





### Comparative Study Between Two Tuning MPC Approaches: Application to a Simulated QTW UAV

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## Outline

- Introduction
- The convertible UAVs
- QTW T-S Fuzzy Modeling
- TSMPC tuning based PSO approch
- TSMPC tuning based Analytical approch
- Comparative Study
- Conclusion & outlook

## Introduction

- Motivations
- Influence of parameters values
- Existing approaches

## The convertible UAVs (1/2)

UAVs are divided in two major categories :

- **The rotary-wing systems :** which lift is provided by the rotation of the propellers
  - ✓ Take-off and landing vertically, perform the hover flight,...
- **The fixed-wing systems :** which lift is provided by the airflow over the wings induced by the own movement of the vehicle
  - ✓ fly forward at high speed, long range, superior endurance,...



## The convertible UAVs (2/2)

### The convertible aircrafts :

- combines the advantages of rotary-wings and fixed-wings aircrafts
- minimizes the energy consumed in forward flights
- can take-off and landing vertically
- transition between the hover flight to forward flight and vice versa

### The transition between the flight phases :

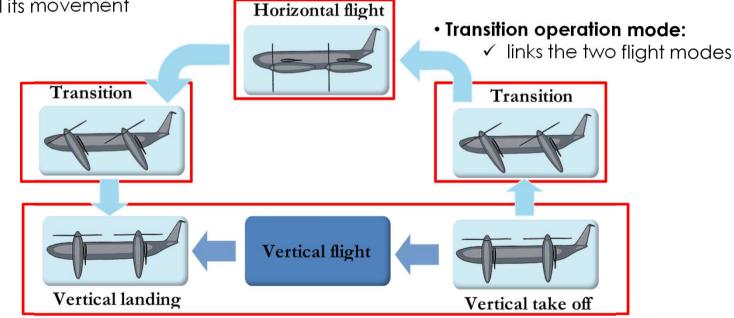
- tilting the full vehicle body (tail-sitter or tilt-body)
- tilting only its rotors using a dedicated mechanism (tilt-rotor or tilt-wing)



## Modeling of the QTW UAV (1/3)

#### • Horizontal flight mode :

- ✓ behaves like a conventional plane
- ✓ the tilt angle of wings almost equal to zero degree
- generates the aerodynamic forces to lift and control its movement



#### • Vertical flight mode:

- ✓ the tilt angles of the wings are nearly equal to 90°
- $\checkmark$  based only on its rotors
- ✓ behaves like a Quadrotors

### Modeling of the QTW UAV (2/3)

The NEWTON-EULER formalism

$$m\dot{V_i} = F_t$$
$$I_c\dot{\Omega}_c = -M_t + \Omega_c \wedge (I_c \times \Omega_c)$$

Translational equations of motion :

 $m\dot{V_i} = F_t$ with  $F_t$  is the total external force :

$$F_t = R_{ib}(F + F_g + F_d + F_a)$$

where :

F : the total thrust forces

 $F_{_{o}}$ : the gravity forces

 $F_d$ : the external disturbances

 $F_a$ : the aerodynamic forces

Rotational equations of motion :

$$I_{c}\dot{\Omega}_{c} = -M_{t} + \Omega_{c} \wedge (I_{c} \times \Omega_{c})$$

with  $M_t$  the total external torque :

$$M_{t} = M + M_{g} + M_{d}$$

where :

