An Analytical Tuning Approach for Adaptive MPC Parameters Applied to LTV-SISO Systems

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Outline

- Introduction
- Remind on MPC considered
- Tuning approach proposed
- Application
- Conclusion & outlook

Introduction

- Motivations
- Influence of parameters values
- Existing approaches

Remind on MPC (1/3)

- State-space model and hypothesis

$$x_m(k+1) = A_m x_m(k) + B_m u(k)$$
$$y(k) = C_m x_m(k)$$

- Augmented state-space model [4]

$$\overbrace{\begin{bmatrix} \Delta x_m(k+1) \\ y(k+1) \end{bmatrix}}^{x(k+1)} = \overbrace{\begin{bmatrix} A_m & 0_m^t \\ C_m A_m & 1 \end{bmatrix}}^{A} x(k) + \overbrace{\begin{bmatrix} B_m \\ C_m B_m \end{bmatrix}}^{B} \Delta u(k)$$
$$y(k) = \overbrace{\begin{bmatrix} 0_m & 1 \end{bmatrix}}^{C} \begin{bmatrix} \Delta x_m(k) \\ y(k) \end{bmatrix}$$

Remind on MPC (2/3)

- Cost fonction [11]

$$J = (Y_{des} - \hat{Y})^t (Y_{des} - \hat{Y}) + \Delta U^t \bar{R} \Delta U$$

Where:

$$\hat{Y} = Fx(k_i) + \Phi \Delta U$$

$$\Delta U = \begin{bmatrix} \Delta u(k) & \Delta u(k+1) & \cdots & \Delta u(k+N_c-1) \end{bmatrix}^t$$

And:

$$F = \begin{bmatrix} CA & CA^2 & CA^3 & \cdots & CA^{N_p} \end{bmatrix}^t$$

$$\Phi = \begin{bmatrix} CB & 0 & 0 & \dots & 0 \\ CAB & CB & 0 & \dots & 0 \\ CA^2B & CAB & CB & \dots & 0 \\ \vdots & & & & \\ CA^{N_p-1}B & CA^{N_p-2}B & CA^{N_p-3}B & \dots & CA^{N_p-N_c}B \end{bmatrix}$$

Remind on MPC (3/3)

- Cost fonction minimization

$$\frac{\partial J}{\partial \Delta U} = -2\Phi^t(Y_{des} - Fx(k)) + 2(\Phi^t\Phi + \bar{R})\Delta U$$

- Resulting control law

1) $\Delta U = (\Phi^t\Phi + \bar{R})^{-1}\Phi^t(Y_{des} - Fx(k))$

2) $\Delta U(k) = K_y Y_{des} - K_{mpc} x(k)$

N_c computation (1/2)

■ <u>The idea</u>: In order to enhance the numerical condition of a process ➔ the dimension of a Hessian matrix H must be reduced.
When the dimension of H is reduced ➔ An optimal control horizon is computed.

- Hessian condition number

$$H = (\Phi^t \Phi + \bar{R})^{-1}$$

$$cond(H(k)) = \|H(k)\|_2 . \|H(k)^{-1}\|_2$$
$$= \frac{\sigma_{max}(k)}{\sigma_{min}(k)}$$

- Condition number & stability

$$N_c \mapsto \infty, N_p \mapsto \infty$$

➔ Relationship between the effective rank of *H* and N_c

N_c computation (2/2)

- Method proposed

1) To initialize $N_c \mapsto \infty$ and $N_p \mapsto \infty$ $(Nc < Np)$.
2) To take $A_{ER} = H$ [16].
3) To evaluate Q defined as follows [16]:

$$Q = min\{M_{ER}, N_{ER}\} = min\{Nc, Nc\} = Nc.$$

4) To decompose H into singular values and to evaluate $\sigma = [\sigma_1 \quad \sigma_2 \quad \cdots \quad \sigma_\infty]^T$:

$$H = U_H D_H V_H$$

$$\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_\infty$$

5) To evaluate the singular value distribution p_k with $k = [1, \quad 2, \quad \cdots, \quad \infty]$.
6) To calculate the Shannon entropy [16].
7) To solve

$$\begin{cases} N_{c_{opt}} = e^{H_{Shannon}(p_1, p_2, \cdots, p_\infty)} \\ \min(N_{c_{opt}}) \\ \min(cond(H(k)) - 1) \end{cases}$$

N_p computation (1/4)

■ <u>The idea</u>: Ensuring the closed-loop stability ➔ applying the concept of eigenvalues location ➔ the Gershgorin circle theorem.

