An MPC Approach to Transient Control of Liquid-Propellant Rocket Engines

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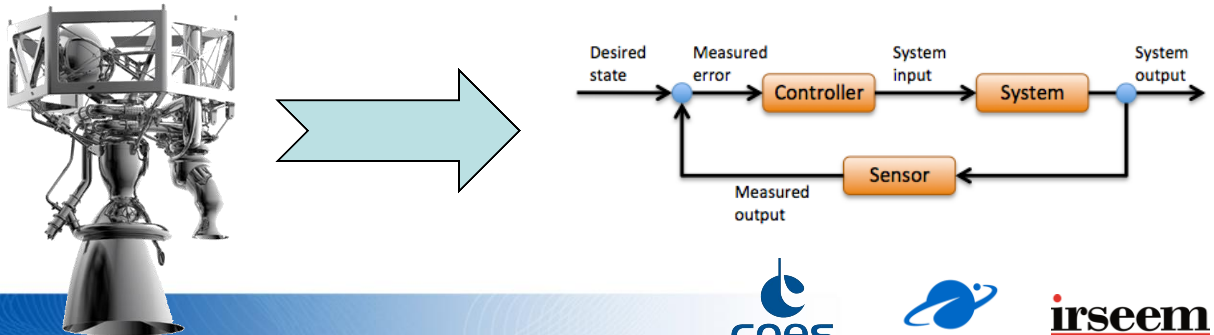
Presentation contents

1. Introduction: problem definition

This article presents:

- A methodology for **controlling** the continuous part of the **transient phases** of **Liquid-Propellant Rocket Engines (LPRE):**
    - LPRE are **complex thermodynamic systems** → nonlinear model-based approach based on fluid mechanics.
    - **Control goal** in this paper:
        - **End-state tracking** in main engine variables.
        - Verification of **operational constraints** during transient.
    - **Controller**:
        - **Nonlinear preprocessor** for reference generation.
        - **Linear MPC** with robustness considerations.

- Progress within the **research trend of improving** the **control performance and robustness** of these devices.



1. Introduction: context

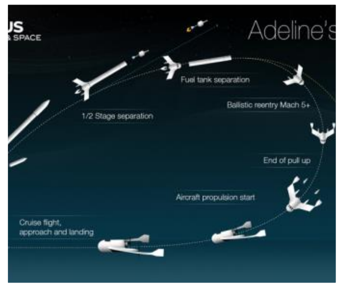
Global trend for affordable access to space
→Reusable launchers+ rocket engines

- ***Reusability race*** after *semi-reusable* Space Shuttle
  - **Demanding control robustness requirements:** multi-restart and thrust-modulation features. [1]
  - Classical steady-state multivariable and linear control **→ reduced throttling envelope (70%-120% thrust)→too narrow for reusable engines** [1] (30% expected in Prometheus).

SpaceX

Blue Origin

PLD Space



Airbus DS Adeline

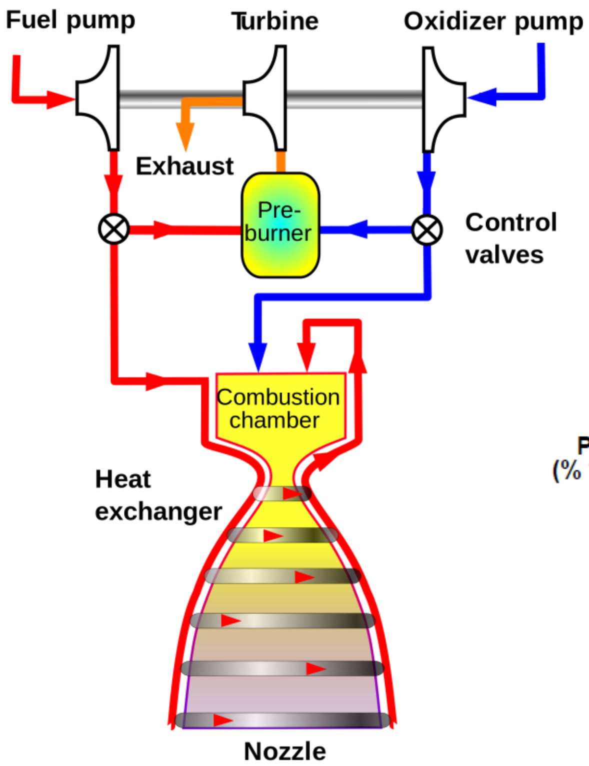
1. Introduction: state of the art

- Most common approaches in literature→**linearised models about operating points**, for synthesising **steady-state controllers:**
  - **PID based** [11]. MIMO are decoupled into dominant SISO subsystems.
  - **Off-line optimisation**: [4].
  - Incorporation of:
    - **Nonlinear** techniques [6]
    - **Hybrid** techniques [10].
    - **Robust** techniques [15].
    - **Reconfiguration** control [10].

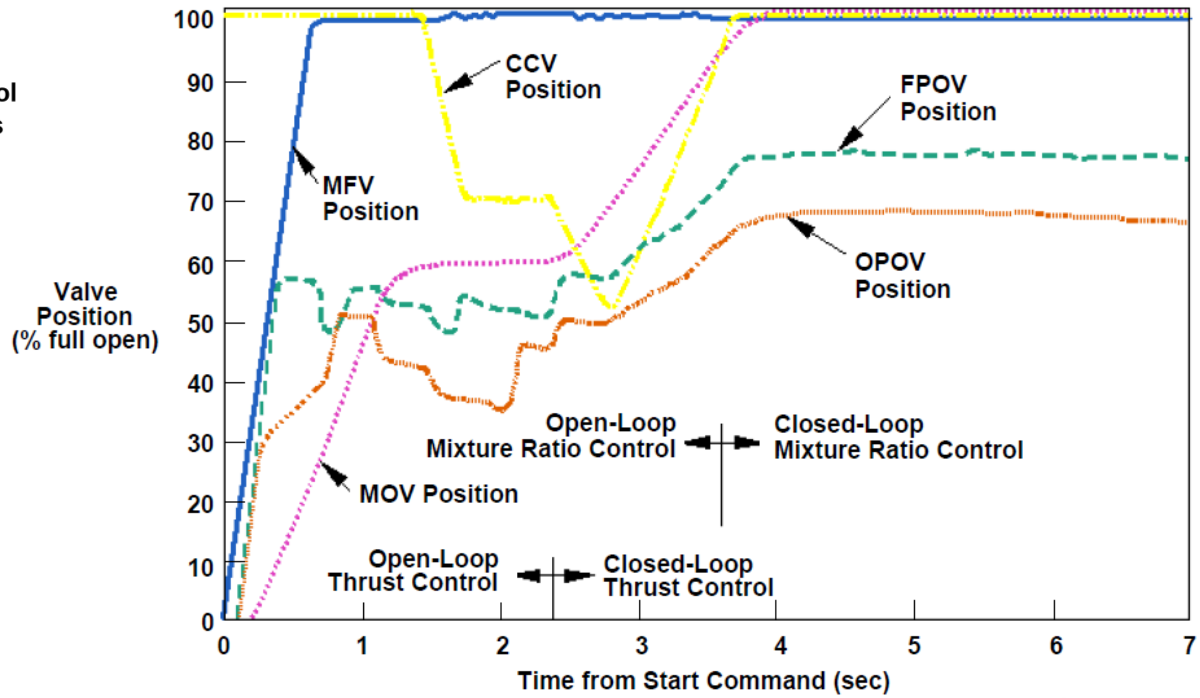
- **No publications** considering **steady-state and transients**, reviewed in [13].