M : the thrust torques

 $M_g$ : the gyroscopic effects

 $M_d$  : the external disturbances

 $M_a$  : The aerodynamic torques

### Modeling of the QTW UAV (3/3)

Dynamics model of a QTW vehicle

$$\begin{cases} \ddot{x} = \frac{1}{m} \Big[ (c\psi c\theta c\gamma - (c\phi s\theta c\psi + s\phi s\psi) s\gamma) T_{t} + (c\psi c\theta) F_{D} + (c\phi s\theta c\psi + s\phi s\psi) F_{L} \\ \ddot{y} = \frac{1}{m} \Big[ (s\psi c\theta c\gamma - (c\phi s\theta c\psi - s\phi c\psi) s\gamma) T_{t} + (s\psi c\theta) F_{D} + (c\phi s\theta s\psi - s\phi c\psi) F_{L} \\ \ddot{z} = \frac{1}{m} \Big[ (-s\theta c\gamma - c\phi c\theta s\gamma) u_{I} - s\theta F_{D} + (c\phi c\theta) F_{L} \Big] + g \\ \ddot{\phi} = \frac{1}{I_{xx}} \Big[ s\gamma \tau_{\phi} + (I_{yy} - I_{zz}) qr - (J_{r}\theta \omega_{p}) s\gamma + M_{\phi} \Big] \\ \ddot{\theta} = \frac{1}{I_{yy}} \Big[ s\gamma \tau_{\theta} + (I_{zz} - I_{xx}) pr + J_{r}\omega_{p} (\phi s\gamma + \psi c\gamma) + M_{\theta} \Big] \\ \ddot{\psi} = \frac{1}{I_{zz}} \Big[ s\gamma \tau_{\psi} + (I_{xx} - I_{yy}) pq - (J_{r}\theta \omega_{p}) c\gamma + M_{\psi} \Big] \end{cases}$$

The control inputs of the QTW :

$$U = \begin{bmatrix} T_t \\ \tau_{\phi} \\ \tau_{\theta} \\ \tau_{\psi} \end{bmatrix} = \begin{bmatrix} k & k & k & k \\ kl_s & -kl_s & kl_s & -kl_s \\ kl_l & kl_l & -kl_l & -kl_l \\ k\lambda & -k\lambda & -k\lambda & k\lambda \end{bmatrix} \begin{bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{bmatrix}$$

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# QTW T-S Fuzzy Modeling (1/4)

### Takagi-Sugeno Fuzzy Models

• T-S models are based on linguistic representation rules such as [Takagi et Sugeno (1985)] :

#### If premise Then consequence

• If we use singleton fuzzifier, product inference engine and center of gravity defuzzification the T-S fuzzy model can be represented as :

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{N} \mu_i(\theta) \left( A_i \ x(t) + B_i \ u(t) \right) \\ y(t) &= \sum_{i=1}^{N} \mu_i(\theta) \ C_i \ x(t) \end{aligned}$$

 $\sum_{i=1}^{N} \mu_i(x(t), u(t)) = 1 \text{ et } 0 \le \mu_i(x(t), u(t)) \le 1$ 

### **Obtaining T-S models**

- Identification [Tanaka et Sugeno (1992)]
- Linearization around different operating points [Johansen et al. (2000)]
- Sectors non linearity approach [Tanaka et al. (1998)] : Convex Polytopic Transformation (CPT)

# QTW T-S Fuzzy Modeling (2/4)

• 1st step : the transformation of a nonlinear dynamic model into an LPV model

$$\begin{cases} \dot{x}(t) = A(\theta) \ x(t) + B(\theta) \ u(t) \\ y(t) = C(\theta) \ x(t) \end{cases}$$

• **2nd step :** the convex polytopic transformation is applied for each premise variables :

$$\theta(x,u) = F_0^j \overline{\theta} + F_1^j(\theta) \underline{\theta} \quad \text{as} \quad \begin{cases} \overline{\theta} = \max\{\theta(x,u)\} \\ \underline{\theta} = \min\{\theta(x,u)\} \end{cases}$$
  
ith 
$$F_1^j = 1 - \mu_1(\theta) = \frac{\overline{\theta} - \theta(x,u)}{\overline{\theta} - \underline{\theta}} \quad \text{and} \quad F_0^j = \frac{\theta(x,u) - \underline{\theta}}{\overline{\theta} - \underline{\theta}}$$

The membership functions :

W

$$\mu_i(\theta(t)) = \frac{w_i(\theta(t))}{\sum_{i=1}^r w_i(\theta(t))} \quad \text{with} \quad w_i(\theta(t)) = \prod_{j=1}^r w_{i_j}^j(\theta_j)$$

• **3nd step :** The local models constituting the T-S models are given by:

$$A_{i} = A(\theta(.))|_{w_{i}(.)=1}, B_{i} = B(\theta(.))|_{w_{i}(.)=1}, C_{i} = C(\theta(.))|_{w_{i}(.)=1}, D_{i} = D(\theta(.))|_{w_{i}(.)=1}$$

