ÉCOLE SUPÉRIEURE D'ÉLECTRICITÉ
Supélec

Un tour d'horizon sur l'invariance positive et ses applications (non seulement) à la commande prédictive

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Outline

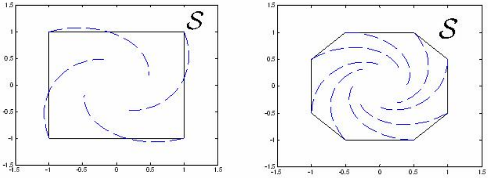
1. Positive invariance: definitions, construction and remarkable classes
   - Autonomous dynamics
   - Robust positive invariance
   - Controlled invariance
   - Relaxed invariance notions
2. Applications
   - Constrained Control
   - Model Predictive Control
   - Interpolation based control
   - Fault detection and isolation
   - Collision avoidance
3. Conclusions

Positive invariance

Definition

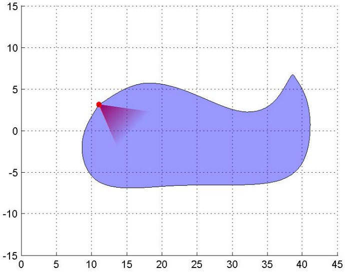
A set $\mathcal{X} \in \mathbb{R}^n$ is positively invariant with respect to the system $\delta x = f(x(t), u(t), w(t))$ if for any $x_0 \in \mathcal{X}$, the solution $x(t, x_0)$ satisfies $x(t, x_0) \in \mathcal{X}$ for $k \in \mathbb{N}$.

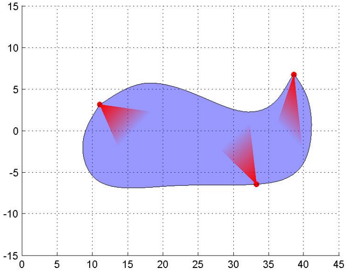
This basic definition is applies for both continous and discrete-time dynamics.



Invariance Conditions for Continous time

Basically, in continuous time the invaraince conditions are related to the cone of the feasible directions on the points on the frontier.



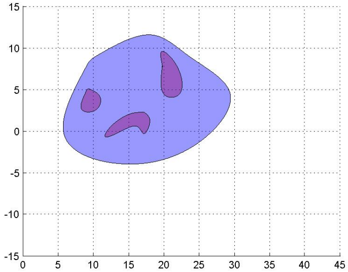


Positive invariance

In discrete-time the invariance cannot be verified by analysing the frontier of the set. In order to obtain geometrical necessary and sufficient conditions the image of the complete set has to be used.

Definition

A set $\mathcal{X} \in \mathbb{R}^n$ is positively invariant with respect to the system $x(k+1) = f(x(k))$ iff $f(\mathcal{X}) \subset \mathcal{X}$.



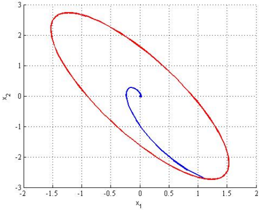
Special Families of Positive Invariant Sets

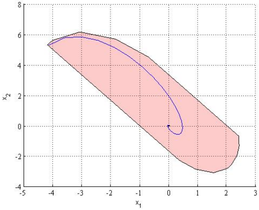
- Ellipsoidal invariant sets.
- Related to the existence of quadratic Lyapunov functions.

$$P = \left\{ x \in \mathbb{R}^n \mid x^T P x < 1 \right\}$$
$$P = P^T > 0$$

- Polyhedral invariant sets.
- Related to the existence of polytopic Lyapunov functions.

$$P = \{ x \in \mathbb{R}^n \mid Cx \leq W \}$$

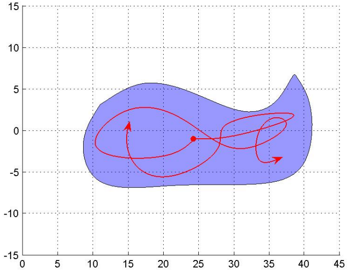




Relationship between invariance and viability (see J.P. Aubin)