- Closed-loop eigenvalues

$$det[\nu I_{(n_A \times n_A)} - (A - BK_{mpc})] = 0$$

- Optimal closed-loop stability

Insensitivity approach ➔

$$\begin{cases} \min \gamma \\ \begin{bmatrix} -\gamma I_{(n_A \times n_A)} & (A - BK_{mpc}) \\ (A - BK_{mpc})^T & -\gamma I_{(n_A \times n_A)} \end{bmatrix} < 0 \end{cases}$$

Yalmip toolbox ➔ $(K_{mpc})^{opt}$

N_p computation (2/4)

- Relationship between K_{MPC} and $\Phi^T\Phi$

From $K_{mpc} = I_{(1\times N_c)}(\Phi^T\Phi + \bar{R})^{-1}\Phi^T F$

Under the assumption $(\Phi^T\Phi)^{-1}\bar{R}$ is nilpotent of order n,

$$
\begin{aligned}
H = {} & (\Phi^T\Phi)^{-1} - (\Phi^T\Phi)^{-1}\bar{R}(\Phi^T\Phi)^{-1} \\
& + [(\Phi^T\Phi)^{-1}\bar{R}]^2(\Phi^T\Phi)^{-1} - \cdots \\
& + (-1)^{n-1}[(\Phi^T\Phi)^{-1}\bar{R}]^{n-1}(\Phi^T\Phi)^{-1}
\end{aligned}
$$

$$
\begin{aligned}
\Rightarrow \quad (\Phi^T\Phi)^{opt} = {} & [-\bar{R} + \bar{R}(\Phi^T\Phi)^{-1}\bar{R} - \cdots \\
& + (-1)^{n-1}(\Phi^T\Phi)^{-1}[(\Phi^T\Phi)^{-1}\bar{R}]^{n-1}] \\
& \times [(K_{mpc})^{opt}F^T(FF^T)^{-1}\Phi - I_{(N_c\times Nc)}]^{-1}
\end{aligned}
$$

N_p computation (3/4)

- Focus on $\Phi^T\Phi$

$$\Phi^T\Phi = \begin{bmatrix} \overbrace{\sum_{i=0}^{N_p-1}(CA^iB)^T(CA^iB)}^{X_{N_p}} & . & . & \cdots & . \\ . & \overbrace{\sum_{i=0}^{N_p-2}(CA^iB)^T(CA^iB)}^{X_{N_p-1}} & . & \cdots & . \\ . & . & \overbrace{\sum_{i=0}^{N_p-3}(CA^iB)^T(CA^iB)}^{X_{N_p-2}} & \cdots & \vdots \\ \vdots & & & \ddots & \\ . & . & . & \cdots & \overbrace{\sum_{i=0}^{N_p-N_c}(CA^iB)^T(CA^iB)}^{X_{N_p-N_c}} \end{bmatrix}$$

With $CA^iB = \sum_{k=0}^{i} C_m A_m^k B_m$

N_p computation (4/4)

- Relationship between $\Phi^T\Phi$ and N_p

$$tr(\Phi^T\Phi) = X_{N_p} + X_{N_p-1} + \cdots + X_{N_p-N_c};$$

$$= \sum_{i=0}^{N_p-1} \sum_{k=0}^{i} (C_m A_m^k B_m)^T (C_m A_m^k B_m) + \sum_{i=0}^{N_p-2} \sum_{k=0}^{i} (C_m A_m^k B_m)^T (C_m A_m^k B_m)$$

$$+ \cdots + \sum_{i=0}^{N_p-N_c} (\sum_{k=0}^{i} (C_m A_m^k B_m)^T (C_m A_m^k B_m)$$

➡ N_p^{opt} is the first integer such as $tr(\Phi^T\Phi) \geq tr((\Phi^T\Phi)^{opt})$

$\bar{R}$ computation (1/2)

■ <u>The idea</u>: minimizing the consumed energy to achieve the desired output, ➔ minimizing the cost of the control.