## QTW T-S Fuzzy Modeling (3/4)

• QTW UAV attitude dynamics during the VTOL mode :

$$\begin{cases} \ddot{\varphi} = \frac{\tau_{\phi}}{I_{xx}} + \frac{I_{yy} - I_{zz}}{I_{xx}} qr - \frac{J_r}{I_{xx}} q\omega_p \\ \ddot{\theta} = \frac{\tau_{\theta}}{I_{yy}} + \frac{I_{zz} - I_{xx}}{I_{yy}} pr + \frac{J_r}{I_{yy}} p\omega_p \quad \text{with} \quad \begin{cases} \phi \in [-\pi/2, +\pi/2[] \\ \theta \in [-\pi/2, +\pi/2[] \\ \theta \in [-\pi/2, +\pi/2[] \\ \psi \in [-\pi, +\pi[] \end{cases} \end{cases} \\ \psi \in [-\pi, +\pi[] \end{cases}$$

• The quasi-LPV system obtained :

$$\begin{cases} \dot{x}(t) = A(\theta) \ x(t) + B \ u(t) \\ y(t) = C \ x(t) \end{cases}$$

with :

$$A(x) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_1 \dot{\theta} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_2 \dot{\phi} \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & a_3 \dot{\theta} & 0 & 0 \end{bmatrix}; B = \begin{bmatrix} 0 & 0 & 0 \\ b_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & b_3 \end{bmatrix}$$
$$a_1 = \frac{I_{yy} - I_{zz}}{I_{xx}}; a_2 = \frac{I_{zz} - I_{xx}}{I_{yy}}; a_3 = \frac{I_{xx} - I_{yy}}{I_{zz}}$$

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# QTW T-S Fuzzy Modeling (4/4)

- Based on the sectors non linearity approach two nonlinear continuous terms  $\left(\dot{\phi},\dot{\theta}
  ight)$  can be observed.
- The nonlinear model is written as four local linear models  $(r = 2^p = 2^2)$  as :

$$A_{1} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & a_{1}\underline{z}_{2} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & a_{2}\underline{z}_{1} \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & a_{3}\underline{z}_{2} & 0 & 0 \end{bmatrix} \quad A_{2} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{1}\overline{z}_{2} \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & a_{3}\overline{z}_{2} & 0 & 0 \end{bmatrix}$$
$$A_{3} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{1}\underline{z}_{2} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{2}\overline{z}_{1} \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & a_{3}\overline{z}_{2} & 0 & 0 \end{bmatrix} \quad A_{4} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{2}\overline{z}_{1} \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & a_{3}\overline{z}_{2} & 0 & 0 \end{bmatrix}$$

### Particle Swarm Optimization(1/3)

- The PSO algorithm was first described in 1995 by **James Kennedy** and **Russell C. Eberhart** inspired by social behavior of bird flocking or fish schooling.
- PSO is an **artificial intelligence (AI)** technique that can be used to find approximate solutions to extremely difficult or impossible numeric maximization and minimization problems.
- PSO algorithm uses a swarm consisting of  $n_p$  particles, randomly distributed in the considered initial search space, to find an optimal solution of a generic **optimization problem**.

### Particle Swarm Optimization(2/3)

- In every iteration, each particle is updated by two "**best**" values:
  - ✓  $p_k^i$  : the best previously obtained position of the ith particle; ✓  $p_g^i$  : the best obtained position in the entire swarm.
- At each algorithm iteration, the ith particle position,  $x^i \in \mathbb{R}^d$ , evolves based on the following update rules:

$$\boldsymbol{x}_{k+1}^{i} = \boldsymbol{x}_{k}^{i} + \boldsymbol{v}_{k+1}^{i}$$
$$\boldsymbol{v}_{k+1}^{i} = w\boldsymbol{v}_{k}^{i} + c_{1}r_{1,k}^{i}\left(\boldsymbol{p}_{k}^{i} - \boldsymbol{x}_{k}^{i}\right) + c_{2}r_{2,k}^{i}\left(\boldsymbol{p}_{k}^{g} - \boldsymbol{x}_{k}^{i}\right)$$

Where

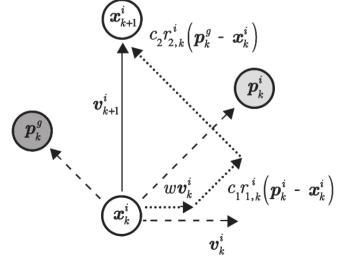
W : the inertia factor;

 $c_1, c_2$ : the cognitive and the social scaling factors;  $r_{1,k}^i, r_{2,k}^i$ : random numbers uniformly distributed.