1. Introduction: objectives

- **Enhance** Liquid-Propellant Rocket Engines **(LPRE) control**.



- **Main target →** control strategy during their **transient phases**, currently carried out as sequences of events in open loop



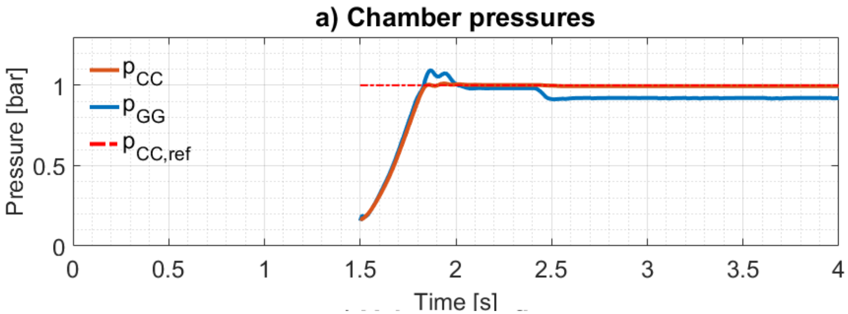
Rocketdyne Propulsion & Power

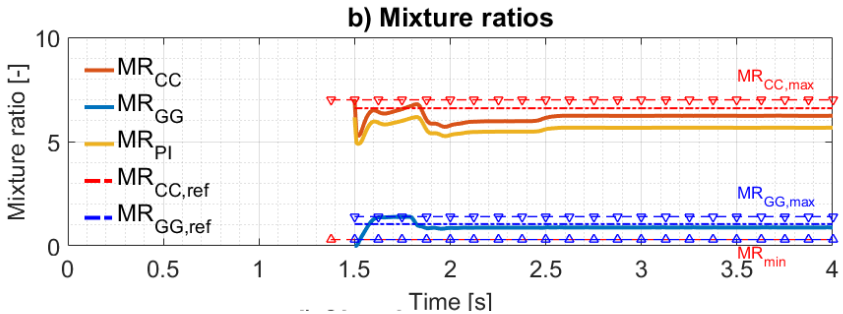
1. Introduction: objectives

– **Goals:**

- **Final-reference tracking** at the end of **start-up transient,** via tuning of **continuous controls (valves).**
  - Main variables to track: **combustion-chamber pressure** (pCC) and **oxidiser-to-fuel mixture ratios** (MR).
- **Constraints verification** (on states and control).
- **Robustness** to **parameters** and **initial-conditions variations** and to **perturbations.**