- Expanded cost fonction [11]

$$J = (Y_{des} - Fx(k))^t (Y_{des} - Fx(k)) - 2\Delta U^t \Phi^T (Y_{des} - Fx(k)) + \Delta U^t (\Phi^t \Phi + \bar{R}) \Delta U$$

- Cost fonction minimization

$$\begin{cases} \frac{\partial J}{\partial \Delta U} = 0 \\ \frac{\partial J}{\partial \bar{R}} = 0 \end{cases}$$

$\bar{R}$ computation (2/2)

- Cost fonction derivative

$$\frac{\partial J}{\partial \bar{R}} = -2(\Phi^T\Phi)^{-1}X_1(\Phi^T\Phi)^{-1}$$
$$- 2(\Phi^T\Phi)^{-1}X_1^T(\Phi^T\Phi)^{-1}$$
$$- 2(\Phi^T\Phi)^{-1}X_2(\Phi^T\Phi)^{-1}$$
$$+ 4(\Phi^T\Phi)^{-1}\bar{R}(\Phi^T\Phi)^{-1}X_1(\Phi^T\Phi)^{-1}$$
$$+ 4(\Phi^T\Phi)^{-1}\bar{R}(\Phi^T\Phi)^{-1}X_1^T(\Phi^T\Phi)^{-1}$$
$$+ 8(\Phi^T\Phi)^{-1}\bar{R}(\Phi^T\Phi)^{-1}X_2(\Phi^T\Phi)^{-1}.$$

$$\bar{R}_{opt} = \frac{1}{2}[X_1 + X_1^T + X_2](\Phi^T\Phi)\times [X_1 + X_1^T + 2X_2]^{-1}$$

In summary

- Multi-objective issue adressed

Yalmip toolbox

$$\min(N_{c_{opt}})$$

$$\min(cond(H(k)) - 1)$$

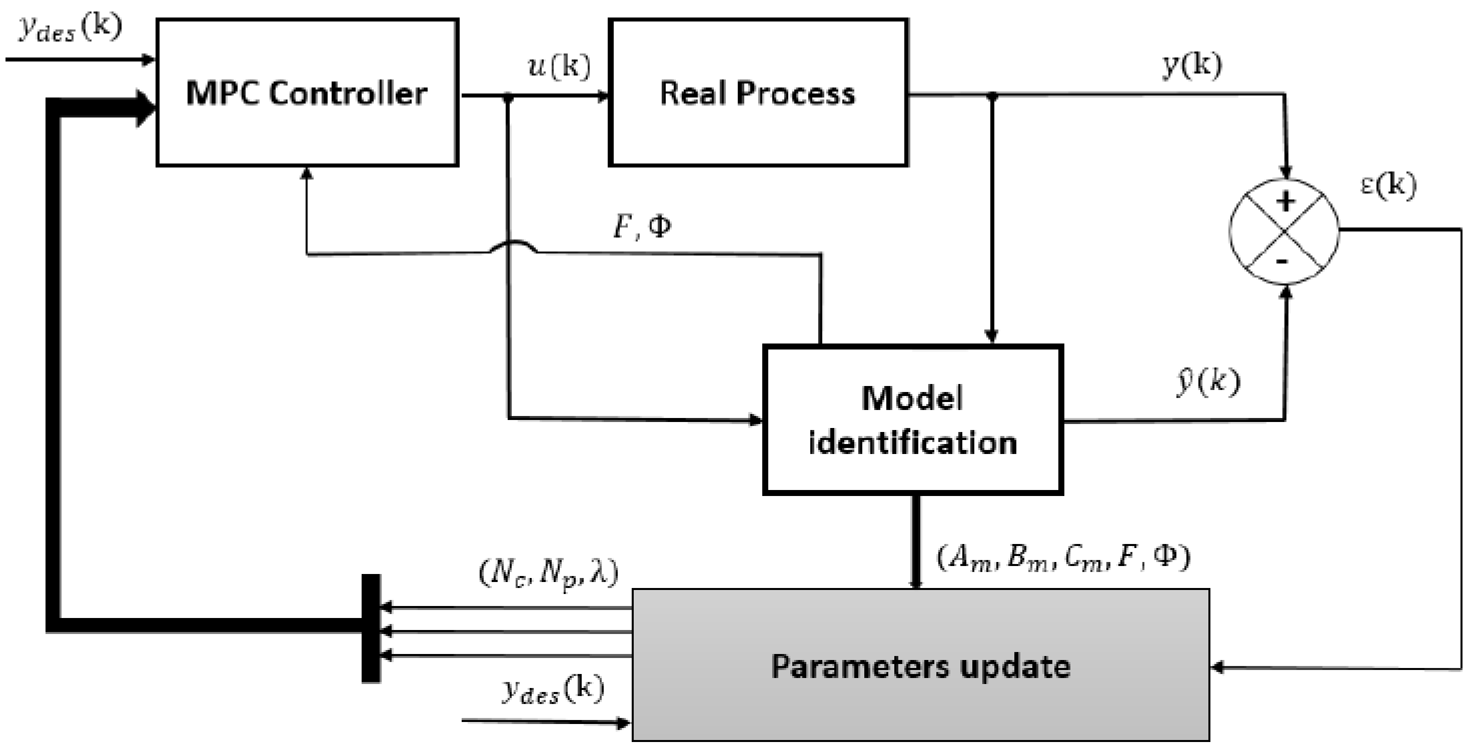
$$N_{c_{opt}} = e^{H_{Shannon}(p_1, p_2, \cdots, p_\infty)}$$

