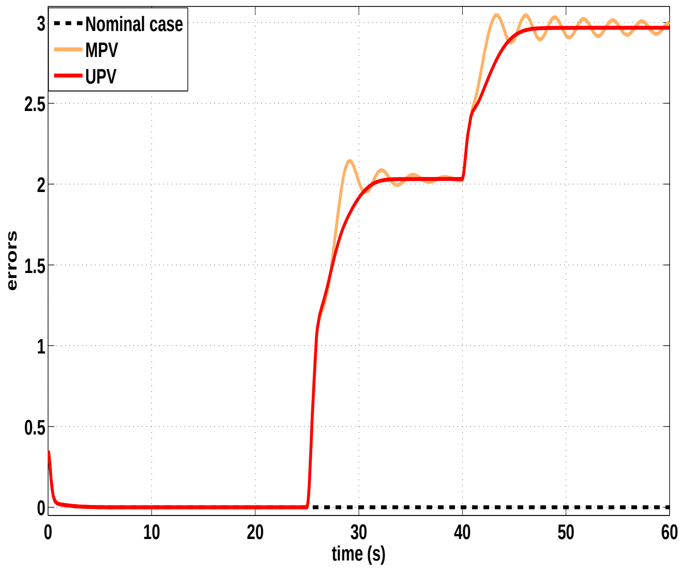
Output response vs.time



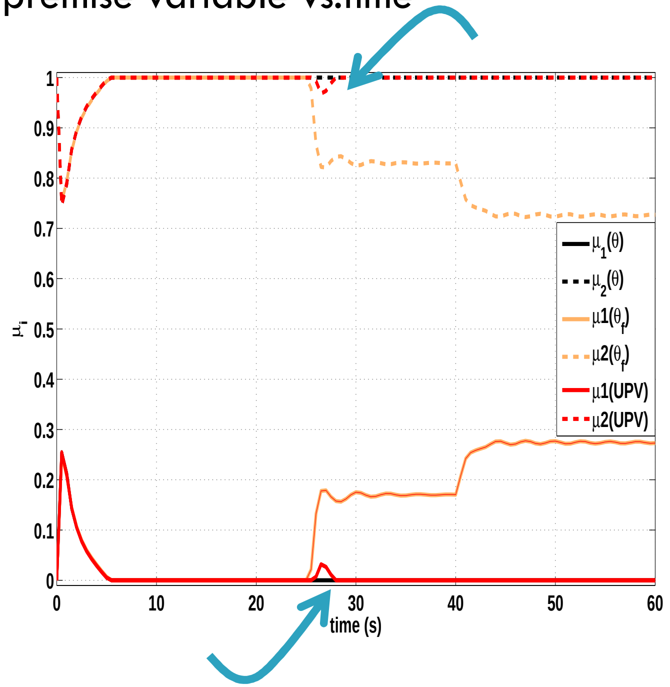
Fuzzy-based Model Predictive Reconfigurable Control (8/10):

Example: System ∑

Estimation error vs.time



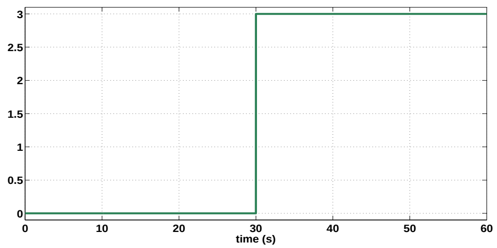
Variation of the activation functions according to the premise variable vs.time



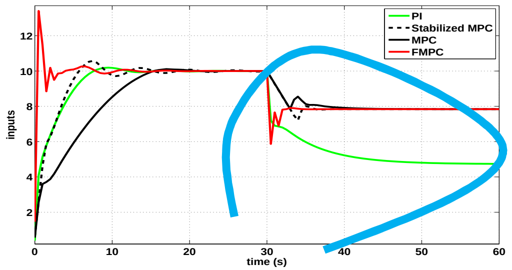
Fuzzy-based Model Predictive Reconfigurable Control (9/10):