Evolutions' viability

- $x(\cdot)$ is viable in K on $[0, T]$ if $\forall t \in [0, T]$ we have $x(t) \in K$
- $x(\cdot)$ is not viable in K on $[0, T]$ if $\exists t \in [0, T]$ for wich $x(t) \notin K$



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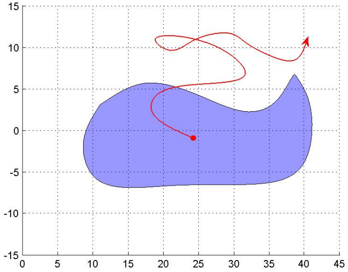

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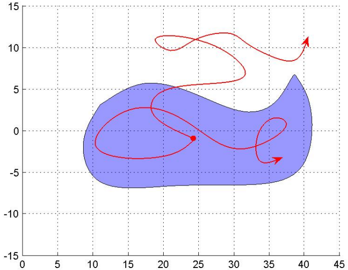
x_0 is viable under K for $[0, T]$ if $\exists x_0(.)$ viable



K is locally viable under F if $\forall x_0 \in K$ and $T > 0$ there exists a viable solution on $[0, T]$ to an evolution starting at x_0

Relationship between invariance and viability (see J.P. Aubin)

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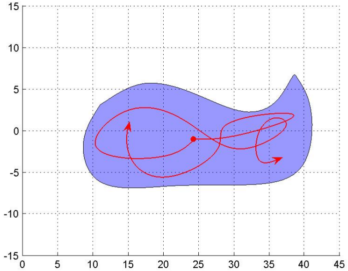
x_0 is invariant under K for $[0, T]$ if $\forall x_0(.)$ is viable



K is locally invariant under F if $\forall x_0 \in K$ and $T > 0$ all solutions on $[0, T]$ to an evolution starting at x_0 are viable

The Maximal Output Admissible Set O_∞

Given a discrete-time linear time-invariant system

$$\begin{aligned} x(k+1) &= Ax(k),\ x(0) = x_0,\ x(k) \in \mathbf{R}^n \\ y(k) &= Cx(k),\ y(k) \in Y = \left\{x : Hx \leq K\right\} \end{aligned}$$

the maximal admissible (and positive invariant) se is defined as

$$O_\infty(A, C, Y) = \left\{x \in \mathbf{R}^n : CA^t x \in Y, \forall t \in \mathcal{I}^+\right\}$$

The question is: can O_∞ practically be constructed?

Finite Determinated O_∞ Set

$$O_t(A, C, Y) = \left\{ x \in \mathbf{R}^n : CA^k x \in Y, \forall k = 0 \cdots t \right\}$$

$O_\infty = O_N$ for a finite N?

Theorem (Gilbert and Tan, '91)

If the following assumptions hold:

1. A *is asymptotically stable*
2. *the pair* (C, A) *is observable*
3. Y *is bounded*
4. $0 \in Int(Y)$

then O_∞ *is finitely determined.*

Similar result hold for the polytopic uncertainties on the linear dynamics (convex difference inclusion).

A major result for invariance verification

Theorem (Bitsoris 1988)

The convex polyhedral set:

$$\mathcal{P} = \{x \in \mathbb{R}^n | Fx \leq w\},$$

with $F \in \mathbb{R}^{r \times n}$, $w \in \mathbb{R}^r$, *is an invariant set with respect to*

$$x(k+1) = Ax(k),$$

with $A \in \mathbb{R}^{n \times n}$, *if and only if there exists a matrix* $H \in \mathbb{R}^{r \times r}$ *with nonnegative elements such that:*

$$FA = HF$$

and

$$Hw \leq w.$$

Classical Results Concerning Polyhedral Set Invariance

Definition (Minkowski functions)

