

Practical implementation of Tube-Based MPC for the stabilization of a quadrotor UAV

Nathan Michel
Doctorant 3^{ème} année

Directeur(s) de thèse : OLARU Sorin (L2S CentraleSupélec)

Co-encadrants : BERTRAND Sylvain (ONERA), VALMORBIDA Giorgio (L2S CentraleSupélec), DUMUR Didier (L2S CentraleSupélec)

Jeudi 13 juin 2019

Outline

- 1 Context
- 2 Tube-Based MPC
- 3 Experimental setup
- 4 System identification and parameter tuning
- 5 Experimental validation
- 6 Conclusion

Outline

- 1 Context
- 2 Tube-Based MPC
- 3 Experimental setup
- 4 System identification and parameter tuning
- 5 Experimental validation
- 6 Conclusion

Context

Motivation

- Increased interest in UAVs to perform automated tasks (exploration, observation).
- The flight area may contain physical obstacles.
- Unknown external forces can act on the system.



Quadrotor



Indoor flight



Disturbance

Context

Problematic of the study

Design a control law that allows to

- fulfill a mission (**stabilisation**, trajectory tracking, ...),
- autonomously (real-time implementable),
- while satisfying physical constraints (position of the obstacles, speed limitation, actuator saturation...),

regardless of the disturbance.



FIGURE – 1. Parrot AR.Drone 2.0. 2. Motion capture system.

State of the art and motivations

Control of the Parrot AR.Drone 2.0

There exists in the literature plenty of strategies for the control of the Parrot AR.Drone 2.0.

- PID and IMC controllers (Hernandez (2013))
- inverse dynamics controller (Santana (2014))
- MPC (Hernandez (2014))

However, these approaches do not guarantee robust constraints satisfaction.

Modelling of the Parrot AR.Drone 2.0

The modelling of the Parrot AR.Drone horizontal dynamics have been extensively studied

- 2-dimensional discrete-time linear system with input delay (Hernandez (2013)),
- 3-dimensional discrete-time linear system with input delay (Engel(2012), Stevek (2016)).

The horizontal dynamics of the Parrot AR.Drone 2.0 can be modelled as a discrete-time linear system.

Proposed approach : Tube-Based Model Predictive Control

Theoretical context

Control of a discrete-time linear system subject to bounded additive disturbance, and state and control input constraints.

Model Predictive Control (MPC)

- Control strategy suited to constraints handling (Mayne et al. (2000))
- Linear dynamics and affine constraints : quadratic programming

Two main approaches to handle the presence of disturbance

- Min-Max MPC (Campo & Morari (1987))
- Tube-Based MPC (Mayne et al. (2006))

Related problems

- Joint design of local controllers and invariant sets (Kolmanovsky (1998), Blanchini (1999), Raković (2007), Oлару (2010))
- Identification of the discrete-time linear system and the disturbance bounds

Outline

- 1 Context
- 2 Tube-Based MPC**
- 3 Experimental setup
- 4 System identification and parameter tuning
- 5 Experimental validation
- 6 Conclusion

Models of the system

Uncertain system

Consider a discrete-time linear system subject to bounded additive disturbance and polytopic state and control input constraints,

$$\xi_{k+1} = A\xi_k + Bu_k + w_k, \xi_k \in \mathcal{X} \subseteq \mathbb{R}^n, u_k \in \mathcal{U} \subseteq \mathbb{R}^m, w_k \in \mathcal{W} \subseteq \mathbb{R}^n, k \in \mathbb{N}.$$

This system will be referred to as the *uncertain system*.

Due to the presence of disturbance, it is impossible to predict the evolution of the uncertain system.

Nominal system

The prediction model of the uncertain system system is given by the disturbance free system

$$\bar{\xi}_{k+1} = A\bar{\xi}_k + B\bar{u}_k, k \in \mathbb{N}.$$

This system will be referred to as the *nominal system*.

Control of the error system

Error system

We define the *error system* as the difference between the uncertain and the nominal systems, $z_k = \xi_k - \bar{\xi}_k$. Its evolution is given by

$$z_{k+1} = A(\xi_k - \bar{\xi}_k) + B(u_k - \bar{u}_k) + w_k, w_k \in \mathcal{W}, k \in \mathbb{N}.$$

Linear state feedback control law

We consider the following linear state feedback control law for the control of the error system,

$$u_k = \bar{u}_k + K(\xi_k - \bar{\xi}_k), k \in \mathbb{N}, K \in \mathbb{R}^{m \times n}$$

leading to the closed-loop system

$$z_{k+1} = (A + BK)z_k + w_k, w_k \in \mathcal{W}, k \in \mathbb{N}.$$

The linear state feedback gain K has to satisfy that $A + BK$ is Schur.

It is possible to characterize a bound for the error z_k using invariant set theory.