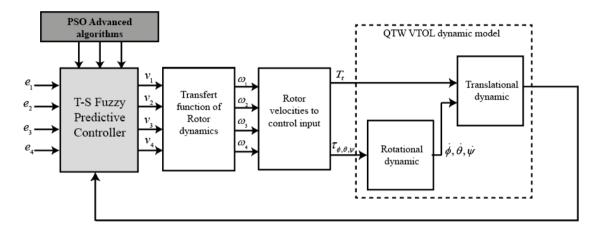
### Particle Swarm Optimization(3/3)

### Particle Swarm Optimization Algorithm

- 1. Define all PSO algorithm parameters
- 2. Randomly initialize the particles positions and velocities. Evaluate the initial population and determine and .
- 3. Increment the iterations . For each particle, apply the update motion equations
- 4. Evaluate the corresponding fitness values :
  - i. if  $\boldsymbol{\varphi}_k^i \leq pbest_k^i$  then  $pbest_k^i = \boldsymbol{\varphi}_k^i$  and  $\boldsymbol{p}_k^i = \boldsymbol{x}_k^i$ ,
  - ii. if  $\boldsymbol{\varphi}_k^i \leq gbest_k$  then  $gbest_k = \boldsymbol{\varphi}_k^i$  and  $\boldsymbol{p}_k^g = \boldsymbol{x}_k^i$ ,
- 5. If the termination criterion is satisfied, the algorithm terminates with the solution. Otherwise, go to step 3.  $(r^i)$



## TSMPC tuning based PSO



• An optimization based approach for design and tuning of a MPC parameters

 $(N_c, N_p, \lambda)$  for each local predictive controller.

• These design parameters present the decision variables of the following multiobjective optimization problem :

$$\begin{cases} \text{minimize } f_i(\mathbf{x}) \\ \mathbf{x} = \begin{pmatrix} N_c, N_p, \mathbf{\lambda} \end{pmatrix}^T \in S \subseteq \square_+^{12} \\ \text{subject to:} \\ g_1(\mathbf{x}) = \delta_{\phi} - \delta_{\phi}^{\max} \leq 0 \\ g_2(\mathbf{x}) = \delta_{\theta} - \delta_{\theta}^{\max} \leq 0 \\ g_3(\mathbf{x}) = \delta_{\psi} - \delta_{\psi}^{\max} \leq 0 \end{cases} \quad \text{where :} \begin{cases} f_i : \square^{15} \to \square \\ \delta_{\phi}, \delta_{\theta}, \delta_{\psi} \end{cases} \text{ : the overshoots of the controlled states} \end{cases}$$

# Analytical tuning approch proposed N<sub>c</sub> computation (1/2)

• Hessian condition number

 $H = (\Phi^t \Phi + \bar{R})^{-1}$ 

 $(n_{in}\dot{Nc} \times n_{in}Nc)$ 

$$\begin{aligned} cond(H(k)) &= \|H(k)\|_2 \cdot \left\|H(k)^{-1}\right\|_2 \\ &= \frac{\sigma_{max}(k)}{\sigma_{min}(k)} \end{aligned}$$

- Condition number & stability  $N_c \mapsto \infty, N_p \mapsto \infty$
- Concept of effective rank

# $N_c$ computation (2/2)

- Method proposed
- 1) To initialize  $N_c \mapsto \infty$  and  $N_p \mapsto \infty$  (Nc < Np).
- 2) To take  $A_{ER} = H$  [16].
- 3) To evaluate Q defined as follows [9]:

$$Q = min\{M_{ER}, N_{ER}\} = min\{n_{in}Nc, n_{in}Nc\}$$
$$= n_{in}Nc.$$

- 4) To decompose *H* into singular values and to evaluate  $\sigma = \begin{bmatrix} \sigma_1 & \sigma_2 & \cdots & \sigma_\infty \end{bmatrix}^T$ :  $H = U_H D_H V_H$  $\sigma_1 \ge \sigma_2 \ge \cdots \ge \sigma_\infty$
- 5) To evaluate the singular value distribution  $p_k$  with  $k = [1, 2, \dots, \infty]$ .
- 6) To calculate the Shannon entropy [16].