$$X_{N_p} = \sum_{i=0}^{N_p^{opt}-1} (\sum_{k=0}^{i} (C_m A_m^k B_m)^T (C_m A_m^k B_m)$$

$$\bar{R}_{opt} = \frac{1}{2}[X_1 + X_1^T + X_2](\Phi^T \Phi) \times [X_1 + X_1^T + 2X_2]^{-1}$$

Case of LTV SISO systems

- Control structure applied



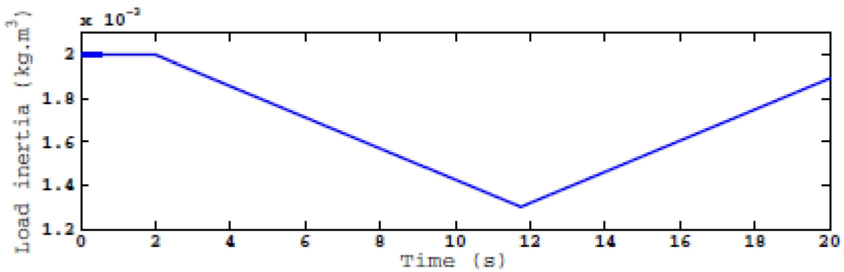
Application : system considered

- Transfert fonction [5]

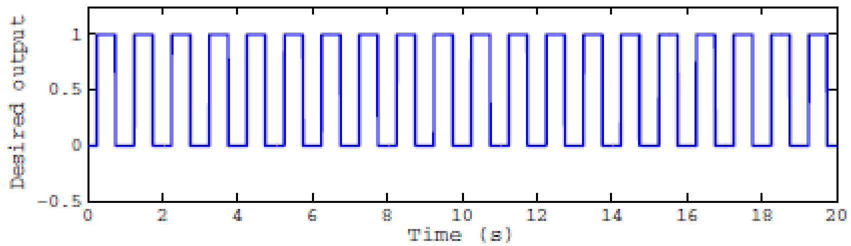
$$H(s) = \frac{\Omega(s)}{V(s)} = \frac{K_n}{J_n(t)s^2 + f_n s + C_n}$$

$$\begin{cases} K_n = & 2 \text{ Nm.rad/s/V;} \\ f_n = & 0.006 \text{ Nm.rad/s;} \\ C = & 2 \text{ Nm.} \end{cases}$$