Robust Positively Invariant set

Robustly Positively Invariant set

The set \mathcal{Z} is a Robustly Positively Invariant (RPI) set for the system $z_{k+1} = (A + BK)z_k + w_k$, $w_k \in \mathcal{W}$ if $\forall z \in \mathcal{Z}$,

$$(A + BK)z + w_k \in \mathcal{Z}, \forall w_k \in \mathcal{W}.$$

Recursive bounding of the error system state

Let $K \in \mathbb{R}^{m \times n}$ be such that $A + BK$ is Schur, and \mathcal{Z} be an associated RPI set. Consider $\xi_0 \in \mathcal{X}$ and $\bar{\xi}_0 \in \mathbb{R}^n$ be such that

$$z_0 = \xi_0 - \bar{\xi}_0 \in \mathcal{Z}.$$

With the control law $u_k = \bar{u}_k + K(\xi_k - \bar{\xi}_k)$, $k \in \mathbb{N}$, the state of the error system satisfies

$$z_k = \xi_k - \bar{\xi}_k \in \mathcal{Z}, \forall w_k \in \mathcal{W}, \forall k \in \mathbb{N}.$$

The control policy keeps the state of the uncertain system close to the state of the nominal system.

Recursive constraints satisfaction

Pontryagin difference

The Pontryagin difference of two sets $\mathcal{S}_1 \subseteq \mathbb{R}^n$ and $\mathcal{S}_2 \subseteq \mathbb{R}^n$ is given by the set

$$\mathcal{S}_1 \ominus \mathcal{S}_2 = \{s \in \mathbb{R}^n \mid \forall s' \in \mathcal{S}_2, s + s' \in \mathcal{S}_1\}.$$

Note that if $\mathcal{S}_2 \not\subseteq \mathcal{S}_1$, then the set $\mathcal{S}_1 \ominus \mathcal{S}_2$ is empty.

Nominal system constraints

We introduce the following state and control input constraints for the nominal system,

$$\bar{\xi} \in \bar{\mathcal{X}} = \mathcal{X} \ominus \mathcal{Z}, \bar{u} \in \bar{\mathcal{U}} = \mathcal{U} \ominus K\mathcal{Z}.$$

Hence, if $\xi_k - \bar{\xi}_k \in \mathcal{Z}, \bar{\xi}_k \in \bar{\mathcal{X}}, \bar{u}_k \in \bar{\mathcal{U}}, \forall k \in \mathbb{N}$, then

$$\xi_k \in \mathcal{X}, u_k \in \mathcal{U} \forall k \in \mathbb{N}.$$

The control policy ensures that the satisfaction of the nominal system constraints implies the satisfaction of the uncertain system constraints.

Control of the nominal system

There is no uncertainty on the evolution of the nominal system state.

Model Predictive Control of the nominal system

We consider the following optimization problem for the control of the nominal system to ensure the recursive satisfaction of its constraints.

$$\begin{aligned}
 P_N(\bar{\xi}) = \quad & \underset{\bar{u}_0, \dots, \bar{u}_{N-1}}{\text{minimize}} && \sum_{i=0}^{N-1} (\|\bar{\xi}_i\|_Q + \|\bar{u}_i\|_R) + \|\bar{\xi}_N\|_P, \\
 & \text{subject to} && \bar{\xi}_{i+1} = A\bar{\xi}_i + B\bar{u}_i, i = \{0, \dots, N-1\}, \\
 & && \bar{\xi}_i \in \bar{\mathcal{X}}, i = \{1, \dots, N-1\}, \\
 & && \bar{u}_i \in \bar{\mathcal{U}}, i = \{0, \dots, N-1\}, \\
 & && \bar{\xi}_0 = \bar{\xi}, \\
 & && \bar{\xi}_N \in \bar{\mathcal{X}}_f.
 \end{aligned}$$

Notation

We define $\|x\|_Q, x \in \mathbb{R}^l, Q \in \mathbb{R}^{l \times l}, Q$ positive definite, as the positive scalar $x^\top Q x \geq 0$.

Stability and change of reference

Stability requirements

The terminal constraints set and terminal cost have to satisfy the usual requirements, namely

$$(A + BK)\bar{\mathcal{X}}_f \subseteq \bar{\mathcal{X}}_f$$

$$\bar{\mathcal{X}}_f \subseteq \bar{\mathcal{X}}$$

$$K\bar{\mathcal{X}}_f \subseteq \bar{\mathcal{U}}$$

$$(A + BK)^\top P(A + BK) + Q + K^\top RK - P \leq 0.$$

For a given linear state feedback gain K , the largest set satisfying these axioms is the Maximal Output Admissible Set (MOAS).

The optimization problem has been established for the stabilization around the origin. For a reference ξ^r, u^r , we consider $\xi' = \xi - \xi^r$, $u' = u - u^r$, $\mathcal{X}' = \mathcal{X} \oplus \{-\xi^r\}$, and $\mathcal{U}' = \mathcal{U} \oplus \{-u^r\}$.

The translation of the constraint sets can invalidate the above axioms. Hence, the terminal set is a function of the chosen reference.