Consider a convex and compact polyhedral set containing the origin:

$$\mathcal{P} = \{x \in \mathbb{R}^n | Fx \leq 1\},$$

with $F \in \mathbb{R}^{r \times n}$, $w \in \mathbb{R}^r$. The polyhedral function associated to $\mathcal{P}$ is called a Minkowski function:

$$V(x) = \max_{j \in \mathbb{Z}_{[1,r]}} \{\max\{\{(Fx)_j\}, 0\}\}.$$

where $\{(Fx)_j\}$ denotes the j^{th} element of Fx. This function can be seen as a vector infinity-norm (Kiendl et al. 1992):

$$V(x) = \|\max\{Fx, 0\}\|_\infty.$$

Classical Results Concerning Polyhedral Set Invariance

Theorem

The Minkowski function of an invaraint set $\mathcal{P}$ for a LTI system can be used as polyhedral Lyapunov candidate (Blanchini, 1995).

Consider $\varepsilon \in \mathbb{R}_{[0,1)}$. One of the statements of the Lyapunov stability theorem is:

$$V(x(k+1)) - \varepsilon V(x(k)) \leq 0$$

If $\varepsilon = 1$ the function $V(x)$ is called a **weak Lyapunov function**. The existence of a weak Lyapunov function does not imply global asymptotic stability but it induces invariant sets (by the).

Other existing results

- A link between the eigenstructure and the complexity of the invaraint set (Molchanov, Bobyleva and Pyatnitskii).
- Conditions for the invariance of the complement of a convex set (related to instability).
- Invariant sets for linear dynamics with polytopic uncertainty.
- Parameterization of the invariant sets for the reference tracking case.

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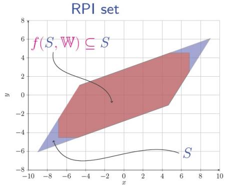
Robust positive invariance

Consider a discrete-time invariant system in $\mathbb{R}^n$ affected by bounded disturbances $w(k) \in \mathbb{W}$

$$x(k+1) = f(x(k), w(k)), \text{ with } f(0,0) = 0.$$

RPI set



Definition (RPI set)

A set S is called Robust Positively Invariant (RPI) iff $\forall x(0) \in S$ and $\forall w(k) \in \mathbb{W}$ then $x(k) \in S$ for $k > 0$.

Definition (mRPI set)

A set Ω_∞ is called minimal Robust Positively Invariant (mRPI) iff it is a RPI set in $\mathbb{R}^n$ contained in every RPI set of the system.

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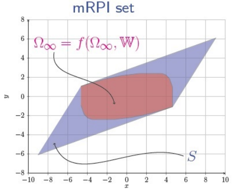

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Ultimate bounds

Theorem (Ultimate bounds – discrete-time case)

Consider the stable system $x^+ = Ax + Bw$. *Let there be the Jordan decomposition* $A = V\Lambda V^{-1}$ *and assume that* $\left|w(k)\right| \leq \bar{w}, \forall k \geq 0$. *Then there exists* $l\left(\epsilon\right)$ *such that for all* $k \geq l$*:*

$$\begin{aligned} |V^{-1}x(k)| &\leq (I - |\Lambda|)^{-1}|V^{-1}B|\bar{w} + \epsilon \\ |x(k)| &\leq |V|(I - |\Lambda|)^{-1}|V^{-1}B|\bar{w} + |V|\epsilon \end{aligned}$$

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Proof (Kofman et al, 2007):
We can write

$$\begin{aligned} x^+ &= Ax + Bw = V\Lambda V^{-1}x + Bw \\ V^{-1}x^+ &= \Lambda V^{-1}x + V^{-1}Bw \\ |z^+| &\leq |\Lambda z + V^{-1}Bw| \leq |\Lambda|z + |V^{-1}B|\bar{w} \end{aligned}$$