- **Constrained enough to consider trajectory tracking unnecessary.**

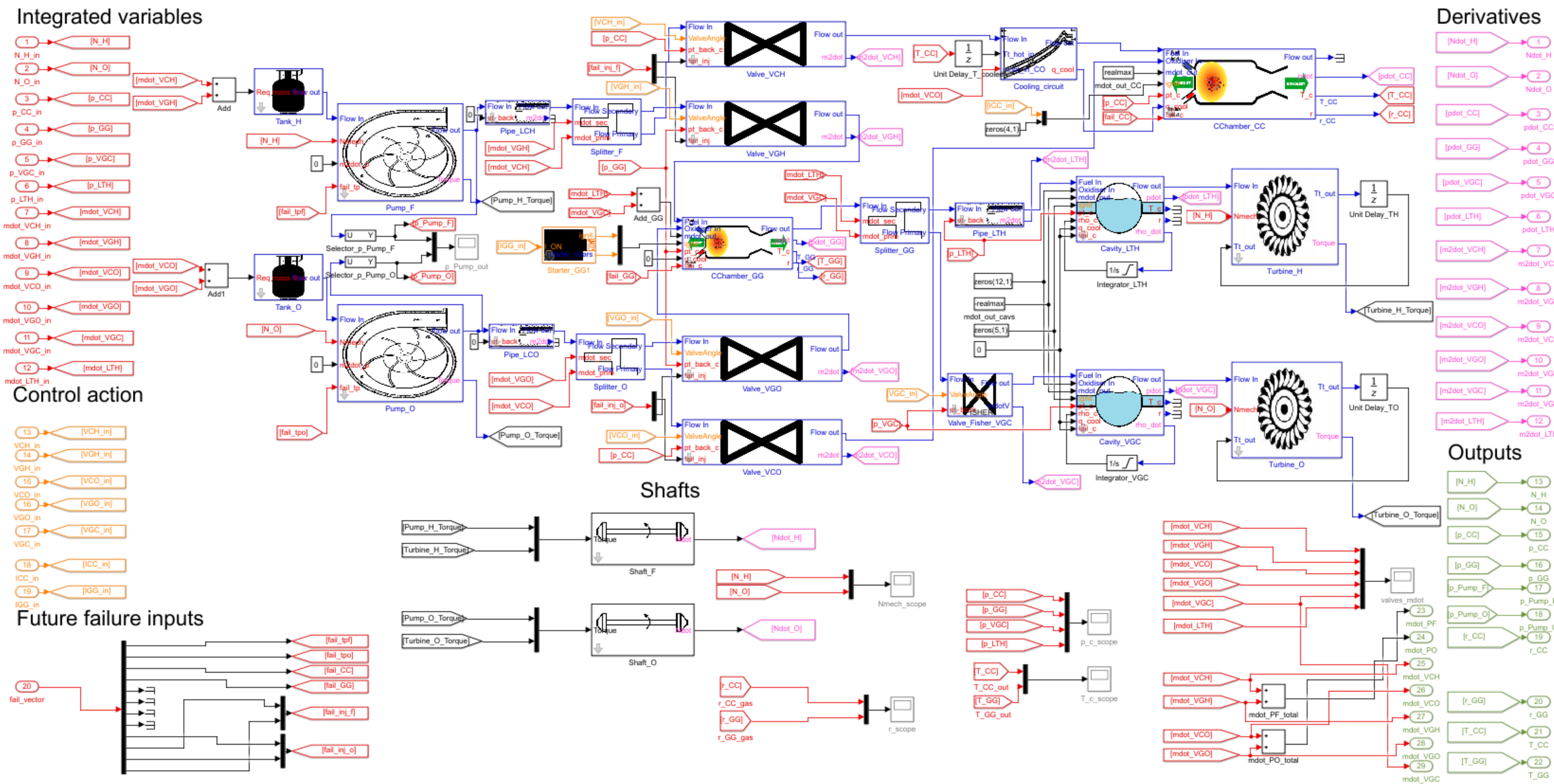




2. Modelling

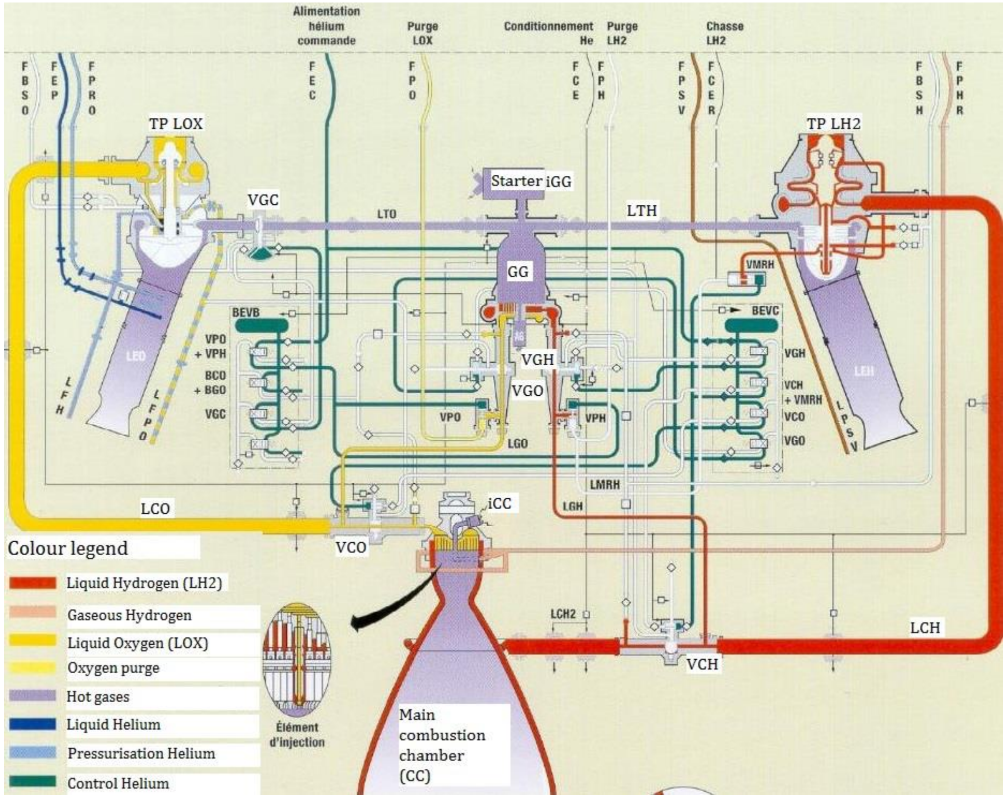
1st steps: development of simulator and state-space models (in [12])

- **T-RETM library: *Vulcain* simulator.**

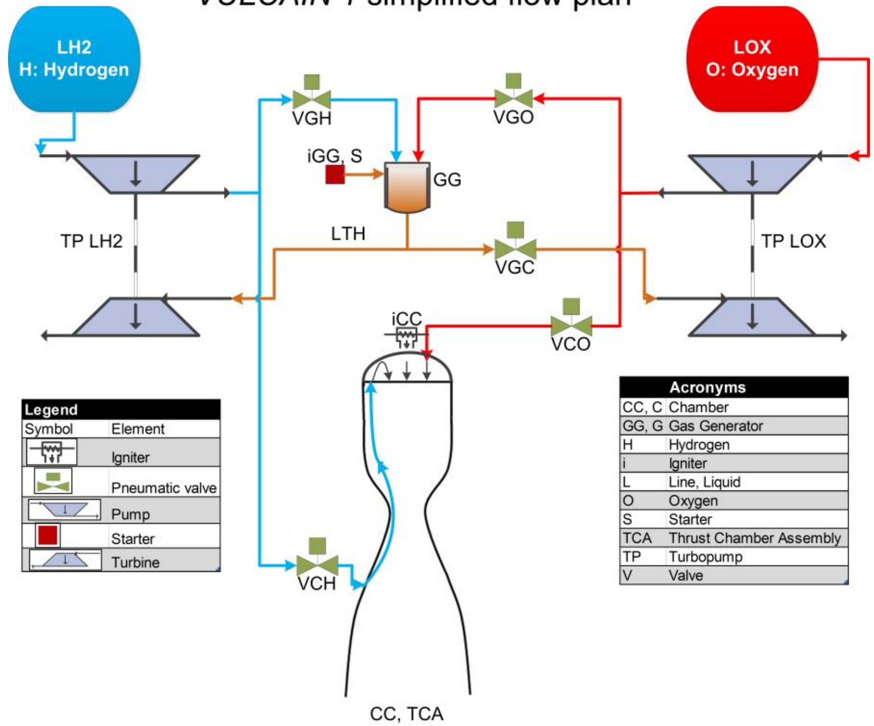


2. Modelling

- **Engine case study**: retired *Vulcain* 1 (AG 1996-2009) **with biased parameters**
  - Gas-generator cycle (GG), LOX/LH2, bi-turbopump.
  - Actuators: 5 continuously-controllable valves (VCH, VCO, VGH, VGO and VGC), 1 binary igniter (i_{CC}) and 1 binary starter (i_{GG}).
  - Valve angles α varied to control chamber pressure p_{CC} and mixture ratio $MR = \frac{\dot{m}_{ox}}{\dot{m}_{fu}}$