Online computation of the terminal set

For a given reference ξ^r , the terminal set $\bar{\mathcal{X}}_f(\xi^r)$ has to satisfy

$$\begin{aligned}(A + BK)\bar{\mathcal{X}}_f(\xi^r) &\subseteq \bar{\mathcal{X}}_f(\xi^r), \\ \bar{\mathcal{X}}_f(\xi^r) &\subseteq \bar{\mathcal{X}} \oplus \{-\xi^r\}, \\ K\bar{\mathcal{X}}_f(\xi^r) &\subseteq \bar{\mathcal{U}} \oplus \{-u^r\}.\end{aligned}$$

The MOAS for the origin, denoted $\bar{\mathcal{X}}_f(0)$, is computed offline.

The terminal set $\bar{\mathcal{X}}_f(\xi^r)$ is chosen as $\lambda(\xi^r)\bar{\mathcal{X}}_f(0)$, where $\lambda(\xi^r) > 0$ is the largest scalar satisfying

$$\begin{aligned}\lambda(\xi^r)\bar{\mathcal{X}}_f(0) &\subseteq \bar{\mathcal{X}} \oplus \{-\xi^r\}, \\ K\lambda(\xi^r)\bar{\mathcal{X}}_f(0) &\subseteq \bar{\mathcal{U}} \oplus \{-u^r\}.\end{aligned}$$

The computation of $\lambda(\xi^r)$ consists in solving online a linear optimization problem.

Nominal predictive control law

Solution of the optimization problem

The solution of the optimal control problem yields the optimal control sequence

$$\{\bar{u}_0^*(\bar{\xi}), \dots, \bar{u}_{N-1}^*(\bar{\xi})\},$$

and the associated optimal state sequence

$$\{\bar{\xi}_0^*(\bar{\xi}), \dots, \bar{\xi}_N^*(\bar{\xi})\}.$$

The implicit model predictive control law is therefore

$$\bar{u}(\bar{\xi}) = \bar{u}_0^*(\bar{\xi}).$$

The evolution of the uncertain system state is given by

$$\xi_{k+1} = A\xi_k + B(\bar{u}_0^*(\bar{\xi}_k) + K(\xi_k - \bar{\xi}_k)) + w_k, w_k \in \mathcal{W}, k \in \mathbb{N}.$$

Recursive state and control input constraints satisfaction is ensured if $\xi_0 - \bar{\xi}_0 \in \mathcal{Z}$ and $\bar{\xi}_0 \in \bar{\mathcal{X}}$.

Challenges of the Tube-Based MPC

System identification and disturbance realizations

The disturbances encountered during the validation flight have to be contained in the disturbance set \mathcal{W} used for the design of the invariant set \mathcal{Z} .

Invariant set design

The invariant set \mathcal{Z}

- defines the sets $\bar{\mathcal{X}} = \mathcal{X} \oplus \mathcal{Z}$ and $\bar{\mathcal{U}} \oplus K\mathcal{Z}$,
- bounds the error $\xi_k - \bar{\xi}_k$.

The set \mathcal{Z} has to be as *small* as possible.

Terminal set constraints and region of attraction

The basin of attraction is a function of the terminal set $\bar{\mathcal{X}}_f$ and the nominal constraints sets $\bar{\mathcal{X}}$ and $\bar{\mathcal{U}}$. The larger is the terminal set, the larger is the basin of attraction of the control law.

Outline

- 1 Context
- 2 Tube-Based MPC
- 3 Experimental setup**
- 4 System identification and parameter tuning
- 5 Experimental validation
- 6 Conclusion

MotionCapture system



FIGURE – 1. Parrot AR.Drone 2.0. 2. Motion capture system.

The MotionCapture system provides an accurate measurement of the position (x, y, z) , speed (v_x, v_y, v_z) and roll, pitch and yaw angles (θ, ϕ, ψ) of the quadrotor.

The rotational speed of the four rotors is not accessible. The quadrotor is equipped with embedded control laws that stabilize the quadrotor with regards to user inputs.

The control law is computed on a PC station and transmitted by WiFi to the quadrotor.

Control of the Parrot AR.Drone 2.0

Embedded control laws

The embedded control laws allow to regulate the roll angle, pitch angle, yaw rate and vertical speed.

We denote the reference values, the inputs to the UAV, by $[\theta^r \quad \phi^r \quad \dot{\psi}^r \quad v_z^r]$.

Assumptions

- The yaw angle and vertical speed are 0.
- Small roll and pitch angles.

The dynamics in the direction x and y can be considered as identical and decoupled. For clarity purposes, we present only the x -direction in the following.