7) To solve 
$$\begin{cases} N_c^{opt} = round(\frac{e^{H_{Shannon}(p_1, p_2, \cdots, p_{\infty})}}{n_{in}})\\ \min(N_c^{opt}); \ \min(cond(H(k)) - 1) \end{cases}$$
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# $N_p$ computation (1/3)

Closed-loop eigenvalues

$$det[\nu I_{(n_A \times n_A)} - (A - BK_{mpc})] = 0$$

• Optimal closed-loop stability

Insensitivity  
approach 
$$\left\{ \begin{array}{cc} \min \gamma & \text{Yalmip} \\ \hline & 1 \\ (A - BK_{mpc})^T & -\gamma I_{(n_A \times n_A)} \end{array} \right\} < 0 \qquad (K_{mpc})^{opt}$$

# $N_p$ computation (2/3)

• Relationship between  $K_{MPC}$  and  $\Phi^T \Phi$ 

From 
$$K_{mpc} = I_{(n_{in} \times N_c)} (\Phi^T \Phi + \bar{R})^{-1} \Phi^T F$$

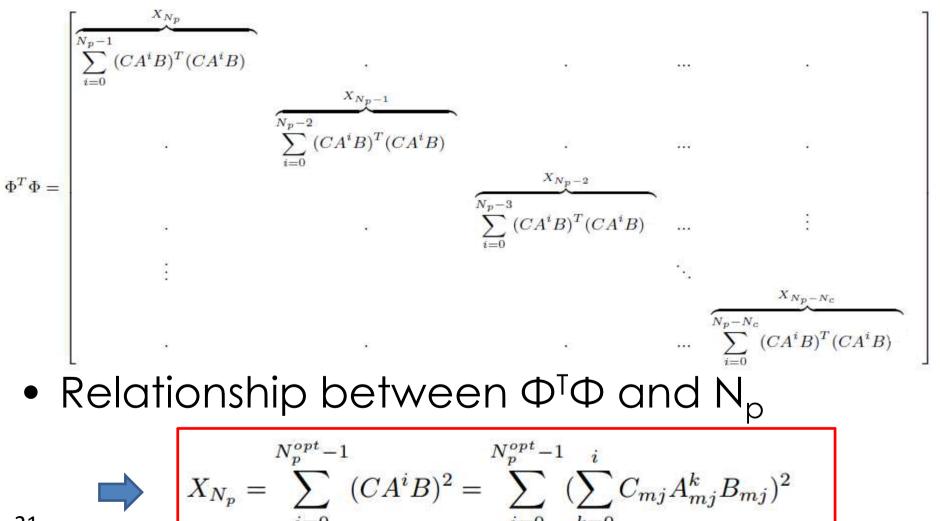
Under the assumption  $(\Phi^T \Phi)^{-1} \overline{R}$  is nilpotent of order n,

$$H = (\Phi^T \Phi)^{-1} - (\Phi^T \Phi)^{-1} \bar{R} (\Phi^T \Phi)^{-1} + [(\Phi^T \Phi)^{-1} \bar{R}]^2 (\Phi^T \Phi)^{-1} - \cdots + (-1)^{n-1} [(\Phi^T \Phi)^{-1} \bar{R}]^{n-1} (\Phi^T \Phi)^{-1}$$

$$(\Phi^{T}\Phi)^{opt} = [-\bar{R} + \bar{R}(\Phi^{T}\Phi)^{-1}\bar{R} - \cdots + (-1)^{n-1}(\Phi^{T}\Phi)^{-1}[(\Phi^{T}\Phi)^{-1}\bar{R}]^{n-1}] \times [(K_{mpc})^{opt}F^{T}(FF^{T})^{-1}\Phi - I_{(n_{in}N_{c}\times n_{in}Nc)}]^{-1}$$

 $N_p$  computation (3/3)

• Focus on  $\Phi^T \Phi$ 



# $\overline{R}$ computation (1/2)

• Expanded cost fonction [11]

$$J = (Y_{des} - Fx(k))^{t} (Y_{des} - Fx(k))$$
$$- 2\Delta U^{t} \Phi^{T} (Y_{des} - Fx(k)) + \Delta U^{t} (\Phi^{t} \Phi + \bar{R}) \Delta U$$

• Cost fonction minimization

$$\begin{cases}
\frac{\partial J}{\partial \Delta U} = 0 \\
\frac{\partial J}{\partial R} = 0
\end{cases}$$