VULCAIN 1 simplified flow plan

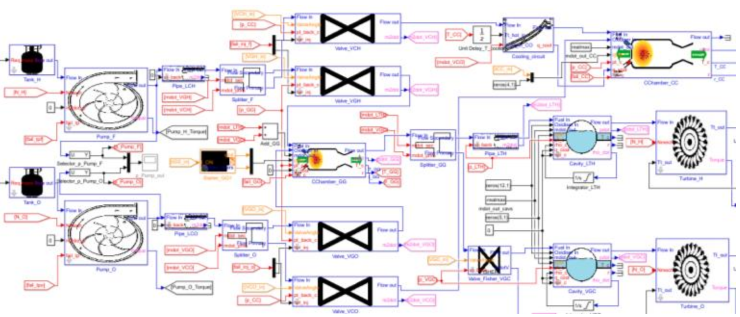


Legend	
Symbol	Element
	Igniter
	Pneumatic valve
	Pump
	Starter
	Turbine

Acronyms	
CC, C	Chamber
GG, G	Gas Generator
H	Hydrogen
i	Igniter
L	Line, Liquid
O	Oxygen
S	Starter
TCA	Thrust Chamber Assembly
TP	Turbopump
V	Valve

Translation into a state-space model

Simulator



Global state-space model

$$\dot{x} = f_c(x, u_c, u_d);$$

$$\begin{bmatrix} \dot{\omega}_{[2\times1]} \\ \dot{p}_{[4\times1]} \\ \ddot{m}_{[6\times1]} \end{bmatrix} = f_c \left(\underbrace{\begin{bmatrix} \omega_{[2\times1]} \\ p_{[4\times1]} \\ \dot{m}_{[6\times1]} \end{bmatrix}}_{\text{12 states}}, \underbrace{\begin{bmatrix} A_{VCH} \\ A_{VGH} \\ A_{VCO} \\ A_{VGO} \\ A_{VGC} \end{bmatrix}}_{\text{5 continuous inputs}}, \underbrace{\begin{bmatrix} i_{CC} \\ i_{GG} \end{bmatrix}}_{\text{2 discrete inputs}} \right);$$

All equations joined via *Maple*

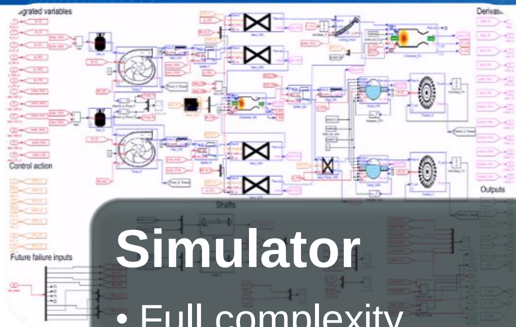
$$\mathbf{x} = [\omega_H \quad \omega_O \quad p_{CC} \quad p_{GG} \quad p_{LTH} \quad p_{VGC} \quad \dot{m}_{LTH} \quad \dot{m}_{VCH} \quad \dot{m}_{VCO} \quad \dot{m}_{VGH} \quad \dot{m}_{VGO} \quad \dot{m}_{VGC}]^T.$$

$$\mathbf{x}_z = [p_{CC} \quad \dot{m}_{VCH} \quad \dot{m}_{VCO} \quad \dot{m}_{VGH} \quad \dot{m}_{VGO}]^T.$$

$$\mathbf{u} = [A_{VCH} \quad A_{VCO} \quad A_{VGH} \quad A_{VGO} \quad A_{VGC}]^T.$$

Greater tracking relevance

2. Modelling



Simulator

- Full complexity

$$\dot{x} = f_c(x, u_c, u_d);$$

$$\begin{bmatrix} \dot{\omega}_{[2\times1]} \\ \dot{p}_{[4\times1]} \\ \ddot{m}_{[6\times1]} \end{bmatrix} = f_c\left(\begin{bmatrix} \omega_{[2\times1]} \\ p_{[4\times1]} \end{bmatrix}, \begin{bmatrix} \alpha_{VCH} \\ \alpha_{VGH} \\ \alpha_{VCO} \\ \alpha_{VGO} \end{bmatrix}, \begin{bmatrix} i_{CC} \\ i_{GG} \end{bmatrix} \right);$$

Complex nonlinear state-space f_c

- 12 states
- Cumbersome equations

4 stages of modelling:

- non-dimensional

- all need $\Delta t=10^{-5}$ s

[12]

$$\dot{x} = f_s(x, u_c, u_d);$$

$$\begin{bmatrix} \dot{\omega}_{[2\times1]} \\ \dot{p}_{[4\times1]} \\ \ddot{m}_{[6\times1]} \end{bmatrix} = f_s\left(\begin{bmatrix} \omega_{[2\times1]} \\ p_{[4\times1]} \end{bmatrix}, \begin{bmatrix} \alpha_{VCH} \\ \alpha_{VGH} \\ \alpha_{VCO} \end{bmatrix}, \begin{bmatrix} i_{CC} \end{bmatrix} \right);$$

Simplified nonlinear state-space f_s

- 12 states
- More tractable and linearisable

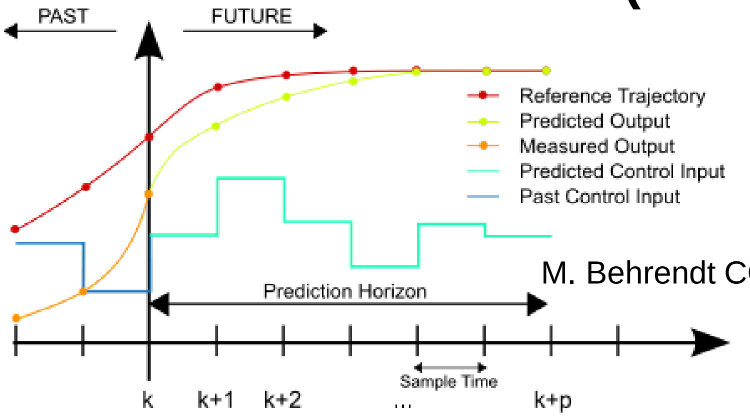
$$\begin{bmatrix} \dot{\omega}_{[2\times1]} \\ \dot{p}_{[4\times1]} \\ \ddot{m}_{[6\times1]} \end{bmatrix} = A \begin{bmatrix} \omega_{[2\times1]} \\ p_{[4\times1]} \end{bmatrix} + B_c \begin{bmatrix} \alpha_{VCH} \\ \alpha_{VGH} \\ \alpha_{VCO} \end{bmatrix};$$