First order regulation

The pitch angle dynamics can be described as the first-order system (Engel (2012))

$$\dot{\phi} = -C(\phi - \phi^r) + \delta.$$

Horizontal dynamics

Continuous-time model

The dynamics in the x -direction can be modelled by the continuous-time linear system

$$\dot{\xi} = A_c \xi + B_c u + \Delta_x,$$

$$\xi = \begin{bmatrix} x \\ v_x \\ \phi \end{bmatrix}, A_c = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & \frac{F}{m} \\ 0 & 0 & -C \end{bmatrix}, B_c = \begin{bmatrix} 0 \\ 0 \\ C \end{bmatrix}, \Delta_x = \begin{bmatrix} 0 \\ f \\ \frac{m}{\delta} \end{bmatrix}.$$

where $u = \phi^r$.

Discrete-time model

A discretized model is considered for the design of the Tube-Based MPC law,

$$\xi_{k+1} = A \xi_k + B u_k + w_k,$$

where $\xi_k = \xi(k\tau) \in \mathbb{R}^3$ is the state, $u_k = \phi^r(k\tau) \in \mathbb{R}$ is the control input, and $w_k \in \mathbb{R}^3$ is the additive disturbance. We chose $\tau = 0.05ms$.

A similar model is obtained for the dynamics in the y -direction.

Delay in the communication

The control inputs $\begin{bmatrix} \theta^r & \phi^r & \psi^r & v_z^r \end{bmatrix}$ are sent by WiFi to the UAV, inducing delay in the transmission of the control signal.

System modelling and state augmentation

Taking into account the presence of delay requires a state augmentation, which

- increases the dimension of the optimization problem.
- complexifies the computation of invariant sets.

Disturbance sources

The disturbance w_k is a joint effect of

- modelling assumptions (small angles and dynamics decoupling)
- external forces (wind, ground effect, ...)
- delay in the communication of the command
- errors in the state measurement.

Code structure

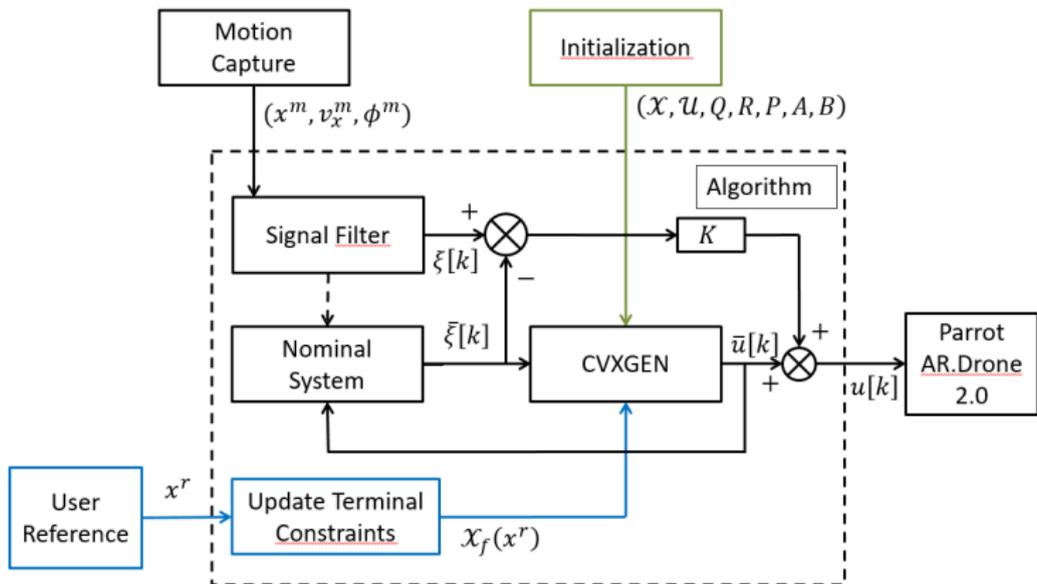


FIGURE – Structure of the Tube-Based Model Predictive Controller.

We use CVXGEN to generate the code for QP-representable convex optimization problems.

Outline

- 1 Context
- 2 Tube-Based MPC
- 3 Experimental setup
- 4 System identification and parameter tuning**
- 5 Experimental validation
- 6 Conclusion

Classical approach

First step : experimental data generation

Perform identification flights with a range of control policies.

Second step : system identification

Identify the model using the experimental data.

Third step : local controller design

Design a local controller K from the model and compute an associated RPI set \mathcal{Z} .

Fourth step : experimental validation

Implement and validate experimentally the Tube-Based MPC law.

Classical approach : arising issues

System identification

The disturbances are function of the control law,

- the disturbance set \mathcal{W} is larger than necessary (conservative).
- the RPI set \mathcal{Z} is larger than necessary (conservative).

Consider experimental data with similar control laws.

Local controller design

Exhibit satisfying closed-loop performances with the real system.

- Once $\bar{\xi} = \xi^r$, we have $u(\xi) = K(\xi - \xi^r)$.

Proposed approach

First step : local controller design

Perform calibration flights with a range of linear control laws $u(\xi) = K\xi$.

Second step : system identification

Identify the model using data from the chosen calibration flight.

