Hybrid approach for linear time-varying systems:

Hovering for impulsive spacecraft rendezvous

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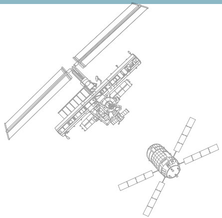
Acknolewdgements



The following researchers are involved in the presented work:

- **M. Brentari (Master thesis)**
- **D. Arzelier,**
- **L. Zaccarian,**

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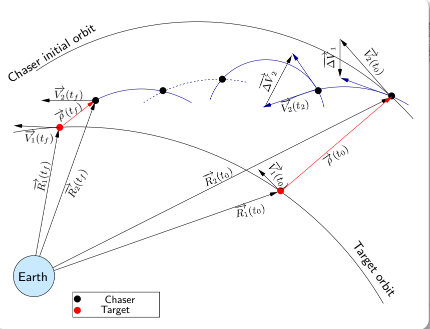
Orbital rendezvous definition

Orbital Rendezvous consists of:

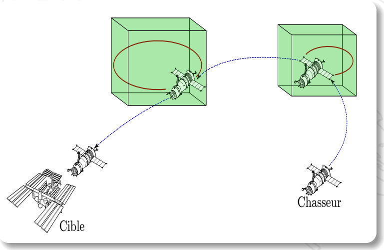
- Keplerian relative motion;
- Relative navigation;
- Passive Target;
- Fixed-time;
- Usage of ergols thrusters :
  - the control is modeled by impulsive signals,
  - Instantaneous velocity change,
  - Sequence of coasting arc limited by thruster impulses;



Rendez-vous problem

- Steering the chaser spacecraft from a state A to state B;
- Assuming some operating constraints (Actuators bounds, safety constraints ...);

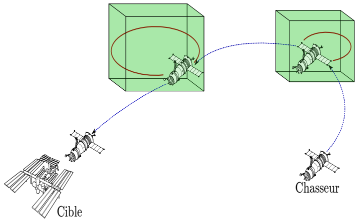
Hovering problem for rendez-vous



Needs of stationkeeping abilities for different purposes

- waiting for orders from the ground station to proceed the mission
- observation mission of passive spacecraft

Hovering problem for rendez-vous



Needs of stationkeeping abilities for different purposes

- waiting for orders from the ground station to proceed the mission
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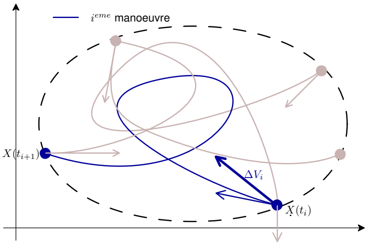
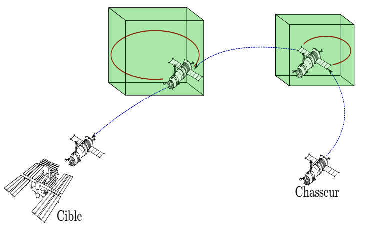
Literature Background:

- Computing maneuver making the spacecraft bouncing on the frontier of the hovering subset.



Hovering problem for rendez-vous



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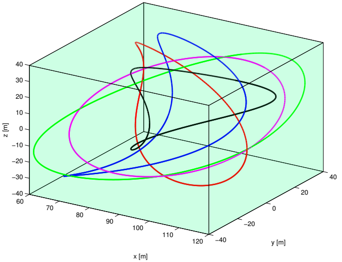
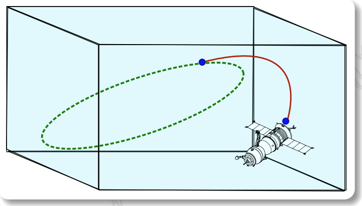


Our approaches:

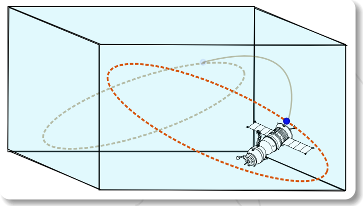
- Taking advantages of natural periodic orbits to decrease the consumption.

Periodic orbits approaches for hovering

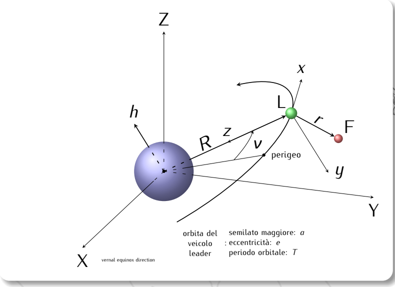
- **Hybrid control approach**
- Stabilisation of a particular admissible periodic orbit



- **MPC control approach**
- Stabilisation toward the set of admissible periodic orbit



Relative motion dynamics I



Space mechanics assumptions

- Keplerian relative motion: perfectly spheric Earth, no aerodynamic drag, no influence from sun and moon
- Passive Target evolving on elliptical orbits;

Non linear dynamics are given by:

$$\ddot{x} = 2\,\dot{\nu}\,\dot{z} + \ddot{\nu}\,z + \dot{\nu}^2 x - \frac{\mu\,x}{\sqrt{(x^2+y^2+(R-z)^2)^3}},$$

$$\ddot{y} = -\frac{\mu\,y}{\sqrt{(x^2+y^2+(R-z)^2)^3}},$$

$$\ddot{z} = -2\,\dot{\nu}\,\dot{x} - \ddot{\nu}\,x + \dot{\nu}^2 z - \frac{\mu(R-z)}{\sqrt{(x^2+y^2+(R-z)^2)^3}} + \frac{\mu}{R^3}.$$

State space vector

$$X(t) = [x,\; y,\; ,z\;,\dot{x}\;,\dot{y}\;,\dot{z}]$$

Relative motion dynamics II

Linearization assumption:

- Relative navigation: distance between satellite is small with respect to the radius of the target spacecraft.