Linearised model

- After sequential events
- About end state

3. Controller design

Model-Predictive Control (MPC):



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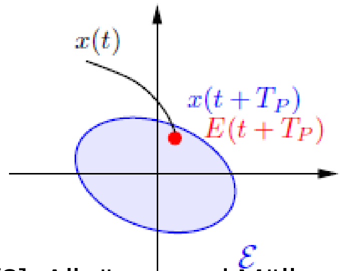
- terminal region constraint $x(t+T_P) \in \mathcal{E}$
- terminal penalty term $E(x(t+T_p))$

$$\min_u J(x(t), u)$$

$$J(\cdot) = \int_t^{t+T_p} F(x(\tau), u(\tau))d\tau + E(x(t+T_p))$$

subject to:	$\dot{x} = f(x,u)$,	system dynamics
	$x(t)$ given	"state feedback"
	$u(\tau) \in \mathcal{U}$	input constraints
	$x(\tau) \in \mathcal{X}$	state constraints
	$x(t+T_p) \in \mathcal{E}$	terminal constraint

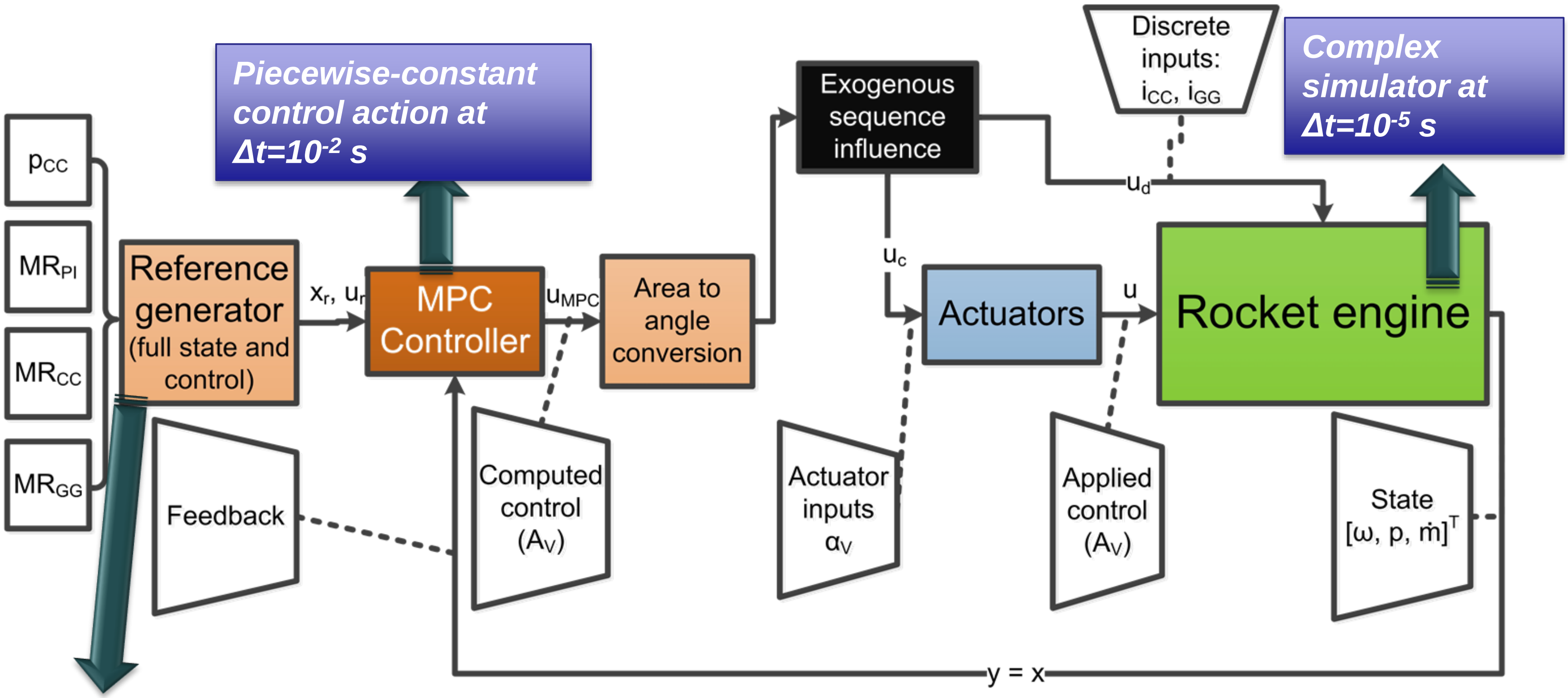


[3], Allgöwer and Müller

- **Constraints** can be well defined and respected with this family of approaches.
- **Robustness** can also be handled.
- **Drawback**: computational time, due to online computation of control law.

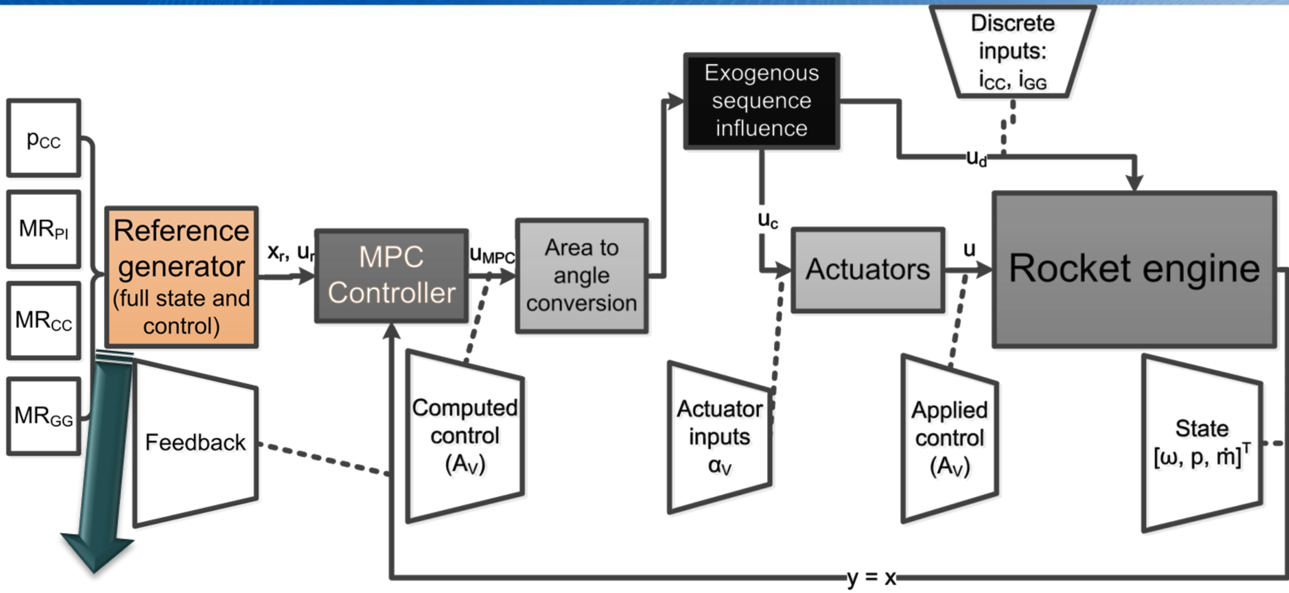
- **Quasi-infinite horizon**: with terminal-set constraint [3] Chen and Allgöwer. The MPC controller drives the system to an end set, where a simple fictitious controller ensures stability and reachability.