- Varying parameter vs. time



- Desired output vs. time



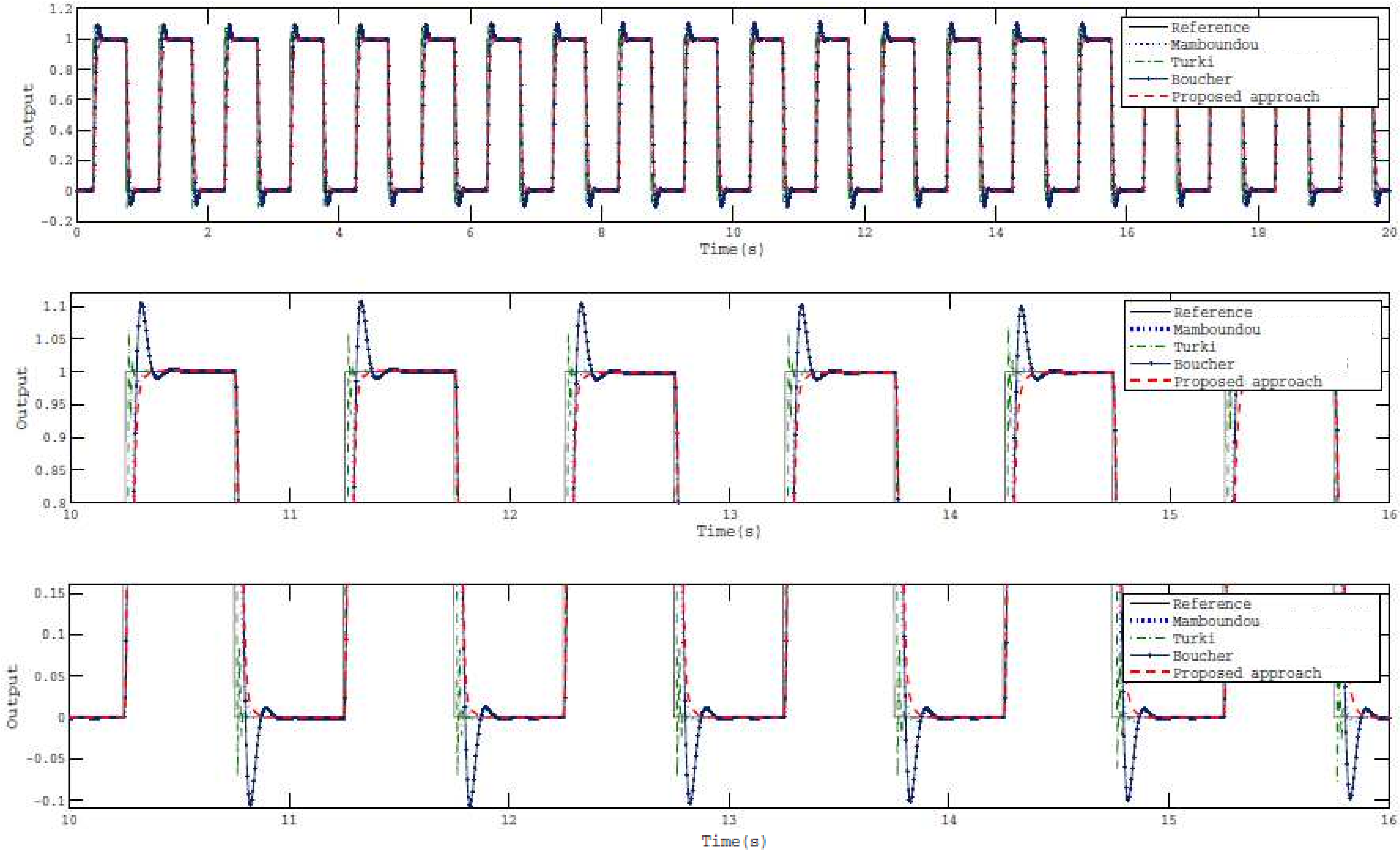
Application : comparative study

- MPCs with constant MPC parameters
  - Boucher et al. [19] (Nc=1, Np=8, λ=0,42)
  - Turki et al. [20] (Nc=6, Np=54, λ=0,4)

- MPC with time-varying parameters
  - Mamboundou et al. [10]