Linearized equation for the relative motion can be obtained:

Tschauner-Hempel equations

$$\ddot{x} = \dot{\nu}\dot{z} + \ddot{\nu}z + \dot{\nu}^2 x - \frac{\mu}{R^3}x$$

$$\ddot{y} = -\frac{\mu}{R^3}x$$

$$\ddot{z} = -2\dot{\nu}\dot{x} - \ddot{\nu}x + \dot{\nu}^2 z + 2\frac{\mu}{R^3}z$$

Substitution of the time variable t by the target true anomaly ν and the change of variable

$$\tilde{X}(\nu) = \begin{bmatrix} (1 + e\cos\nu)\mathbb{I}_3 & 0_3 \\ -e\sin\nu\mathbb{I}_3 & \frac{(1+e\cos\nu)}{\dot{\nu}}\mathbb{I}_3 \end{bmatrix} X(t) \quad \text{with} \quad \dot{\nu} = \frac{2\pi}{T}\frac{(1 + e\cos\nu)^2}{(1 - e^2)^{3/2}}$$

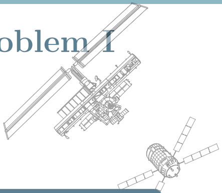
Simplified Tschauner-Hempel eqs.

$$\tilde{x}'' = 2\tilde{z}'$$

$$\tilde{y}'' = -\tilde{y}$$

$$\tilde{z}'' = -2\tilde{x}' + \frac{3}{1 + e\cos\nu}\tilde{z}$$

Hybrid Modeling for the rendezvous problem I



Time-varying model

$$\tilde{X}' = A(\nu)\tilde{X} \qquad \text{in free motion}$$

$$\tilde{X}^+ = \tilde{X} + \begin{bmatrix} 0_{3\times 3} \\ I_3 \end{bmatrix} u \qquad \text{at impulse execution}$$

$$A(\nu) = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \dfrac{3}{1+e\cos\nu} & -2 & 0 & 1 \end{bmatrix}$$

Hybrid Modeling for the rendezvous problem II

Floquet-Lyapunov transformation

$\xi = G(\nu)\tilde{X} := S(\nu)\,C(\nu)\tilde{X}$,

- $$C(\nu) := \begin{bmatrix} 0 & c_\nu & 0 & 0 & -s_\nu & 0 \\ 0 & s_\nu & 0 & 0 & c_\nu & 0 \\ 1 & 0 & -\frac{3es_\nu(1+\rho)}{\rho(e^2-1)} & \frac{es_\nu(1+\rho)}{e^2-1} & 0 & \frac{\rho^2-ec_\nu-3}{e^2-1} \\ e & 0 & -3s_\nu & s_\nu(1+\rho) & 0 & c_\nu\rho \\ 0 & 0 & \frac{3(c_\nu+e)}{e^2-1} & -\frac{c_\nu(1+\rho)+e}{e^2-1} & 0 & s_\nu\rho \\ 0 & 0 & -\frac{3(3ec_\nu+e^2+2)}{e^2-1} & \frac{3\rho^2}{e^2-1} & 0 & -\frac{3es_\nu\rho}{e^2-1} \end{bmatrix}$$

Trajectories parametrization
$c_\nu = \cos(\nu)$
$s_\nu = \sin(\nu)$

- $$S(\nu) := \left[\begin{array}{cc|cccc} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 & 0 & \frac{\sigma(\nu)}{(1-e^2)^{3/2}} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right], \quad \sigma(\nu) = (\nu-\nu_f) - n(t-t_f)$$

Floquet-Lyapunov

Hybrid Modeling for the rendezvous problem III

New coordinates ξ dynamics

LTI flow dynamics: $\xi' = A_\xi \xi$ in free motion
$\xi' = A_\xi \xi + B_\xi(\nu) u$ at impulse execution

$$A_\xi = (1\text{-}e^2)^{-3/2} \left[\begin{array}{cc|cccc} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right], \; B_\xi(\nu) = \frac{1}{k^2\rho^2(1\text{-}e^2)} \begin{bmatrix} 0 & -(1\text{-}e^2)\rho s_\nu & 0 \\ 0 & (1\text{-}e^2)\rho c_\nu & 0 \\ -e(1+\rho)\rho s_\nu - \frac{3\sigma\rho^3}{(1\text{-}e^2)^{3/2}} & 0 & \frac{3\sigma e \rho^2 s_\nu}{(1\text{-}e^2)^{3/2}} - \rho^3 + \rho^2 - 2\rho \\ (1\text{-}e^2)(1+\rho)\rho s_\nu & 0 & (1\text{-}e^2)\rho^2 c_\nu \\ (1+\rho)\rho c_\nu + e\rho & 0 & -\rho^2 s_\nu \\ -3\rho^3 & 0 & 3e\rho^2 s_\nu \end{bmatrix}$$

Equilibrium and periodicity

- $\xi_6 = 0$ implies that ξ is constant
- Constant $\xi \iff$ Periodic orbits

Hybrid Control laws

New hybrid model

$(\xi,\nu,\tau)\in\mathcal{C}$	$(\xi,\nu,\tau)\in\mathcal{D}_\nu$	$(\xi,\nu,\tau)\in\mathcal{D}_u$
$\left\{\begin{array}{l}\xi' = A_\xi \xi,\\ \nu' = 1,\\ \tau' = -1,\end{array}\right.$;	$\left\{\begin{array}{l}\xi^+ = \xi,\\ \nu^+ = 0,\\ \tau^+ = \tau,\end{array}\right.$;	$\left\{\begin{array}{l}\xi^+ = \xi + B_\xi(\nu)\gamma_u(\xi,\nu)\\ \nu^+ = \nu,\\ \tau^+ = \gamma_\tau(\xi,\nu),\end{array}\right.$;

where

- $\mathcal{C} := (\mathbb{R}^6 \times [0,2\pi] \times [0,2\pi]) \setminus \mathcal{D}_\nu \cap \mathcal{D}_u$
- $\mathcal{D}_\nu := \mathbb{R}^6 \times \{2\pi\} \times [0,2\pi]$,
- $\mathcal{D}_u := \mathbb{R}^6 \times [0,2\pi] \times \{0\}$,

The degree of freedom this hybrid approach are

Control law $\gamma_u(\cdot)$;

Trigger law $\gamma_\tau(\cdot)$.