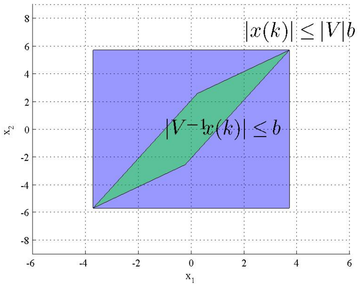
and, then:

$$|V^{-1}x| \leq (I - |\Lambda|)^{-1}|V^{-1}B|\bar{w} + \epsilon$$

Exemplification for a $\mathbb{R}^2$ case

$$x(k+1) = Ax(k) + Bw(k)$$

where $\left|w(k)\right| \leq 1$



$$A = \begin{bmatrix} 0.0241 & 0.4184 \\ -0.7869 & 1.2759 \end{bmatrix} \qquad b = \begin{bmatrix} 0.90 \\ 1.75 \end{bmatrix}$$

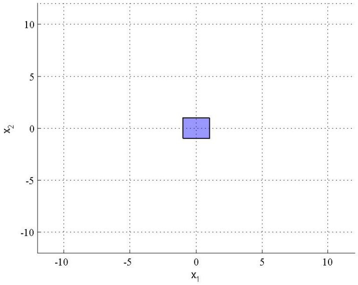
$$B = \begin{bmatrix} 0.8462 \\ 0.5252 \end{bmatrix} \qquad |V|b = \begin{bmatrix} 3.70 \\ 5.68 \end{bmatrix}$$

mRPI ϵ inner invariant approximations

The mRPI is not finetly determined : $\Omega = \bigoplus_{i=0}^{i=\infty} A^i \Delta$

The iterative computation of an inner RPI approximation

$$\Phi_{k+1} = A\Phi_k \oplus \Delta, \quad \Phi_0 = \{0\}$$



Theorem (Rakovic et al)

For any $\epsilon \geq 0$ *it exists* $s \in \mathbb{N}^+$ *such that*

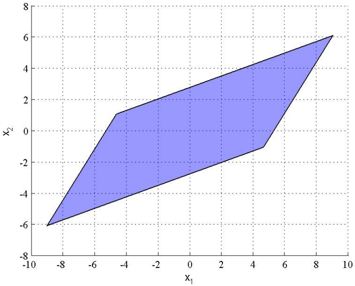
$$\Phi_s \subset \Omega \subset (1-\alpha(s))^{-1} \Phi_s(\epsilon)$$

mRPI ϵ outter invariant approximations

The mRPI is not finetly determined : $\Omega = \bigoplus_{i=0}^{i=\infty} A^i \Delta$

The iterative computation of an outter RPI approximation

$$\Phi_{k+1} = A\Phi_k \oplus \Delta, \quad \Phi_0 = \Psi$$



Theorem (Olaru et al)

For any $\epsilon \geq 0$ it exists $s \in \mathbb{N}^+$ such that

$$\Omega \subset \Phi_s \subset \Omega \oplus \mathbb{B}_p^n(\epsilon)$$

The construction of robust invaraint sets for nonlinear case

- The idea for nonlinear dynamics in general

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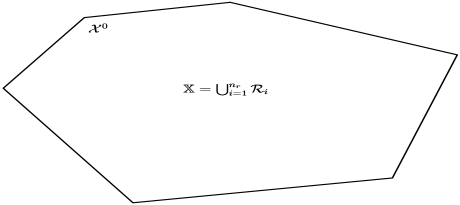

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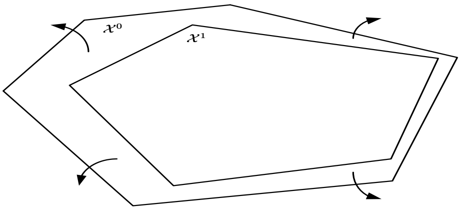

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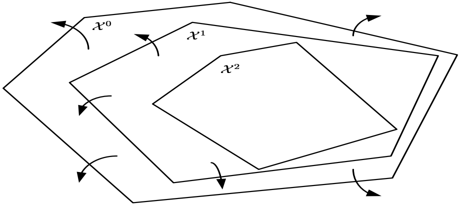

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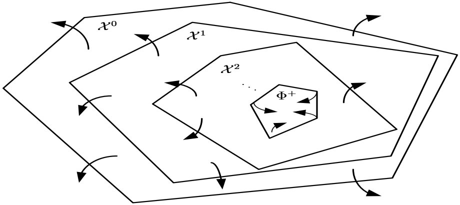