# $\overline{R}$ computation (2/2)

• Cost fonction derivative

$$\frac{\partial J}{\partial \bar{R}} = (\Phi^T \Phi)^{-1} G^T G (\Phi^T \Phi)^{-1}$$
$$- 2(\Phi^T \Phi)^{-1} \bar{R} (\Phi^T \Phi)^{-1} G^T G (\Phi^T \Phi)^{-1}$$

with 
$$\begin{cases} G = \Psi^T \Phi \\ \Psi = Y_{des} - Fx(k) \end{cases}$$

$$\bar{R}_{opt} = \frac{1}{2} (\Phi^T \Phi) \qquad \implies \quad \bar{R}_{opt} = \begin{bmatrix} \lambda_1^{opt} & 0 & \cdots & 0 \\ 0 & \lambda_2^{opt} & \cdots & \vdots \\ \vdots & 0 & \cdots & 0 \\ 0 & \cdots & 0 & \lambda_{n_{in}N_c}^{opt} \end{bmatrix}$$

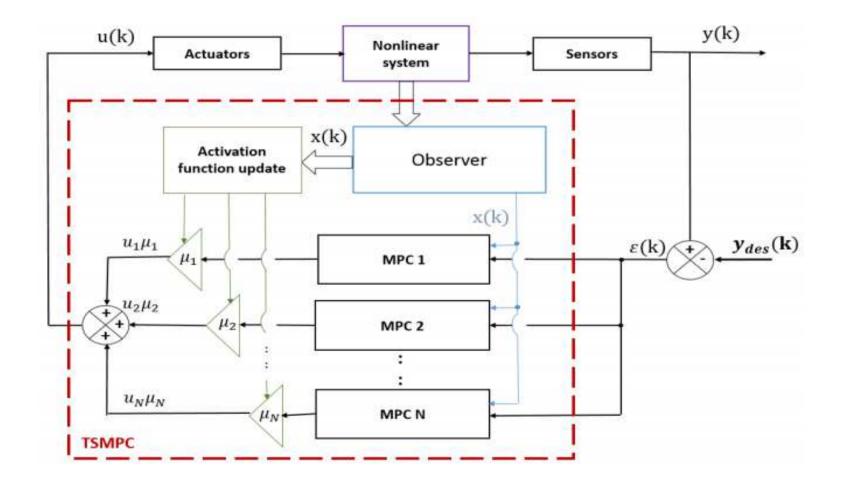
## In summary

• Multi-objective issue adressed

$$\begin{split} \min(N_c^{opt}) \\ \min(cond(H(k)) - 1) \\ N_c^{opt} = round(\frac{e^{H_{Shannon}(p_1, p_2, \dots, p_\infty)}}{n_{in}}) \\ X_{N_p} &= \sum_{i=0}^{N_p^{opt} - 1} (\sum_{k=0}^i (C_m A_m^k B_m)^T (C_m A_m^k B_m)) \\ \bar{R}_{opt} &= \frac{1}{2} (\Phi^T \Phi) \end{split}$$

## Case of Nonlinear MIMO System

• MPC Control structure



## Application : comparative study

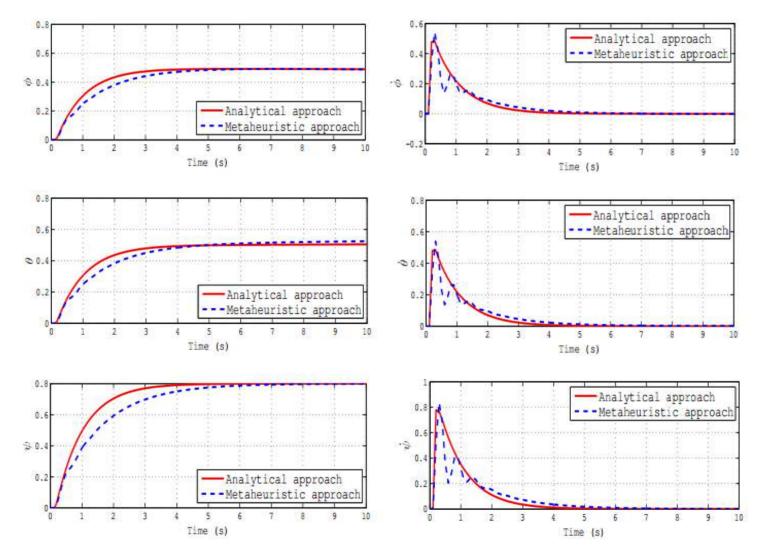
- Computing the weighting factor based on
  - Our metaheuristic approch
  - Our analytical approach