3. Controller design



- **Reference generator:** reconstruction of whole state from input configuration (4 data).
  - Least squares with complex NLSS f_c at equilibrium (*lsqnonlin* MATLAB).
- **MPC:**
  - **Optimisation NL** under constraints with *IPOPT* [16].
  - $\Delta t = 10^{-2} s$, Np = $10\Delta t$, Nu = $5\Delta t$

3. Controller design



- **Reference generator:** reconstruction of whole state from input command from launcher (4 data).
  - Least squares with complex NLSS f_c at equilibrium (*lsqnonlin* MATLAB).
  - Forced by the unavailability of analytic solution.
  - Verification of static mixture ratios.

$$\begin{cases} \dot{\mathbf{x}} = f_c(\mathbf{x}_r, \mathbf{u}_r) = 0 \quad \setminus (\dot{p}_{CC} = 0) \\ \frac{\tilde{\dot{m}}_{VCO} + \tilde{\dot{m}}_{VGO}}{\tilde{\dot{m}}_{VCH} + \tilde{\dot{m}}_{VGH}} = MR_{PI,r} \\ \frac{\tilde{\dot{m}}_{VGO}}{\tilde{\dot{m}}_{VGH}} = MR_{GG,r} \\ \tilde{\dot{m}}_{VGH} + \tilde{\dot{m}}_{VGO} = \tilde{\dot{m}}_{LTH} + \tilde{\dot{m}}_{VGC}. \end{cases}$$

3. Controller design

- **Standard MPC (with integrator term)**: **Robust MPC:**
  - Quadratic cost function, with terminal-cost term:

$$J(\mathbf{x},\mathbf{u},\mathbf{z}) = \left(\sum_{k=1}^{N_p} \mathbf{x}_k^T Q \mathbf{x}_k + \sum_{k=1}^{N_u} \mathbf{u}_k^T R \mathbf{u}_k + \sum_{k=1}^{N_p} \mathbf{z}_k^T S \mathbf{z}_k\right) \Delta t + \mathbf{x}_{N_p+1}^T P \mathbf{x}_{N_p+1}$$

Q, R optimised via black-box Kriging [8]

- Minimax problem: worst-case scenario for given perturbation w [9]:

$$\min_{\mathbf{u}} \max_{\mathbf{w}} \quad J(\mathbf{x},\mathbf{u})$$
$$\text{s.t.} \quad \mathbf{x} \in X \quad \forall \mathbf{w} \in \mathbb{W}^n$$
$$\mathbf{u} \in U \quad \forall \mathbf{w} \in \mathbb{W}^n$$

- Minimax optimisation too costly →rewriting as min of scalar $\gamma \in \mathbb{R}^+$ for given perturbation cases (equivalent epigraph formulation) [5]
- Smooth convex NLP (nonlinear programme).

$$\min_{\mathbf{x}_i,\mathbf{u},\mathbf{z}_i,\gamma} \quad \gamma$$
$$\text{s.t.} \quad J(\mathbf{x}_i,\mathbf{u},\mathbf{z}_i) \leq \gamma \quad \forall i \in I$$
$$\mathbf{x}_i \in X, \quad \mathbf{u} \in U \quad \forall i \in I$$
$$A_{ineq}[\mathbf{x}_i \quad \mathbf{u}]^T \leq \mathbf{b}_{ineq} \quad \forall i \in I$$
$$A_{eq}[\mathbf{x}_i \quad \mathbf{u}]^T = \mathbf{b}_{i,eq} \quad \forall i \in I$$
$$\mathbf{x}_{i,N_p+1}^T P \mathbf{x}_{i,N_p+1} \leq \alpha_P \quad \forall i \in I$$
$$\mathbf{z}_{i,k+1} = \mathbf{z}_{i,k} + \Delta t K_I \mathbf{x}_{z,i,k} \quad \forall i \in I, k \in [0, N_p].$$

$$\mathbf{x}_i = [\mathbf{x}_{i,1}, ..., \mathbf{x}_{i,k}, ..., \mathbf{x}_{i,N_p+1}]^T, \quad i \in I$$
$$\mathbf{x}_{i,k} = A_d \mathbf{x}_{i,k} + B_d \mathbf{u}_k + \mathbf{w}_{i,k}, \quad k \in [0, N_p+1],$$

- **Constraints**:
  - State and control bounds [7]
  - Linear MR+ actuators ($\dot{u}_{max}$) inequalities.
  - Linear dynamics.
  - Nonlinear for terminal region [3].
  - Integrator dynamics [14]

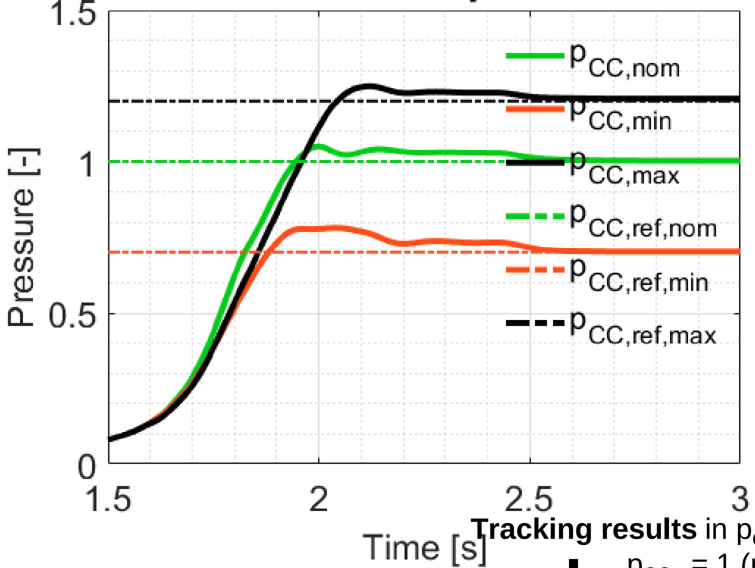
Matrices A, B at reference end point, to **converge to x_r=0** in the linear case.