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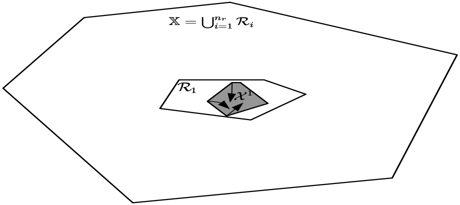

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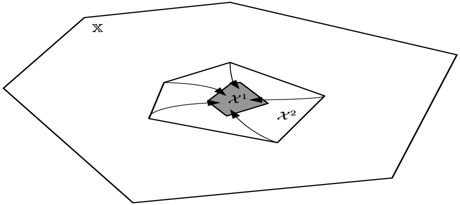

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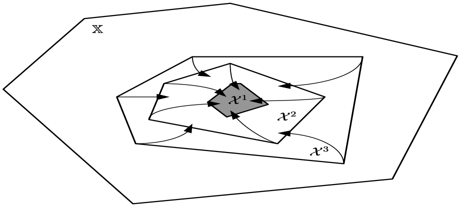

The construction of robust invaraint sets for nonlinear case

- The idea for nonlinear dynamics in general
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- Approach:
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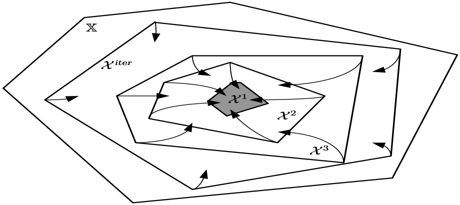
  - Use the Hausdorff distance to analyse the convergence



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Controlled invariance

Definition

A set $\mathcal{X} \in \mathbb{R}^n$ is controlled positively invariant with respect to the dynamical system $x^+ = f(x(t), u(t))$ if for any $x_0 \in \mathcal{X}$, there exists a control sequence $u(t), \forall t \geq 0$ such that the solution $x(t)$ satisfies $x(t) \in \mathcal{X}$ for all $k \in \mathbb{N}$.

Definition

A set $\mathcal{X} \in \mathbb{R}^n$ is robust controlled positively invariant with respect to the dynamical system $x^+ = f(x(t), u(t), w(t))$ if for any $x_0 \in \mathcal{X}$, there exists a control sequence $u(t), \forall t \geq 0$ such that the solution $x(t)$ satisfies $x(t) \in \mathcal{X}$ for all $k \in \mathbb{N}$, $\forall w(t) \in W$ where W is the set of admissible disturbances.

Controlled invariance - remarks and construction

- Different formulations with respect to the information available for control (analogy with the game theoretic approaches)
- The controlled invariance will be a useful concept in view of the control design (see MPC, vertex control, etc.)
- Closely related to the reachability analysis and the dynamic programming

Controlled invariance - remarks and construction

For the constraction of the *controlled invaraint sets* the main idea will be to use the reachability analysis with the backward set dynamics as a main tool.

- For LTI dynamics $x^+ = Ax + Bu$ in presence of state $(x_t \in X)$ and input $(u_t \in U)$ constraints if functioning well via set iterates. For example in the discrete-time this leads to:

$$\mathcal{S}_k = (A^{-1}\mathcal{S}_{k-1} \oplus A^{-1}B(-U)) \cap X \tag{1}$$

- The finite determination of the maximal controlled invariant set is not guaranteed as long as this might not be a closed set. However a ϵ- approximation can be obtained.
- For the construction of *robust controlled invariant sets* the set iteration needs to use the *Pontryagyn difference*. A special attention should be given to this operation as long as this is not representing the inverse of the Minkowski sum.

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Alternative invariance notions

Definition (Periodic invariance - Lee and Kouvaritakis)

The set $\Omega \subset \mathbb{R}^n$ containing the origin is called *periodic invariant* with respect to the system $x^+ = f(x)$ if there exists a positive number $p \in \mathbb{Z}_+$ such that for any $x_k \in \Omega$ it holds that $x_{k+p} \in \Omega$.