• MPC parameters

	Metaheuristic approach				Analytical approach			
	OA1	OA2	OA3	OA4	OA1	OA2	OA3	OA4
$N_c$	5	5	5	5	5	5	5	5
$N_p$	29	29	29	29	29	29	29	29
•	0.9450	0.0495	0.1655	3	0.0095	0.0033	0.0133	0.051
$\lambda$	1	1.6982	2.2678	2.9962	0.008	0.0024	0.065	1.022
	0.0297	3	0.0018	2.9807	0.0067	0.005	0.0065	0.0065

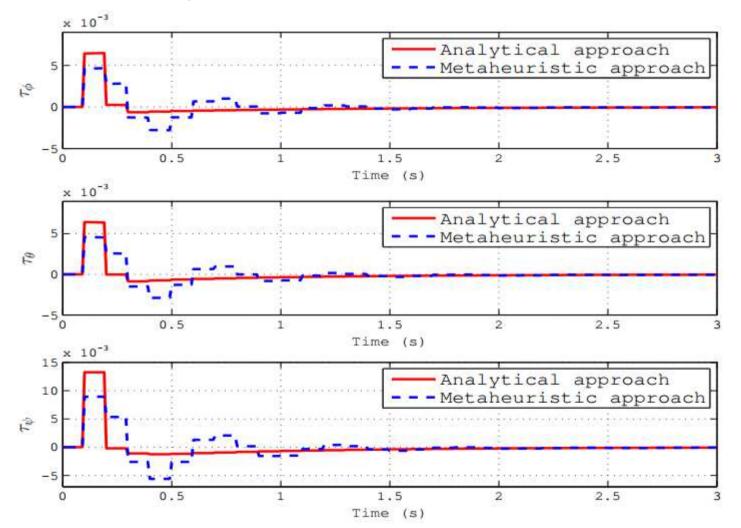
## Application : simulation results

• Desired output and system output vs. time



## Application : simulation results

• Control output vs. time



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### Application : perf. comparison (2/3)

• Performance obtained y1

	Metaheuristic approach	Analytical approach
RT (s)	2.1895	1.9747
ST (s)	3.8713	2.9816
OV (%)	1.2457	0
SE	0.0285	0.0091
SDI (%)	1.7764	1.1102
VARU $(e^-7)$	5.4067	5.1076
CSE $(e^-4)$	1.6959	1.3961
<b>CEE</b> $(e^{-4})$	2.9198	2.7208

• Performance obtained y2

	Metaheuristic approach	Analytical approach
RT (s)	2.0893	1.3022
ST (s)	3.8225	2.6328
OV (%)	2.1384	1.4898
SE	0.0149	0.0214
SDI (%)	3.1086	3.1086
VARU $(e^-7)$	5.3987	5.1076
CSE $(e^-4)$	1.6935	1.3961
<b>CEE</b> $(e^{-4})$	2.9121	2.7208

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### Application : perf. comparison (3/3)

• Performance obtained y3

	Metaheuristic approach	Analytical approach
RT (s)	2.0358	1.3415
ST (s)	3.7998	2.6161
OV (%)	2.0401	0
SE	0.0245	0.0002
SDI (%)	1.9984	1.2273
VARU $(e^-7)$	5.4068	5.4026
CSE $(e^-4)$	1.6959	1.5961
<b>CEE</b> $(e^{-4})$	2.9200	2.9107

## Conclusion & Outlook

#### **PSO Tuning approach:**

- Advantages
- Applicable to NL MIMO systems
- Efficient search for the optimal MPC controller parameters
- Easy implementation

#### Disadvantages

- Offline optimization
- Number of iteration
- Don't take into account the disturbances

#### Analytical Tuning approach:

#### Advantages

- Applicable to NL MIMO systems
- Optimal closed-loop stability
- Energy consumption reduced

#### Disadvantages

- Requiert a representative model
- Requiert an important computational effort to be applied online







### Thank you



PREDIRE est cofinancé par l'Union européenne. L'Europe s'engage en Normandie avec le Fonds européen de développement régional GT CPNL ONERA Châtillon le 04/06/2018