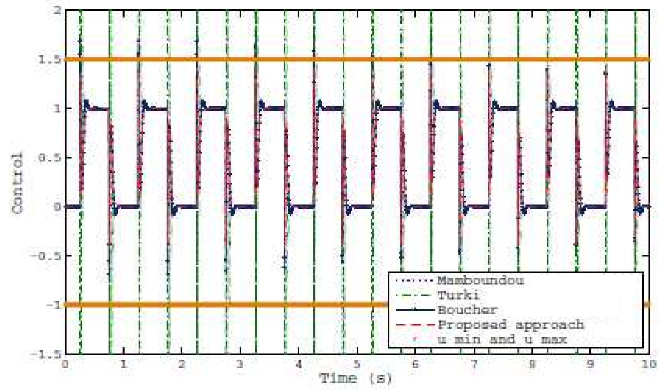
Application : simulation results

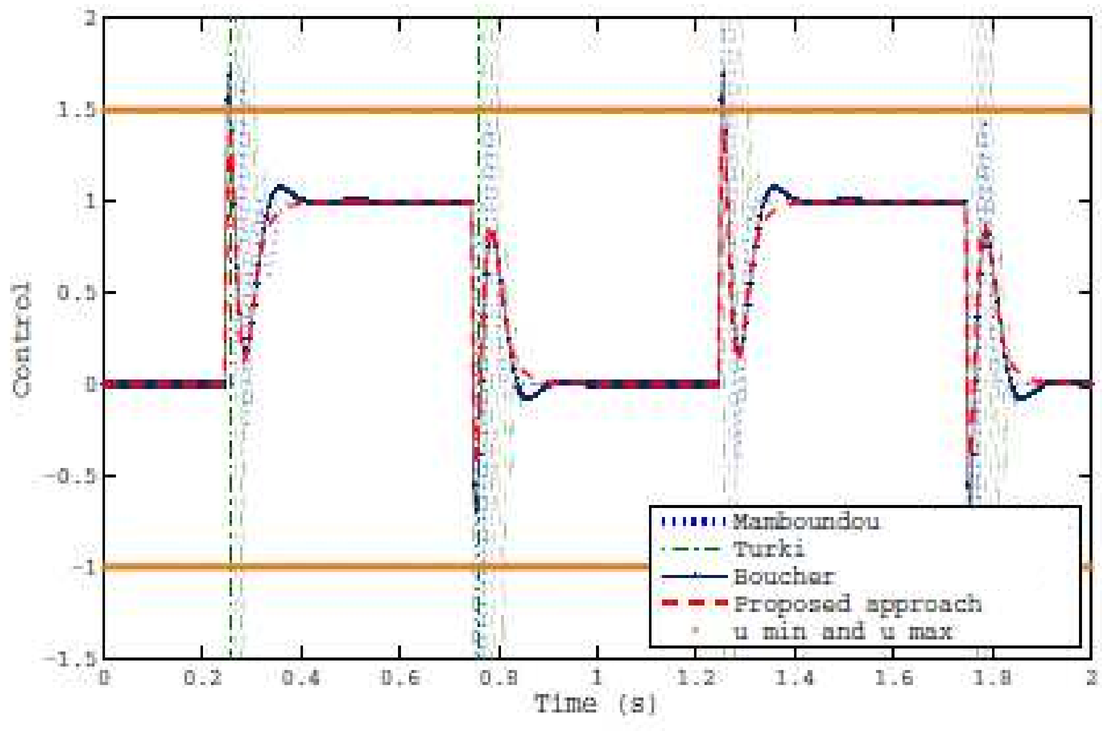
- Desired output and system output vs. time