Definition (Cyclic invariance - Lombardi et al)

The ordered family of sets containing the origin $\mathcal{O} = \{\Omega_1, \ldots, \Omega_p\} \subseteq \{\mathbb{R}^n\}^p$ is called *cyclic invariant* with respect to $x^+ = f(x)$ if for any $x_k \in \Omega_i$, $i \in \mathbb{Z}_{[1,p]}$ it holds that $x_{k+p} \in \Omega_i$ and $x_{k+j} \in \Omega_{i+j-\left\lfloor \frac{i+j}{p} \right\rfloor p}$ for $j \in \mathbb{Z}_{[1,p-1]}$.

Definition (Invariant family of sets - Rakovic et al)

The family of sets containing the origin $\mathcal{O} = \{\Omega_1, \ldots, \Omega_r\} \subseteq \{\mathbb{R}^n\}^r$ with $r \in \mathbb{Z}_+$ is called *invariant* with respect to $x^+ = f(x)$ if for any $x_k \in \Omega_i$, $i \in \mathbb{Z}_{[1,p]}$ it holds that $x_{k+p} \in \Omega_i$ and $x_{k+l} \in \bigcup_{j=1}^{r} \Omega_j$ for $l \in \mathbb{Z}_{[1,p-1]}$.

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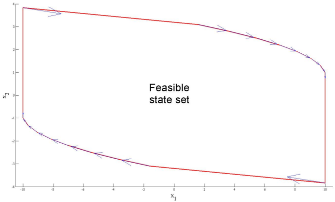
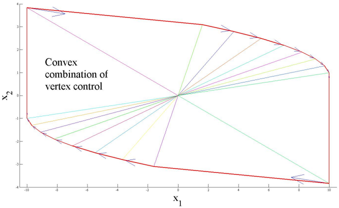
Vertex control based on a controlled invariant set

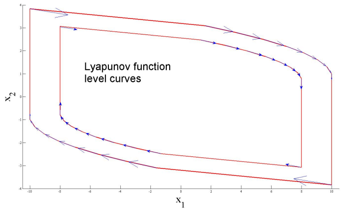
- Gutman and Cwikel (1986)
- The necessary and sufficient condition for stabilizing a linear discrete time system with polyhedral state and control constraints is that at each vertex of the feasible set P_N there exists a feasible control signal $u \in U$ that brings the state to *int(*P_N*)*.





Vertex control based on a controlled invariant set

A stabilizing controller is given by the convex combination of vertex controls. There exists a polyhedral Lyapunov function given by shrunken images of P_N.



- Blanchini (1992) extended to the uncertain plant case.
- The extension to the nonlinear homogenous dynamics is possible as long as a convex (polyhedral) controlled invariant set exists.

Outline

1 Positive invariance: definitions, construction and remarkable classes
- Autonomous dynamics
- Robust positive invariance
- Controlled invariance
- Relaxed invariance notions

2 Applications
- Constrained Control
- Model Predictive Control
- Interpolation based control
- Fault detection and isolation
- Collision avoidance

3 Conclusions

The MPC design

Consider the discrete-time system:

$$x_{k+1} = g(x_k, u_k) \tag{2}$$

where $g(0,0) = 0$, $u_k \in U$ and $x_k \in X$ with U and X a convex, compact subsets of $\mathbb{R}^m$ and $\mathbb{R}^n$ respectively containing the origin in their interiors. Consider also the optimization problem

$$\begin{array}{l} \min\limits_{u_k,\dots,u_{k+N-1}} \sum\limits_{i=k}^{k+N-1} l(x_i, u_i) + T(x_{k+N}) \\ x_{i+1} = g(x_i, u_i), \forall i \in \mathbb{Z}_{[k,k+N-1]} \\ x_i \in X,\ u_i \in U,\ x_{k+N} \in \Omega \end{array} \tag{3}$$

MPC

The set $\Omega \in \mathbb{R}^n$ contains the origin in the interior and represents the terminal constraint.