Third step : iterative refinement of the system identification

- Implement a Tube-Based MPC law
- Perform identification flights
- Identify a novel model from the identification flights

Iterate until a Tube-Based MPC law is validated in flight.

Tuning of the linear state feedback gain

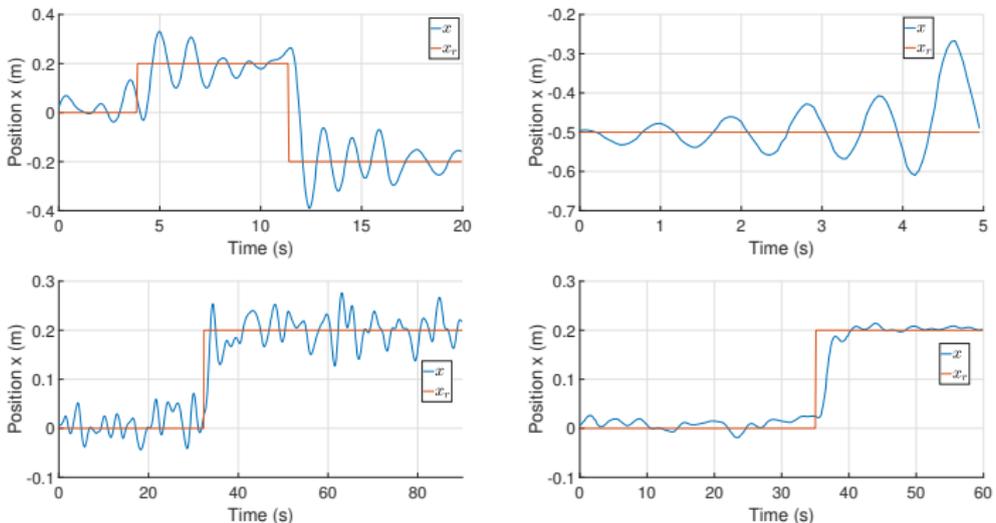


FIGURE – Time evolution of the position x (blue) and reference x^r (red) for $K = \begin{bmatrix} -0.233 & -0.4 & 0 \end{bmatrix}$ (top-left), $\begin{bmatrix} -0.6 & -0.7 & -0.3 \end{bmatrix}$ (top-right), $\begin{bmatrix} -0.4 & -0.5 & -0.7 \end{bmatrix}$ (bottom-left), and $\begin{bmatrix} -0.4 & -0.5 & -1 \end{bmatrix}$ (bottom-right).

We present the closed-loop behavior of the quadrotor for four linear state feedback gains. In the following, we choose $K = \begin{bmatrix} -0.4 & -0.5 & -1 \end{bmatrix}$.

Identification flights and disturbance realizations

Disturbance realizations

The disturbance set \mathcal{W} is defined from flight tests, where the state and control inputs are measured over $L + 1$ points,

$$\{\xi_k^m, u_k^m\}, k = \{1, \dots, L + 1\}.$$

The disturbance realizations are defined as the mismatch between the measured data and the one-step state prediction,

$$w_k^m = \xi_{k+1}^m - A\xi_k^m - Bu_k^m, k = \{1, \dots, L\}.$$

Disturbance set

The disturbance set \mathcal{W} has to be a convex set containing all the possible disturbance realizations,

$$\mathcal{W} = \text{conv} \{w_k^m, k = \{1, \dots, L\}\}.$$

The identification of the matrices A and B fully characterizes the disturbance set \mathcal{W} from the experimental data.

System identification and invariant set design

Minimal Robustly Positively Invariant set

For a given pair (A, B) , a given disturbance set \mathcal{W} and a given linear state feedback gain K , the minimal RPI (mRPI) set is given by

$$\mathcal{Z}_{\infty}(A, B, K, \mathcal{W}) = \bigoplus_{i=0}^{\infty} (A + BK)^i \mathcal{W}.$$

With the previous considerations, the linear state feedback gain K is fixed and the disturbance set \mathcal{W} is a function of the pair (A, B) .

Goal of the identification

The method presented further seeks to find the pair (A, B) that leads to an mRPI set that is as *small* as possible.

Constrained Least Mean Square

The mRPI set depends on the successive powers of the matrix $A + BK$ and on the disturbance set \mathcal{W} .

Constrained Least Mean Square algorithm

The approach proposed here consists in bounding the eigenvalues of the matrix A while minimizing the quadratic norm of the disturbance signal

$$\begin{aligned}
 (A, B) = \arg \quad & \underset{A, B}{\text{minimize}} \quad \sum_{k=0}^L \|w_k^m\|_2^2, \\
 & \text{subject to} \quad w_k^m = \xi_{k+1}^m - A\xi_k^m - Bu_k^m, k = \{1, \dots, L\} \\
 & A = \begin{bmatrix} 1 & a_{xv} & a_{x\phi} \\ 0 & a_v & a_{v\phi} \\ 0 & 0 & a_\phi \end{bmatrix}, B = \begin{bmatrix} b_x \\ b_v \\ b_\phi \end{bmatrix}, \\
 & a_{xv}, a_{x\phi}, a_{v\phi}, b_x, b_v, b_\phi \geq 0, \\
 & 0 \leq a_v \leq \alpha, 0 \leq a_\phi \leq \beta.
 \end{aligned}$$

The parameters $\alpha > 0$ and $\beta > 0$ are to be tuned by comparing the resulting mRPI sets.