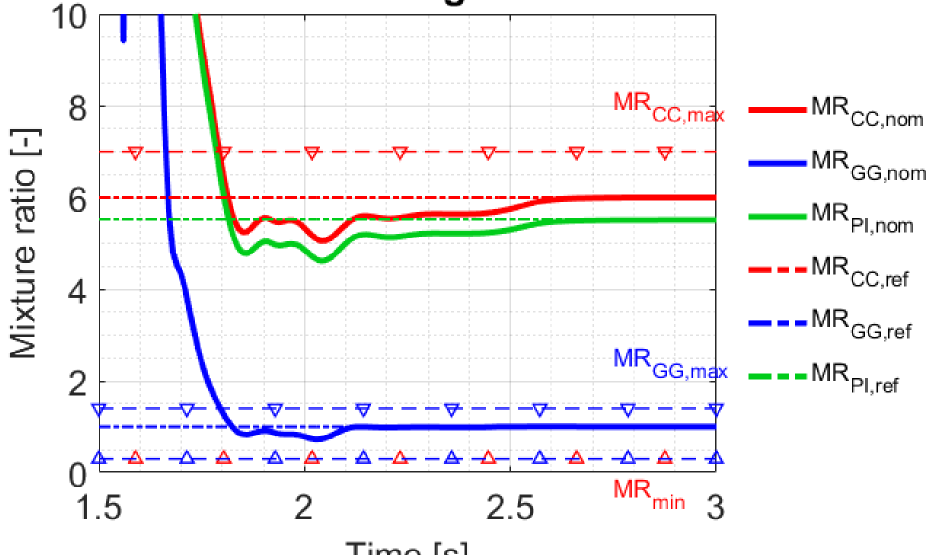
$w_{i,k}$ selected according to eigenvectors of A_d [2,17]

I = {1,2,3}

4. Analysis of results

- **Results of robust MPC for a set perturbed cases.**





Tracking results in p_{CC} for

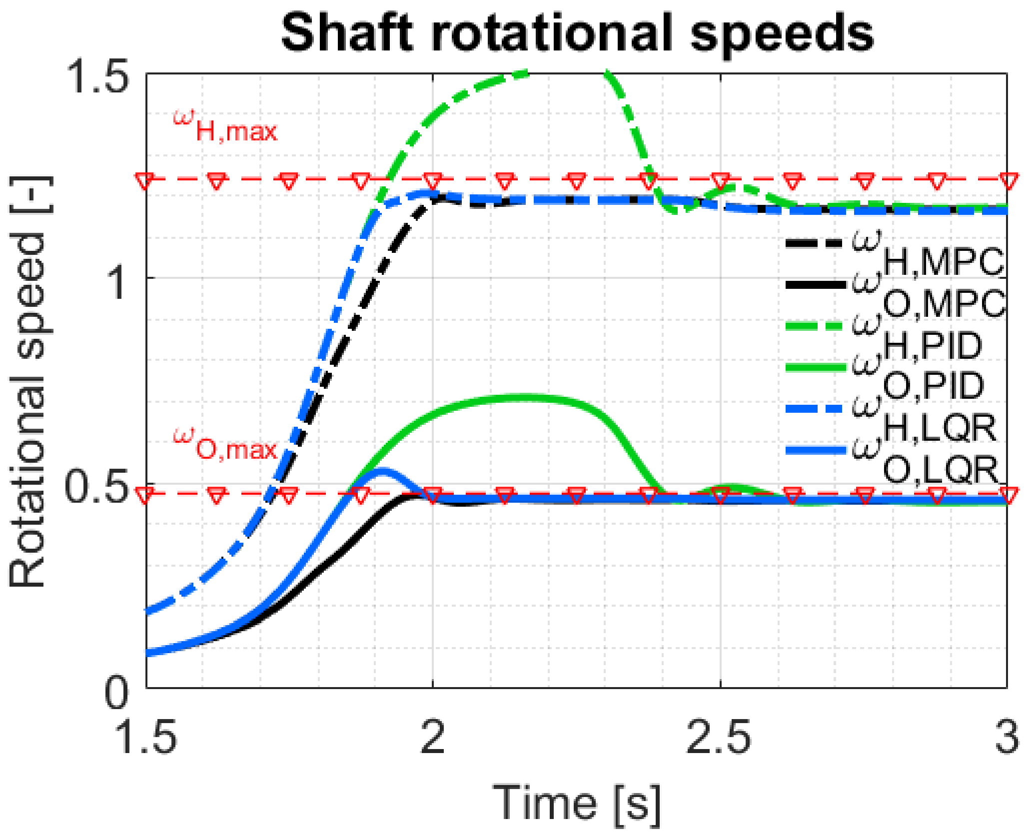
- $p_{CC,r}$ = 1 (nominal)
- $p_{CC,r}$ = 0.7 (minimum)
- $p_{CC,r}$ = 1.2 (maximum)

- **MR constraints** relaxed at beginning, due to initial conditions outside feasible area.