1. **Periodic bi-impulsive control:**
   - $\gamma_u(\cdot)$ based on transition inversion;
   - $\gamma_\tau(\cdot)$ fixed *a priori*.

2. **Periodic norm-minimizing control:**
   - $\gamma_u(\cdot)$ minimizes the 2-norm of impulse;
   - $\gamma_\tau(\cdot)$ fixed *a priori*.

3. **Non periodic bi-impulsive control:**
   - $\gamma_u(\cdot)$ based on transition inversion;
   - $\gamma_\tau(\cdot)$ optimized over one period.

Periodic bi-impulsive control law

Control law γ_u

- γ_u computes $u(\nu_i)$ and $u(\nu_i + \bar{\nu})$ is such that:

$$\xi_{ref} - \xi^{+}(\nu_i + \bar{\nu}) = \tilde{\xi}^{+}(\nu_i + \bar{\nu}) = 0$$

- γ_u is given by

$$\boxed{\gamma_u(\xi(\nu_i), \bar{\nu}) = [-I \quad 0] M(\nu, \bar{\nu})^{-1} \tilde{\xi}.}$$

where $M(\nu, \bar{\nu}) := \left[\hat{B}(\nu) \quad \Phi(-\bar{\nu})\hat{B}(\nu + \bar{\nu})\right]$ $\quad \Phi(\mu) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & \mu(1-e^2)^{-3/2} \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

- Matrix M is inversible $\iff \bar{\nu} \neq k\pi \;, \quad k \in \mathbb{Z}$

Trigger law γ_τ

Periodic trig of the control law: $\gamma_\tau = \bar{\nu}$

Theorem

The reference periodic orbit ξ_{ref} is stable and attractive

Periodic bi-impulsive norm-minimizing control law

Control law γ_u

- γ_u computes $u(\nu_i)$ such that:

$$u^* = \underset{u}{\operatorname{argmin}} \, |\tilde{\xi}^+|^2, \qquad \text{such that:} \quad \tilde{\xi}^+ = \tilde{\xi} + \widehat{B}(\nu)u, \quad \tilde{\xi}_6^+ = 0$$

- γ_u is given by

$$\boxed{\gamma_u(\tilde{\xi}, \nu) = u_6 - \hat{B}_6^{\perp}(\nu)(\widehat{B}(\nu)\hat{B}_6^{\perp}(\nu))^{-L}(\tilde{\xi} + \widehat{B}(\nu)u_6) \qquad \text{with } u_6 = -\frac{\hat{b}_6(\nu)}{|\hat{b}_6(\nu)|^2}\tilde{\xi}_6,}$$

where

$$\hat{b}_6(\nu) = \begin{bmatrix} \frac{3\rho^3}{e^2-1} \\ 0 \\ \frac{3\rho^2 e \sin(\nu)}{1-e^2} \end{bmatrix}, \; \hat{B}_6^{\perp}(\nu) = \begin{bmatrix} e\sin(\nu) & 0 \\ 0 & 1 \\ \rho(\nu) & 0 \end{bmatrix}, \; \hat{b}_6(\nu)\hat{B}_6^{\perp}(\nu) = 0$$

Trigger law γ_τ

Periodic trig of the control law:$\gamma_\tau = \bar{\nu}$

Theorem

The reference periodic orbit ξ_{ref} is stable (attractivity non guaranteed)

Aperiodic bi-impulsive control law

Control law γ_u

- γ_u computes $u(\nu_i)$ and $u(\nu_i + \bar{\nu})$ is such that:

$$\xi_{ref} - \xi^{+}(\nu_i + \bar{\nu}) = \tilde{\xi}(\nu_i + \bar{\nu}) = 0$$

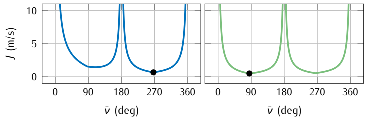
- γ_u is given by

$$\boxed{\gamma_u(\xi(\nu_i), \bar{\nu}) = [-I \quad 0] M(\nu, \bar{\nu})^{-1} \tilde{\xi}.}$$

with $M(\nu, \bar{\nu}) := \left[\hat{B}(\nu) \quad \Phi(-\bar{\nu})\hat{B}(\nu + \bar{\nu})\right] \qquad \bar{\nu} \neq k\pi \;, \quad k \in \mathbb{Z}$

Trigger law γ_τ

- Search for the best moment to trig:
  $\gamma_\tau(\tilde{\xi}, \nu) = \underset{\bar{\nu} \in [0, 2\pi]}{\operatorname{argmin}} \left| M(\nu, \bar{\nu})^{-1} \tilde{\xi} \right|_1$



Theorem

The reference periodic orbit ξ_{ref} is stable and attractive.

Numerical studies first example

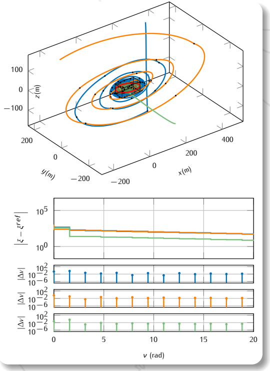
mission 1

$a = 7011$**km**, $e = 0.1$, $\color{red}{\bullet}\,\xi^{ref} = [5.2, 17.7, 98.0, 22.5, \text{-}17.6, 0]$,