Application : simulation results

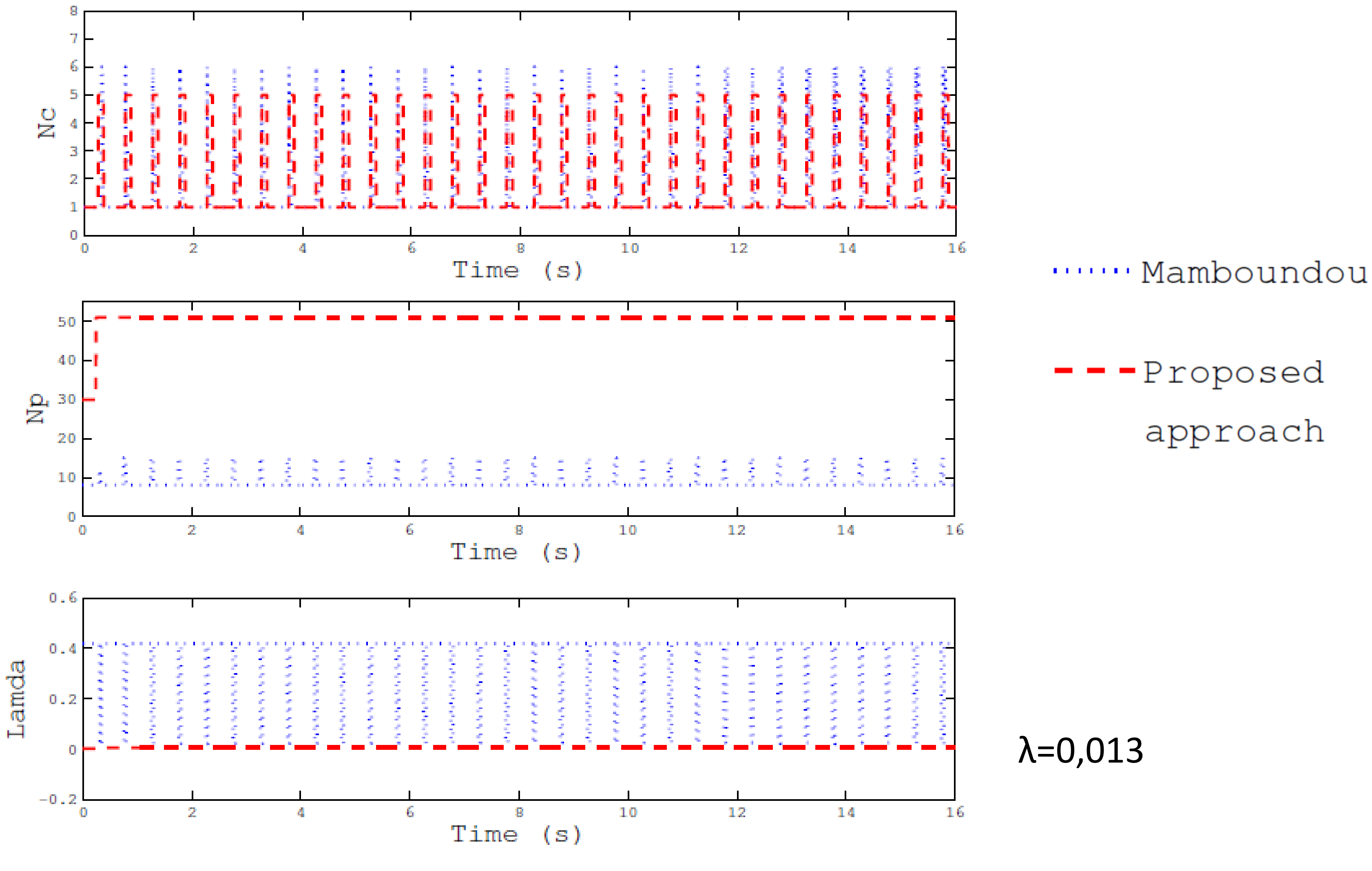
- Control output vs. time





Application : simulation results

- MPC parameters vs. time



λ=0,013

Application : perf. comparison (1/2)

- Performance criteria considered

- Rise Time (RT) from 10% to 90%
- Settling Time (ST) within 2%
- Overshoot (OV)
- Static Error (SE)
- Control Signal Variance (VARU)
- Control Signal Energy (CSE)
- Control Effort Energy (CEE)
- Computational Load (CL) [9]
- Condition Number of H

$$VARU(k) = \overline{u(k)^2} - \left[\overline{u(k)}\right]^2$$

$$CSE = \sum_{k=0}^{ST} u^2(k)$$

$$CEE = \sum_{k=0}^{ST} \Delta u^2(k)$$

Application : perf. comparison (2/2)

- Performance obtained

	Boucher [19]	Mamboundou [10]	Turki [20]	Proposed approach
VARU	0.2448	873.01	21.9565	0.2327
RT (s)	0.0017	0.0154	0.0006	0.0029
ST (s)	19.8560	19.8035	19.7920	19.8520
OV (%)	0.1076	0.0090	0.0869	0.0014
Static-error	3.6241	6.2455	0.0013	0.0005
CSE $(e+04)$	1.9734	2.7382	8.8843	1.8770
CEE	98.3452	97.8325	2254.8	75.8808
CL	3.8065	25.6039	33.0177	0.4901
Cond(H)	1	6.4881	6.4579	1.1940

Conclusion & Outlook

- **Approach interests :**
  - Applicability regardless the system order
  - Optimal closed-loop stability

- **Future work :**
  - MPC taking contraints into account
  - Case of NL MIMO systems

Thank you

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