Constrained Least Mean Square

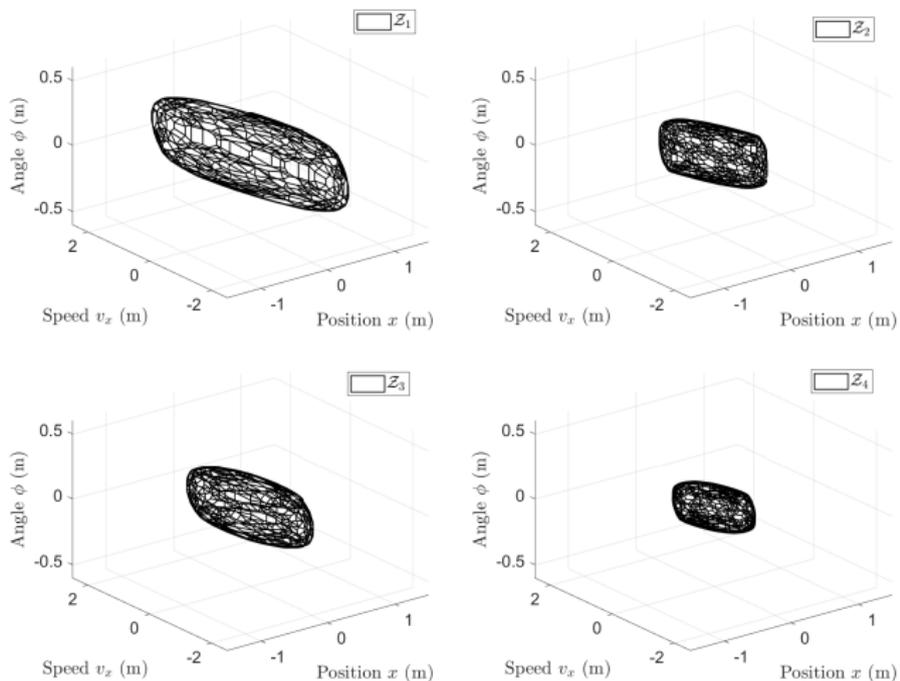


FIGURE – Approximation of the resulting mRPI set for $\alpha = 2$ (left), and $\alpha = 0.97$ (right) and $\beta = 2$ (top) and $\beta = 0.94$ (bottom).

System identification and invariant set design

Parameter tuning of the Constrained LMS

The tuning of the parameters α and β led to $\alpha = 0.97$, and $\beta = 0.94$,

$$A = \begin{bmatrix} 1 & 0.0490 & 0.0524 \\ 0 & 0.9700 & 0.4294 \\ 0 & 0 & 0.9400 \end{bmatrix}, B = \begin{bmatrix} 0.0050 \\ 0.0100 \\ 0.0683 \end{bmatrix}.$$

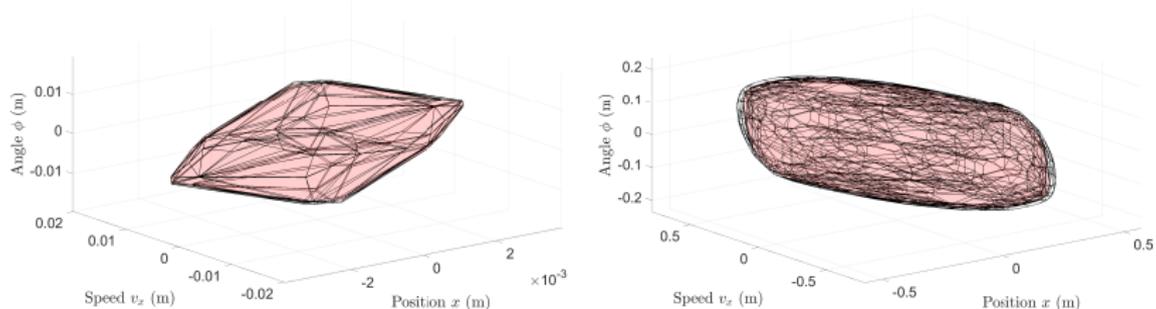


FIGURE – State-space representation of the disturbance set (left) and the mRPI set approximation (right).

Remaining parameters of the Tube-Based MPC

The horizon prediction length is chosen as $N = 30$ ($N\tau = 1.5s$).