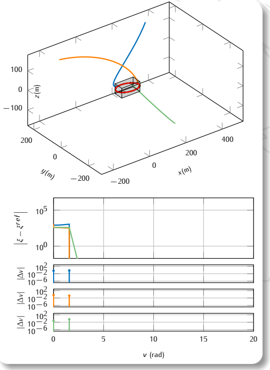
$\color{blue}{\bullet}\,\zeta_{01} = [500, 400, 10, 0, 0, 0]$, $\color{orange}{\bullet}\,\zeta_{02} = [-200, 100, 200, 0, 0, 0]$, $\color{green}{\bullet}\,\zeta_{03} = [100, -350, -20, 0, 0, 0]$,

with $\nu_{01} = 10^{\circ}$, $\nu_{02} = 150^{\circ}$, $\nu_{03} = 60^{\circ}$

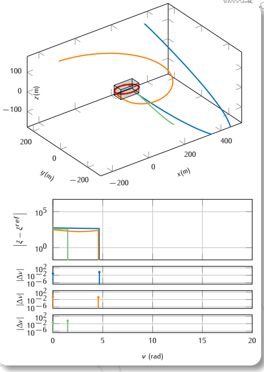
Periodic Norm-minimizing Control, $\bar{\nu} = \pi/2$



Periodic Bi-impulsive Control, $\bar{\nu} = \pi/2$



Aperiodic Bi-impulsive Control, $\bar{\nu}$ opt



$\color{blue}{\bullet}$ ζ_{01}	J	(m/s)	**1.1192**
	$\max(\lvert\xi_{1..3}\rvert_{\mathcal{B}})$	(m)	**513.9965**
	$\nu_{\mathcal{B}}$	(deg)	**1547.5395**
$\color{orange}{\bullet}$ ζ_{02}	J	(m/s)	**1.0188**
	$\max(\lvert\xi_{1..3}\rvert_{\mathcal{B}})$	(m)	**362.0635**
	$\nu_{\mathcal{B}}$	(deg)	**1300.7956**
$\color{green}{\bullet}$ ζ_{03}	J	(m/s)	**0.5311**
	$\max(\lvert\xi_{1..3}\rvert_{\mathcal{B}})$	(m)	**326.6401**
	$\nu_{\mathcal{B}}$	(deg)	**87.1149**

$\color{blue}{\bullet}$ ζ_{01}	J	(m/s)	**1.5206 (+35.86%)**
	$\max(\lvert\xi_{1..3}\rvert_{\mathcal{B}})$	(m)	**512.9571 (-0.20%)**
	$\nu_{\mathcal{B}}$	(deg)	**88.7732 (-94.26%)**
$\color{orange}{\bullet}$ ζ_{02}	J	(m/s)	**0.6550 (-35.71%)**
	$\max(\lvert\xi_{1..3}\rvert_{\mathcal{B}})$	(m)	**314.2451 (-13.21%)**
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$\color{green}{\bullet}$ ζ_{03}	J	(m/s)	**0.5184 (-2.39%)**
	$\max(\lvert\xi_{1..3}\rvert_{\mathcal{B}})$	(m)	**326.6401 (+0.00%)**
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$\color{blue}{\bullet}$ ζ_{01}	J	(m/s)	**0.6552 (-41.46%)**
	$\max(\lvert\xi_{1..3}\rvert_{\mathcal{B}})$	(m)	**514.1301 (+0.03%)**
	$\nu_{\mathcal{B}}$	(deg)	**265.8172 (-82.82%)**
$\color{orange}{\bullet}$ ζ_{02}	J	(m/s)	**0.6098 (-40.15%)**
	$\max(\lvert\xi_{1..3}\rvert_{\mathcal{B}})$	(m)	**314.2451 (-13.21%)**
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$\color{green}{\bullet}$ ζ_{03}	J	(m/s)	**0.4877 (-8.17%)**
	$\max(\lvert\xi_{1..3}\rvert_{\mathcal{B}})$	(m)	**325.0000 (-0.50%)**
	$\nu_{\mathcal{B}}$	(deg)	**81.7291 (-6.18%)**

Numerical studies first example

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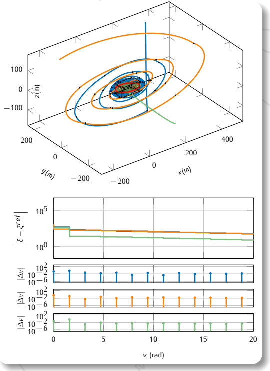
Periodic Norm-minimizing Control, $\bar{\nu} = \pi/2$



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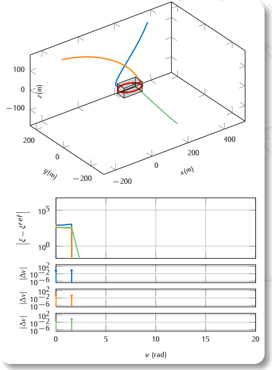
Periodic Bi-impulsive Control, $\bar{\nu} = \pi/2$



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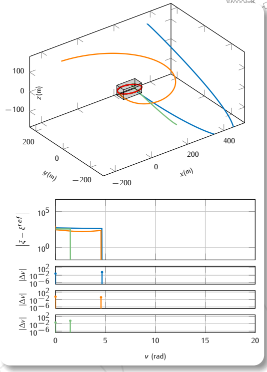
Aperiodic Bi-impulsive Control, $\bar{\nu}$ opt



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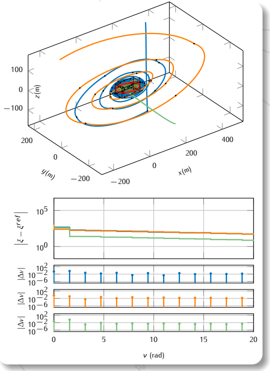

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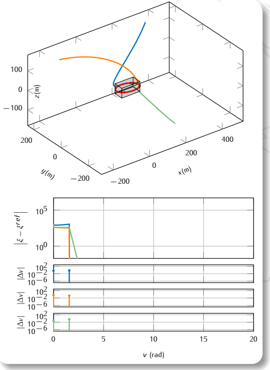
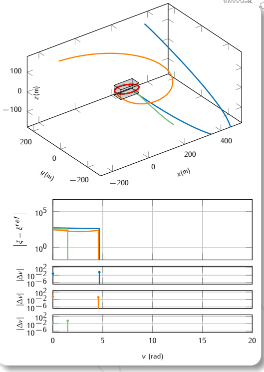
ζ_{03}	J	(m/s)	**0.5311**
	$\max(\lvert\xi_{1..3}\rvert_{\mathcal{B}})$	(m)	**326.6401**
	$\nu_{\mathcal{B}}$	(deg)	**87.1149**