A predictive control applies the first component of the optimal sequence $u_{mpc} = u_k^*$ and restarts the optimization.

The classical MPC stability proof relies on invariance

The next results underlines the importance of the positive invariance for the stability of the resulting closed-loop system.

Theorem

If there exists a function $\mathcal{K} : \Omega \rightarrow U$ *such that:*

1. $g(x, \mathcal{K}(x)) \in \Omega, \forall x \in \Omega$ *(controlled positive invariance of* Ω*)*
2. $T(g(x, \mathcal{K}(x))) + l(x, \mathcal{K}(x)) - T(x) \leq 0$ *(local Lyapunov function)*

then the control law u_{mpc} *obtained by solving the receding horizon optimization* (3) *guarantees the asymptotic stability of the resulting closed-loop system* $x_{k+1} = g(x_k, u_{mpc}(x_k))$.

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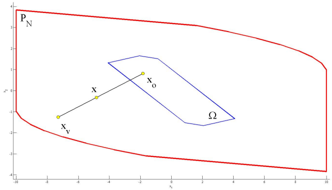
Interpolation based control - Nguyen et al

Let $x^+ = Ax + Bu$ in presence of input and state constraints. Given P_N a controlled invariant set and Ω the MOAS for $x^+ = (A + BK)x$ then $\forall x \in P_N$ can be rewritten as a convex combination $x = cx_v + (1 - c)x_o$ with $x_v \in P_N$, $x_o \in \Omega$



Theorem

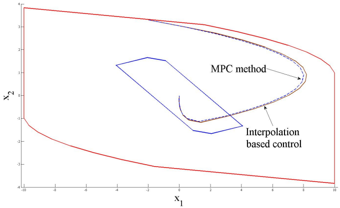
$u = cu_v + (1 - c)u_o$ is recursively feasible for all $x \in P_N$.

Theorem

$c^*(x) = min(c)$ is a positive and non-increasing Lyapunov function for the system in closed loop with $u_{interp} = c^* u_v + (1 - c^*)u_o$.

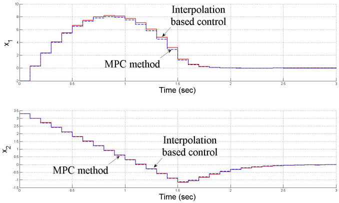
Interpolation based control - Nguyen et al

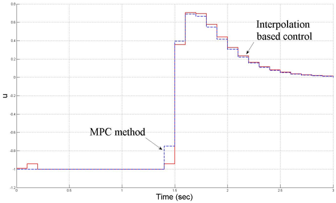
- Low complexity for the control based on optimization
- Similar performances with MPC



Interpolation based control - Nguyen et al

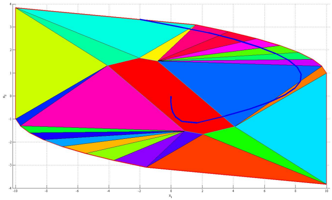
- Low complexity for the control based on optimization
- Similar performances with MPC

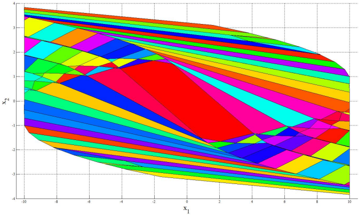




Interpolation based control - Nguyen et al

- Low complexity for the control based on optimization
- Similar performances with MPC
- Attractive explicit solution