State and control input constraints

We consider the following state and control input constraints (\mathcal{X} and \mathcal{U})

$$-0.6m \leq x \leq 1.8m, |v_x| \leq 1m/s, |\phi| \leq 0.3rad, |u_x| \leq 0.56.$$

This defines the nominal constraints ($\bar{\mathcal{X}}$ and $\bar{\mathcal{U}}$),

$$-0.1m \leq \bar{x} \leq 1.3m, |\bar{v}_x| \leq 0.321m/s, |\bar{\phi}| \leq 0.094rad, |u_x| \leq 0.332.$$

Weighting matrices

The weighting matrices are chosen as

$$Q = \begin{bmatrix} 0.8 & 0 & 0 \\ 0 & 1.2 & 0 \\ 0 & 0 & 1 \end{bmatrix}, R = 5, P = \begin{bmatrix} 26.99 & 9.924 & 27.45 \\ 9.924 & 15.15 & 36.25 \\ 27.45 & 36.25 & 137.3 \end{bmatrix},$$

where P is the solution of the Riccati equation $A_K^T P A_K - P + Q + K^T R K = 0$.

Outline

- 1 Context
- 2 Tube-Based MPC
- 3 Experimental setup
- 4 System identification and parameter tuning
- 5 Experimental validation**
- 6 Conclusion

Trajectory of the validation flight

We present the results from a validation flight of the Parrot AR.Drone 2.0 with a Tube-Based MPC law.

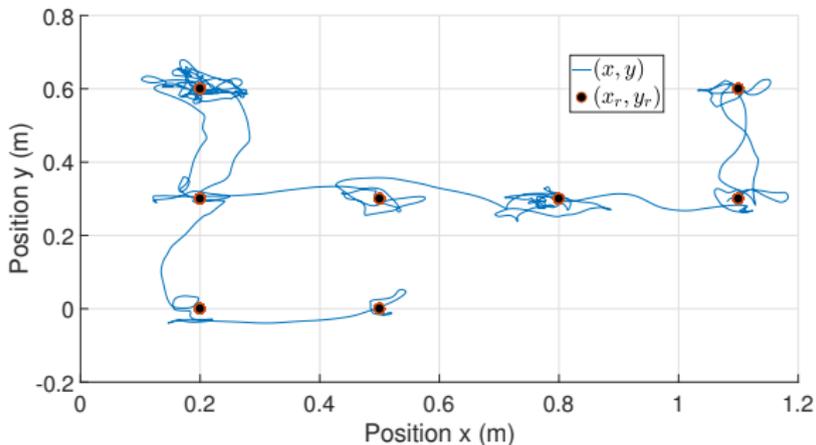


FIGURE – Horizontal trajectory of the Parrot AR.Drone 2.0 during the validation flight.

The references are chosen close to each-other to ensure feasibility of the optimization problem.

Validation flight

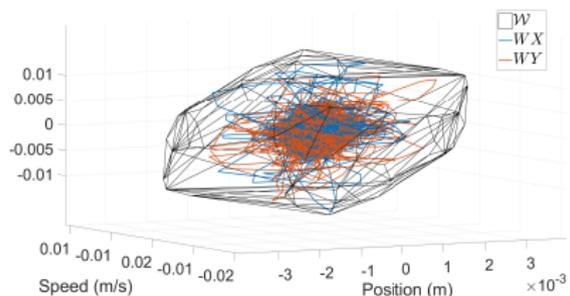


FIGURE – State-space representation of the disturbance set \mathcal{W} and the disturbance realizations encountered in flight.

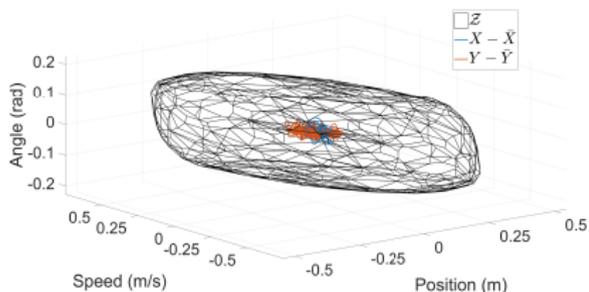


FIGURE – State-space representation of the RPI set \mathcal{Z} and the state of the error system $\xi - \tilde{\xi}$ during the flight.

The disturbance realizations w_k^m and the error system states $z_k = \xi_k - \tilde{\xi}_k$ are presented for both directions x (blue) and y (red).

The disturbance realizations are contained in the disturbance set \mathcal{W} throughout the flight.

As a consequence, the error system state is remains in the invariant set \mathcal{Z} .