Periodic Bi-impulsive Control, $\bar{\nu} = \pi/2$



ζ_{01}	J	(m/s)	**1.5206 (+35.86%)**
	$\max(\lvert\xi_{1..3}\rvert_{\mathcal{B}})$	(m)	**512.9571 (-0.20%)**
	$\nu_{\mathcal{B}}$	(deg)	**88.7732 (-94.26%)**
ζ_{02}	J	(m/s)	**0.6550 (-35.71%)**
	$\max(\lvert\xi_{1..3}\rvert_{\mathcal{B}})$	(m)	**314.2451 (-13.21%)**
	$\nu_{\mathcal{B}}$	(deg)	**80.4853 (-93.81%)**
ζ_{03}	J	(m/s)	**0.5184 (-2.39%)**
	$\max(\lvert\xi_{1..3}\rvert_{\mathcal{B}})$	(m)	**326.6401 (+0.00%)**
	$\nu_{\mathcal{B}}$	(deg)	**87.1149 (+0.00%)**

Aperiodic Bi-impulsive Control, $\bar{\nu}$ opt



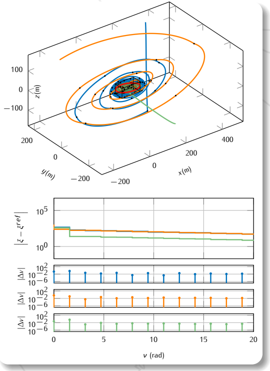
ζ_{01}	J	(m/s)	**0.6552 (-41.46%)**
	$\max(\lvert\xi_{1..3}\rvert_{\mathcal{B}})$	(m)	**514.1301 (+0.03%)**
	$\nu_{\mathcal{B}}$	(deg)	**265.8172 (-82.82%)**
ζ_{02}	J	(m/s)	**0.6098 (-40.15%)**
	$\max(\lvert\xi_{1..3}\rvert_{\mathcal{B}})$	(m)	**314.2451 (-13.21%)**
	$\nu_{\mathcal{B}}$	(deg)	**246.2420 (-81.07%)**
ζ_{03}	J	(m/s)	**0.4877 (-8.17%)**
	$\max(\lvert\xi_{1..3}\rvert_{\mathcal{B}})$	(m)	**325.0000 (-0.50%)**
	$\nu_{\mathcal{B}}$	(deg)	**81.7291 (-6.18%)**

Numerical studies first example

mission 1

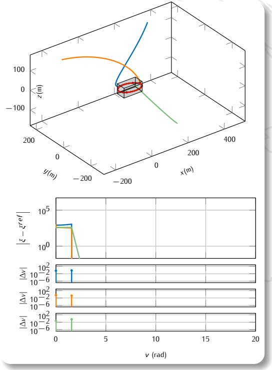
$a = 7011$**km**, $e = 0.1$, $\color{red}{\bullet}\,\xi^{ref} = [5.2, 17.7, 98.0, 22.5, \text{-}17.6, 0]$,
$\color{blue}{\bullet}\,\zeta_{01} = [500, 400, 10, 0, 0, 0]$, $\color{orange}{\bullet}\,\zeta_{02} = [-200, 100, 200, 0, 0, 0]$, $\color{green}{\bullet}\,\zeta_{03} = [100, -350, -20, 0, 0, 0]$,
with $\nu_{01} = 10^\circ$, $\nu_{02} = 150^\circ$, $\nu_{03} = 60^\circ$

Periodic Norm-minimizing Control, $\bar{\nu} = \pi/2$



ζ_{01}	J	(m/s)	**1.1192**
	$\max(\lvert\xi_{1..3}\rvert_{\mathcal{B}})$	(m)	**513.9965**
	$\nu_{\mathcal{B}}$	(deg)	**1547.5395**
ζ_{02}	J	(m/s)	**1.0188**
	$\max(\lvert\xi_{1..3}\rvert_{\mathcal{B}})$	(m)	**362.0635**
	$\nu_{\mathcal{B}}$	(deg)	**1300.7956**
ζ_{03}	J	(m/s)	**0.5311**
	$\max(\lvert\xi_{1..3}\rvert_{\mathcal{B}})$	(m)	**326.6401**
	$\nu_{\mathcal{B}}$	(deg)	**87.1149**

Periodic Bi-impulsive Control, $\bar{\nu} = \pi/2$



ζ_{01}	J	(m/s)	**1.5206 (+35.86%)**
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ζ_{03}	J	(m/s)	**0.5184 (-2.39%)**
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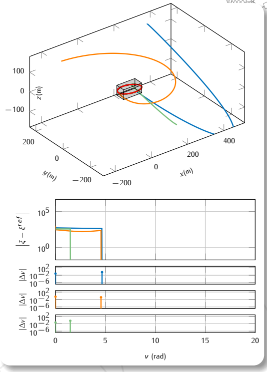
	$\nu_{\mathcal{B}}$	(deg)	**87.1149 (+0.00%)**