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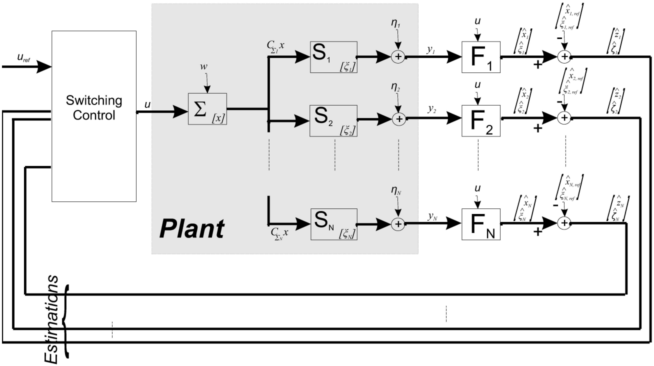
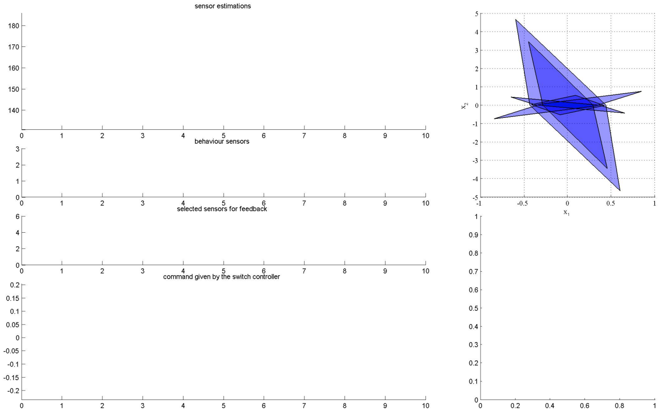
Multisensor fault detection usign invariant sets



see the results of Seron, De Dona, Stoican and Olaru

Multisensor control of the inter-vehicle distance



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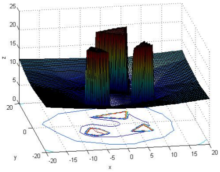
Invariant sets for collision avoidance (Prodan et al)

Consider an agent described by

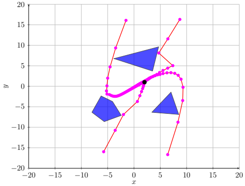
$$A_i = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{\mu_i}{m_i} & 0 \\ 0 & 0 & 0 & -\frac{\mu_i}{m_i} \end{bmatrix}, \; B_i = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{m_i} & 0 \\ 0 & \frac{1}{m_i} \end{bmatrix},$$

modeling the pedestrian flow, with $x_i(k) = [p_i^T(k) \; v_i^T(k)]^T$.

Potential field constructed based on the approximation of mRPIs



Agent governed by MPC with collision avoidance constraints



Other control application

- Reference tracking (reference governor design)
- Hybrid system modeling/design
- Probabilistic analysis: prababilistic invariant sets (DeDona et al 2013)
- Decentralized control: via invariant families of sets
- Delay independent stability results
- ...

Conclusions

The invariant sets are useful in the analysis and the design of control!

Much is to be done from the construction point of view.

Thanks to F. Stoican and I. Prodan for the artwork and some of the slides used in this presentation.

Search Server time: Monday, March 24, 2014 23:16:07 PDT Time left to final submissions deadline: 35:00:43:52 Day Hr Min Sec

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Workshop # 5

Prev Next

Interpolation-based techniques for constrained control: from improved vertex control to robust model predictive control alternatives.

Organizers: Hoai-Nam Nguyen, Sorin Olaru and Per-Olof Gutman

The present workshop intends to present the latest developments on a topic with a renewed interest in the last years: the interpolation based control. Here, the strength of interpolation based control will be demonstrated for a large class of fundamental control problems covering the state/output feedback control of uncertain linear systems, including time varying and parameter varying systems with a specific attention to the presence of constraints on the outputs, states, and control inputs. The one-day event will present historical elements of constrained control and bring gradually the auditorium to the recent challenges in the optimization-based control design related topic. The auditorium will be introduced to the theoretical foundations of a generic interpolation scheme and subsequently will be exposed to the analysis of structural implications and the computational aspects of the resulting control law for various classes of dynamical systems. The participants will have the opportunity to compare the features of novel methodologies with respect to classical alternatives as vertex control or Model Predictive Control in their implicit or explicit formulations. In addition, the participants will attain the basis to apply computational tools to the design of robust interpolating controllers for uncertain linear systems with state and control constraints.

Speakers
Duration and date
Location
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Final submission & Early Bird registration
open: March 13, 2014
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Best Student Award nominations
begin: March 13, 2014
end: May 10, 2014

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