Example: System ∑

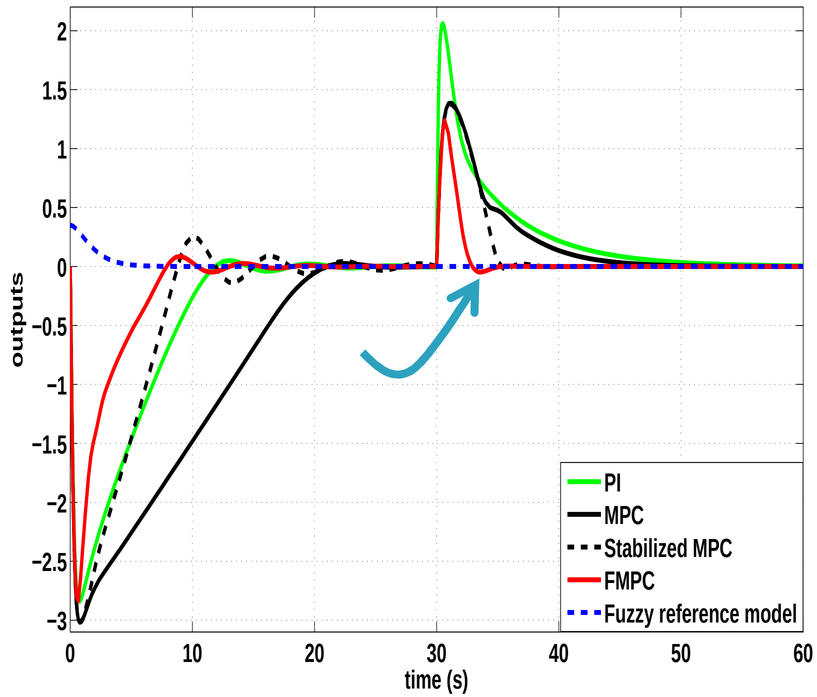
Sensor fault: UPV



Control signal vs.time

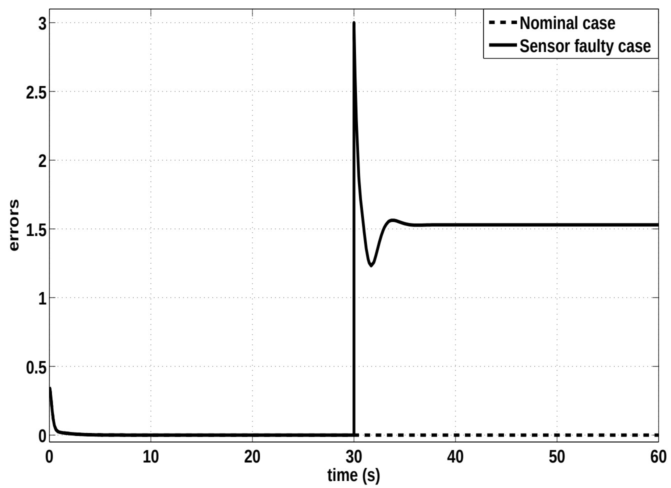


Output response vs.time

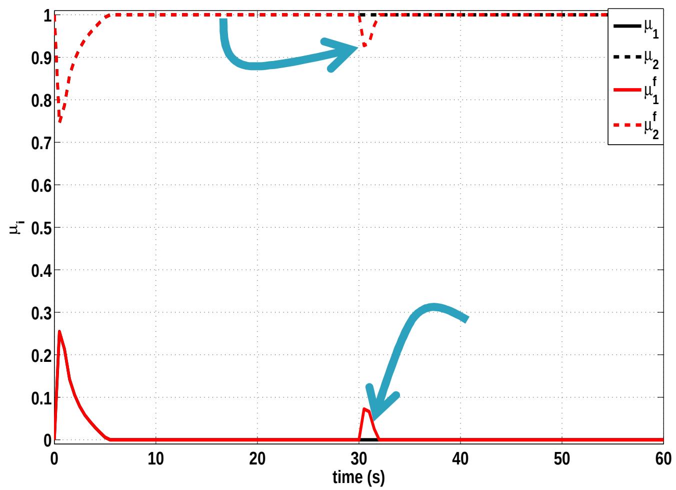


Fuzzy-based Model Predictive Reconfigurable Control (10/10): Example: System ∑

Estimation error vs.time



Variation of the activation functions according to the premise variable vs.time



Conclusion

❖A new FTC strategy for nonlinear processes is proposed. It integrates the state estimation and the dynamic optimizer based on interpolation control to guaranty the stabilization of the faulty plant.

❖The proposed FTC design scheme cancels faults (Sensors & Actuators) properly.

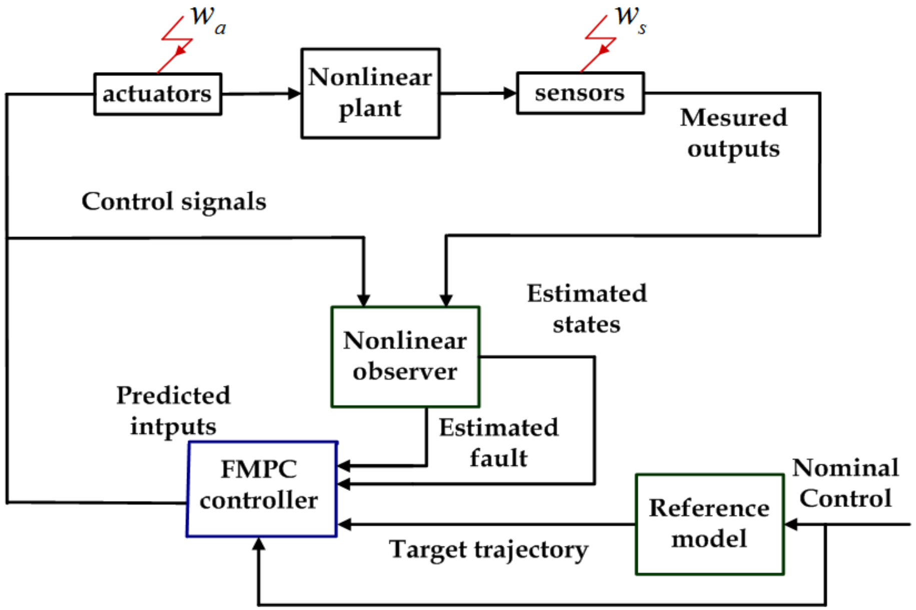
❖The reconfigurable FMPC preserves stability conditions in the nominal and faulty cases.

❖The method proposed to obtain a convex hull is a very conservative embedding procedure.

Outlook: FTC strategy

Lyapunov-Based Nonlinear Observers (T-S observer):

- Estimate the fault, and the unmeasured states using a T-S observer.
- The Gain of the controller , and the gains of the observer are obtained by solving a linear matrix inequality (LMI) derived from the Lyapunov theory.
- Application of the proposed approach FMPC to an electric vehicle fitted with an extension of autonomy.



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Fault tolerant control: a Takagi-Sugeno (T-S) -model based predictive approach

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