Aperiodic Bi-impulsive Control, $\bar{\nu}$ opt



ζ_{01}	J	(m/s)	**0.6552 (-41.46%)**
	$\max(\lvert\xi_{1..3}\rvert_{\mathcal{B}})$	(m)	**514.1301 (+0.03%)**
	$\nu_{\mathcal{B}}$	(deg)	**265.8172 (-82.82%)**
ζ_{02}	J	(m/s)	**0.6098 (-40.15%)**
	$\max(\lvert\xi_{1..3}\rvert_{\mathcal{B}})$	(m)	**314.2451 (-13.21%)**
	$\nu_{\mathcal{B}}$	(deg)	**246.2420 (-81.07%)**
ζ_{03}	J	(m/s)	**0.4877 (-8.17%)**
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Numerical studies first example

mission 1

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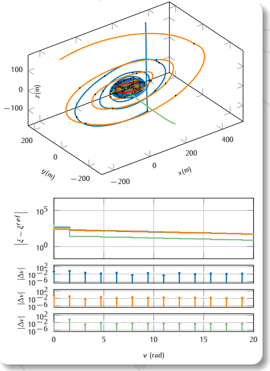
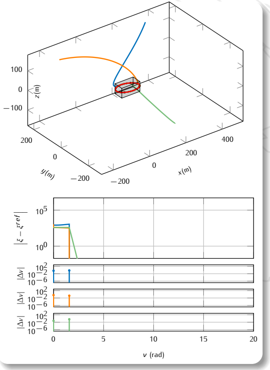
$\bullet\,\zeta_{01} = [500, 400, 10, 0, 0, 0]$, $\bullet\,\zeta_{02} = [-200, 100, 200, 0, 0, 0]$, $\bullet\,\zeta_{03} = [100, -350, -20, 0, 0, 0]$,
with $\nu_{01} = 10^{\circ}$, $\nu_{02} = 150^{\circ}$, $\nu_{03} = 60^{\circ}$

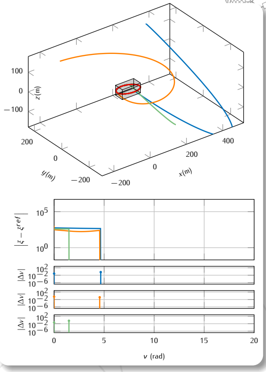
Periodic Norm-minimizing Control, $\bar{\nu} = \pi/2$



Periodic Bi-impulsive Control, $\bar{\nu} = \pi/2$



Aperiodic Bi-impulsive Control, $\bar{\nu}$ opt



$\bullet\ \zeta_{01}$	J	(m/s)	**1.1192**
	$\max(\lvert\xi_{1..3}\rvert_{\mathcal{B}})$	(m)	**513.9965**
	$\nu_{\mathcal{B}}$	(deg)	**1547.5395**
$\bullet\ \zeta_{02}$	J	(m/s)	**1.0188**
	$\max(\lvert\xi_{1..3}\rvert_{\mathcal{B}})$	(m)	**362.0635**
	$\nu_{\mathcal{B}}$	(deg)	**1300.7956**
$\bullet\ \zeta_{03}$	J	(m/s)	**0.5311**
	$\max(\lvert\xi_{1..3}\rvert_{\mathcal{B}})$	(m)	**326.6401**
	$\nu_{\mathcal{B}}$	(deg)	**87.1149**

$\bullet\ \zeta_{01}$	J	(m/s)	**1.5206 (+35.86%)**
	$\max(\lvert\xi_{1..3}\rvert_{\mathcal{B}})$	(m)	**512.9571 (-0.20%)**
	$\nu_{\mathcal{B}}$	(deg)	**88.7732 (-94.26%)**
$\bullet\ \zeta_{02}$	J	(m/s)	**0.6550 (-35.71%)**
	$\max(\lvert\xi_{1..3}\rvert_{\mathcal{B}})$	(m)	**314.2451 (-13.21%)**
	$\nu_{\mathcal{B}}$	(deg)	**80.4853 (-93.81%)**
$\bullet\ \zeta_{03}$	J	(m/s)	**0.5184 (-2.39%)**
	$\max(\lvert\xi_{1..3}\rvert_{\mathcal{B}})$	(m)	**326.6401 (+0.00%)**
	$\nu_{\mathcal{B}}$	(deg)	**87.1149 (+0.00%)**

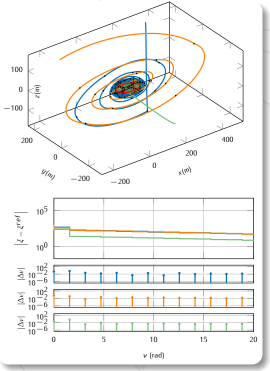
$\bullet\ \zeta_{01}$	J	(m/s)	**0.6552 (-41.46%)**
	$\max(\lvert\xi_{1..3}\rvert_{\mathcal{B}})$	(m)	**514.1301 (+0.03%)**
	$\nu_{\mathcal{B}}$	(deg)	**265.8172 (-82.82%)**
$\bullet\ \zeta_{02}$	J	(m/s)	**0.6098 (-40.15%)**
	$\max(\lvert\xi_{1..3}\rvert_{\mathcal{B}})$	(m)	**314.2451 (-13.21%)**
	$\nu_{\mathcal{B}}$	(deg)	**246.2420 (-81.07%)**
$\bullet\ \zeta_{03}$	J	(m/s)	**0.4877 (-8.17%)**
	$\max(\lvert\xi_{1..3}\rvert_{\mathcal{B}})$	(m)	**325.0000 (-0.50%)**
	$\nu_{\mathcal{B}}$	(deg)	**81.7291 (-6.18%)**

Numerical studies first example

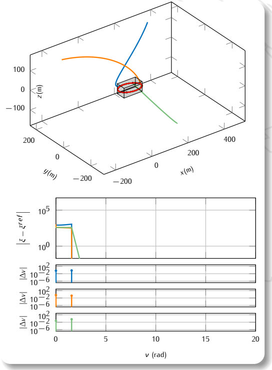
mission 1

$a = 7011$**km**, $e = 0.1$, $\bullet\xi^{ref} = [5.2, 17.7, 98.0, 22.5, \text{-}17.6, 0]$,
$\bullet\zeta_{01} = [500, 400, 10, 0, 0, 0]$, $\bullet\zeta_{02} = [-200, 100, 200, 0, 0, 0]$, $\bullet\zeta_{03} = [100, -350, -20, 0, 0, 0]$,
with $\nu_{01} = 10^{\circ}$, $\nu_{02} = 150^{\circ}$, $\nu_{03} = 60^{\circ}$