Nominal system constraints

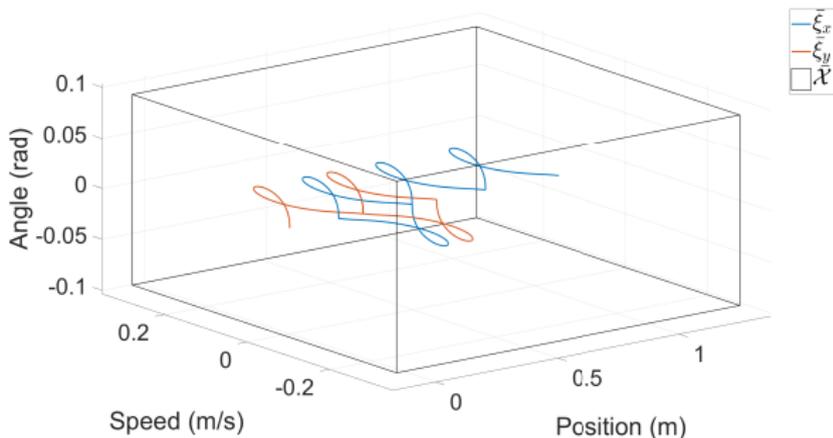


FIGURE – Trajectory of the nominal system state for both directions ξ_x (blue) and ξ_y (red) in the state space representation, along with the nominal system state constraints \bar{X} (white).

The MPC law ensures recursive satisfaction of the nominal system constraints.

Uncertain system trajectory

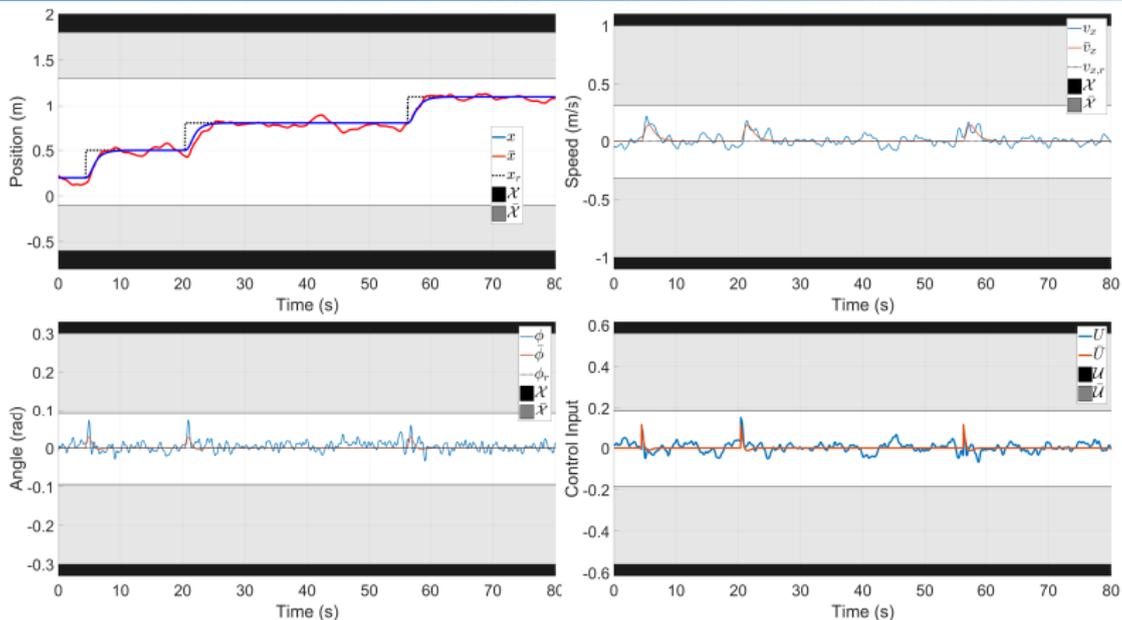


FIGURE – Time-evolution of the state and control input of the nominal (blue) and the uncertain (red) system and their respective constraints (grey and black) during the validation flight.

The Tube-Based MPC law ensures recursive constraints satisfaction of the uncertain system constraints.

Outline

- 1 Context
- 2 Tube-Based MPC
- 3 Experimental setup
- 4 System identification and parameter tuning
- 5 Experimental validation
- 6 Conclusion**

Conclusion

System modelling

- The horizontal dynamics of the Parrot AR.Drone 2.0 quadrotor are modelled by a discrete-time linear system subject to additive disturbance.
- The disturbance term consist in the modelling error, external forces, and the presence of delay in the communication of the control.
- The choice to not take the delay into account in the modelling is motivated by the complexity of invariant set computation for high-dimension systems.

Linear state feedback gain

- Once the nominal system has converged to the reference, the Tube-Based MPC law is equivalent to the linear control law.
- The linear state feedback gain was tuned from calibration flights.

Conclusion

System identification

- We identified the model using experimental data from identification flights.
- We proposed an identification method that seeks to minimize the mRPI set.
- This method consists in minimizing the disturbance signal while bounding the eigenvalues of the model.

Experimental validation of the Tube-Based MPC

- We introduced constraints on the state and control input of the system.
- We performed validation flights with the Tube-Based MPC law for the stabilization of the horizontal position.
- The disturbance set used for the implementation of the control law contained all the disturbance realizations encountered during the validation flight.
- The state and control input constraints of the uncertain and the nominal systems were satisfied.