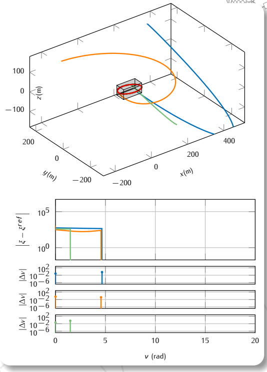
Periodic Norm-minimizing Control, $\bar{\nu} = \pi/2$



Periodic Bi-impulsive Control, $\bar{\nu} = \pi/2$



Aperiodic Bi-impulsive Control, $\bar{\nu}$ opt



ζ_{01}	J	(m/s)	**1.1192**
	$\max(\lvert\xi_{1..3}\rvert_{\mathcal{B}})$	(m)	**513.9965**
	$\nu_{\mathcal{B}}$	(deg)	**1547.5395**
ζ_{02}	J	(m/s)	**1.0188**
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	$\nu_{\mathcal{B}}$	(deg)	**1300.7956**
ζ_{03}	J	(m/s)	**0.5311**
	$\max(\lvert\xi_{1..3}\rvert_{\mathcal{B}})$	(m)	**326.6401**
	$\nu_{\mathcal{B}}$	(deg)	**87.1149**

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	$\max(\lvert\xi_{1..3}\rvert_{\mathcal{B}})$	(m)	**512.9571 (-0.20%)**
	$\nu_{\mathcal{B}}$	(deg)	**88.7732 (-94.26%)**
ζ_{02}	J	(m/s)	**0.6550 (-35.71%)**
	$\max(\lvert\xi_{1..3}\rvert_{\mathcal{B}})$	(m)	**314.2451 (-13.21%)**
	$\nu_{\mathcal{B}}$	(deg)	**80.4853 (-93.81%)**
ζ_{03}	J	(m/s)	**0.5184 (-2.39%)**
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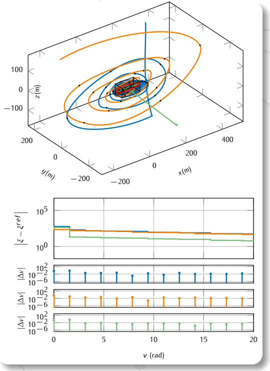
	$\nu_{\mathcal{B}}$	(deg)	**246.2420 (-81.07%)**
ζ_{03}	J	(m/s)	**0.4877 (-8.17%)**
	$\max(\lvert\xi_{1..3}\rvert_{\mathcal{B}})$	(m)	**325.0000 (-0.50%)**
	$\nu_{\mathcal{B}}$	(deg)	**81.7291 (-6.18%)**

Numerical studies: second example

mission 2

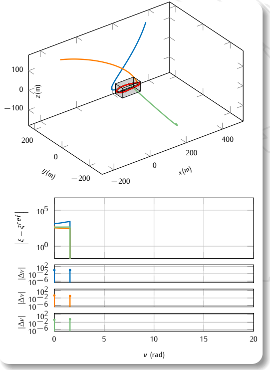
$a = 7011\,\mathbf{km}$, $e = 0.4$, $\bullet\xi^{ref} = [7.7, 17.7, 87.8, 33.0, \text{-}15.8, 0]$,
$\bullet\zeta_{01} = [500, 400, 10, 0, 0, 0]$, $\bullet\zeta_{02} = [-200, 100, 200, 0, 0, 0]$, $\bullet\zeta_{03} = [100, -350, -20, 0, 0, 0]$,
with $\nu_{01} = 10^\circ$, $\nu_{02} = 150^\circ$, $\nu_{03} = 60^\circ$

Periodic Norm-minimizing Control, $\bar{\nu} = \pi/2$



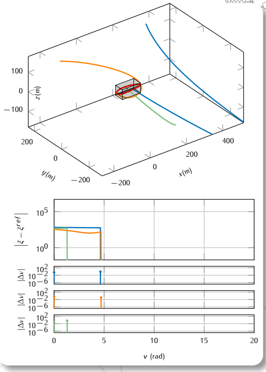
ζ_{01}	J	(m/s)	**1.9248**
	$\max(\lvert\xi_{1..3}\rvert_{\mathcal{B}})$	(m)	**515.5433**
	$\nu_{\mathcal{B}}$	(deg)	**1122.9619**
ζ_{02}	J	(m/s)	**0.9448**
	$\max(\lvert\xi_{1..3}\rvert_{\mathcal{B}})$	(m)	**340.5700**
	$\nu_{\mathcal{B}}$	(deg)	**1285.4598**
ζ_{03}	J	(m/s)	**0.7094**
	$\max(\lvert\xi_{1..3}\rvert_{\mathcal{B}})$	(m)	**343.3386**
	$\nu_{\mathcal{B}}$	(deg)	**87.4921**

Periodic Bi-impulsive Control, $\bar{\nu} = \pi/2$



ζ_{01}	J	(m/s)	**2.6658 (+38.50%)**
	$\max(\lvert\xi_{1..3}\rvert_{\mathcal{B}})$	(m)	**512.9571 (-0.50%)**
	$\nu_{\mathcal{B}}$	(deg)	**89.2876 (-92.05%)**
ζ_{02}	J	(m/s)	**0.4566 (-51.67%)**
	$\max(\lvert\xi_{1..3}\rvert_{\mathcal{B}})$	(m)	**314.2451 (-7.73%)**
	$\nu_{\mathcal{B}}$	(deg)	**82.2925 (-93.60%)**
ζ_{03}	J	(m/s)	**0.6953 (-1.99%)**
	$\max(\lvert\xi_{1..3}\rvert_{\mathcal{B}})$	(m)	**343.3386 (+0.00%)**
	$\nu_{\mathcal{B}}$	(deg)	**87.4921 (+0.00%)**

Aperiodic Bi-impulsive Control, $\bar{\nu}$ opt



ζ_{01}	J	(m/s)	**1.3189 (-31.48%)**
	$\max(\lvert\xi_{1..3}\rvert_{\mathcal{B}})$	(m)	**925.3879 (+79.50%)**
	$\nu_{\mathcal{B}}$	(deg)	**264.4981 (-76.45%)**
ζ_{02}	J	(m/s)	**0.4291 (-54.58%)**
	$\max(\lvert\xi_{1..3}\rvert_{\mathcal{B}})$	(m)	**314.2451 (-7.73%)**
	$\nu_{\mathcal{B}}$	(deg)	**222.4380 (-82.70%)**
ζ_{03}	J	(m/s)	**0.5891 (-16.96%)**
	$\max(\lvert\xi_{1..3}\rvert_{\mathcal{B}})$	(m)	**325.0004 (-5.34%)**
	$\nu_{\mathcal{B}}$	(deg)	**71.5639 (-18.21%)**

Numerical studies: second example

mission 2

$a = 7011$**km**, $e = 0.4$, $\bullet\xi^{ref} = [7.7, 17.7, 87.8, 33.0, \text{-}15.8, 0]$,
$\bullet\zeta_{01} = [500, 400, 10, 0, 0, 0]$, $\bullet\zeta_{02} = [-200, 100, 200, 0, 0, 0]$, $\bullet\zeta_{03} = [100, -350, -20, 0, 0, 0]$,
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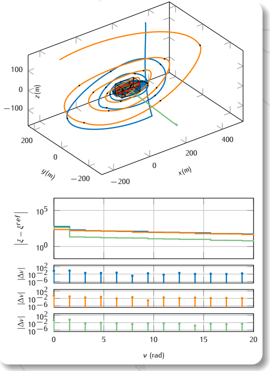
Periodic Norm-minimizing Control, $\bar{\nu} = \pi/2$



ζ_{01}	J (m/s)		**1.9248**
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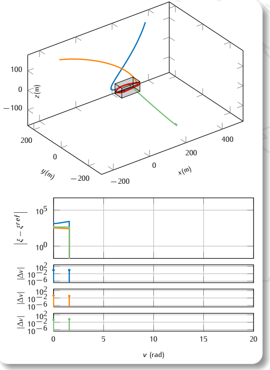
	$\nu_{\mathcal{B}}$	(deg)	**1285.4598**
ζ_{03}	J (m/s)		**0.7094**
	$\max(\lvert\xi_{1..3}\rvert_{\mathcal{B}})$	(m)	**343.3386**
	$\nu_{\mathcal{B}}$	(deg)	**87.4921**

Periodic Bi-impulsive Control, $\bar{\nu} = \pi/2$



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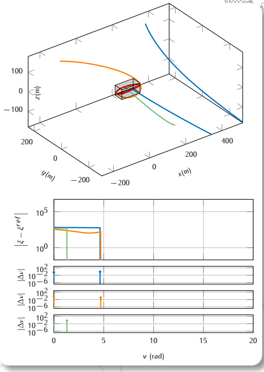
	$\max(\lvert\xi_{1..3}\rvert_{\mathcal{B}})$	(m)	**512.9571 (-0.50%)**
	$\nu_{\mathcal{B}}$	(deg)	**89.2876 (-92.05%)**
ζ_{02}	J (m/s)		**0.4566 (-51.67%)**
	$\max(\lvert\xi_{1..3}\rvert_{\mathcal{B}})$	(m)	**314.2451 (-7.73%)**
	$\nu_{\mathcal{B}}$	(deg)	**82.2925 (-93.60%)**
ζ_{03}	J (m/s)		**0.6953 (-1.99%)**
	$\max(\lvert\xi_{1..3}\rvert_{\mathcal{B}})$	(m)	**343.3386 (+0.00%)**
	$\nu_{\mathcal{B}}$	(deg)	**87.4921 (+0.00%)**

Aperiodic Bi-impulsive Control, $\bar{\nu}$ opt



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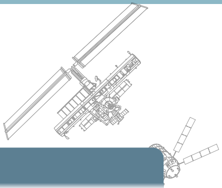
	$\nu_{\mathcal{B}}$	(deg)	**71.5639 (-18.21%)**

Conclusions



Submitted paper

54th *Conference on Decision and Control* IEEE
"A hybrid control framework for impulsive control of satellite rendezvous".

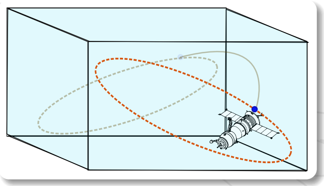
Achievements

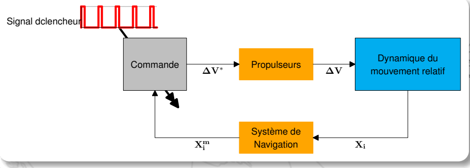
- Use of Floquet-Lyapunov to obtain LTI flow;
- Hybrid description of the hovering problem;
- Stability and convergence (in most of cases) is guaranteed.

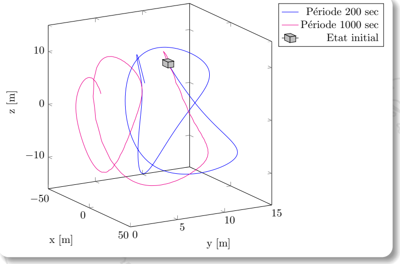
Future works

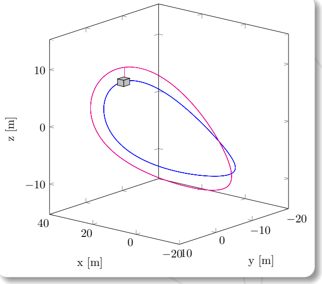
- Accounting for thrusters saturations;
- Evaluate the robustness through non-linear simulations;
- Address the robust stability properties of the presented control law.

Model Predictive Control for hovering station zone









Constrained periodic orbits SDP description

$$S^p(H,V) = \left\{ D(w_0) \in \mathbb{R}^6 \;\middle|\; \begin{array}{l} d_3(t) = 0 \\ \exists\, Y_i \succeq 0 \text{ t.q. } \gamma_i^p = \Lambda^*(Y_i) \end{array} \right\}$$