Indicator	p_{CC} tracking error (%)		MR tracking errors (CC, GG, PI) (%)		Overshoot (% in p_{CC})	
Operating point	OL	CL	OL	CL	OL	CL
70%	2,8	0,26	2,58; 1,31; 2,84	1,38; 0,69; 0,85	15,1	11,46
100%	0,25	0,26	0,17; 1,39 1,43	0,01; 0,05; 0,3	6,31	5,04
120%	0,34	0,67	3,18; 1,23; 3,41	1,37; 0,59; 1,64	3,34	4,04

4. Analysis of results

- PID and LQR achieve tracking, but MPC also ensures constraints verification :



5. Conclusion

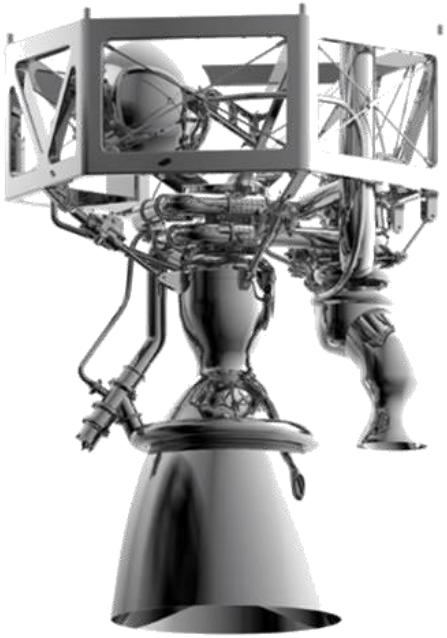
- Evolving **operating requirements** of pump-fed rocket engines, currently related to **reusability scenarios** → force the **improvement of their control algorithm, in OL during transients.**

- **Control of continuous phase of LPRE transients:**
  - **Full-state reference generation** via complex nonlinear state-space model.
  - **Linear MPC with integral action:** end-state tracking while verifying constraints.
    - Robustness considerations for a given set of perturbation scenarios.

- **Perspectives:**
  - Investigation of other ways of posing robustness globally.
  - Trajectory generation and tracking.
  - Extensive validation and sensitivity study.

Thank you for your attention.

Any questions or suggestions?



References

1. **Baiocco P. and Bonnal C. (2016).** Technology demonstration for reusable launchers. Acta Astronautica, 120, 43-58.
2. **Calafiore G.C. and Fagiano L. (2013).** Robust Model Predictive Control via Scenario Optimization. IEEE Transactions on Automatic Control, 58(1), 219-224.
3. **Chen H. and Allgoewer F. (1998).** A Quasi-Infinite Horizon Nonlinear Model Predictive Control Scheme with Guaranteed Stability. Automatica, 34(10), 1205-1217.
4. **Dai X. and Ray A. (1996).** Damage-Mitigating Control of a Reusable Rocket Engine: Part II-Formulation of an Optimal Policy. Journal of Dynamic Systems, Measurement, and Control, 118(3), 409-415.
5. **Loefberg J. (2003).** Minimax Approaches to Robust Model Predictive Control. Number No. 812 in Linkoeping Studies in Science and Technology Dissertations. UniTryck.
6. **Lorenzo C.F., Ray A., and Holmes M.S. (2001).** Nonlinear control of a reusable rocket engine for life extension. Journal of Propulsion and Power, 17(5), 998-1004.
7. **Luo Y., Serrani A, Yurkovich S., Doman D.B., and Oppenheimer M.W. (2004)**. Model predictive dynamic control allocation with actuator dynamics. In Proceedings of the 2004 American Control Conference, volume 2, 1695-1700. Boston, USA.
8. **Marzat J., Walter E., Piet-Lahanier H., and Damongeot F. (2010).** Automatic tuning via Kriging-based optimization of methods for fault detection and isolation. In 2010 Conference on Control and Fault-Tolerant Systems (SysTol), 505-510. Nice, France.
9. **Mayne D.Q., Rawlings J.B., Rao C.V., and Scokaert P.O.M. (2000).** Constrained model predictive control: Stability and optimality. Automatica, 36(6), 789-814.
10. **Musgrave J.L., Guo T.H., Wong E., and Duyar A. (1996).** Real-time accommodation of actuator faults on a reusable rocket engine. IEEE transactions on control systems technology, 5(1), 100-109.
11. **Nemeth E., Anderson R., Ols J., and Olsasky M. (1991).** Reusable rocket engine intelligent control system framework design, phase 2. Technical Report NASA Contractor Report 187213, Rockwell International, Canoga Park, California.
12. **Pérez-Roca S., Langlois N., Marzat J., Piet-Lahanier H., Galeotta M., Farago F., and Le Gonidec S. (2018).** Derivation and Analysis of a State-Space Model for Transient Control of Liquid-Propellant Rocket Engines. In 2018 9th International Conference on Mechanical and Aerospace Engineering (ICMAE), 58-67. Budapest, Hungary.
13. **Pérez-Roca S., Marzat J., Piet-Lahanier H., Langlois N., Farago F., Galeotta M., and Le Gonidec S. (2019).** A survey of automatic control methods for liquid-propellant rocket engines. Progress in Aerospace Sciences, 107, 3-84.
14. **Santos L.O., Afonso P.A.F.N.A., Castro J.A.A.M., Oliveira N.M.C., and Biegler L.T. (2001).** On-line implementation of nonlinear MPC: an experimental case study. Control Engineering Practice, 9(8), 847-857.
15. **Saudemont R. and Le Gonidec S. (2000).** Study of a robust control law based on H infinity for the Vulcain rocket engine ArianeGroup internal report, ArianeGroup, ESTACA.
16. **Waechter A. and Biegler L.T. (2006).** On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming. Mathematical Programming, 106(1), 25-57.
17. **Yedavalli R.K. (1985).** Improved measures of stability robustness for linear state space models. IEEE Transactions on Automatic Control, 30(6), 577-579.

Backup slides: State-space model

State-space model

$$\dot{x} := f(x, u_c, u_d);$$

$$\dot{x} := \left[\dot{\widetilde{\omega_H}}, \dot{\widetilde{\omega_O}}, \dot{\widetilde{p_{CC}}}, \dot{\widetilde{p_{GG}}}, \dot{\widetilde{p_{LTH}}}, \dot{\widetilde{p_{VGC}}}, \ddot{\widetilde{m_{LTH}}}, \ddot{\widetilde{m_{VCH}}}, \ddot{\widetilde{m_{VCO}}}, \ddot{\widetilde{m_{VGH}}}, \ddot{\widetilde{m_{VGO}}}, \ddot{\widetilde{m_{VGC}}}\right]^T;$$

$$x := \left[\widetilde{\omega_H}, \widetilde{\omega_O}, \widetilde{p_{CC}}, \widetilde{p_{GG}}, \widetilde{p_{LTH}}, \widetilde{p_{VGC}}, \widetilde{\dot{m}_{LTH}}, \widetilde{\dot{m}_{VCH}}, \widetilde{\dot{m}_{VCO}}, \widetilde{\dot{m}_{VGH}}, \widetilde{\dot{m}_{VGO}}, \widetilde{\dot{m}_{VGC}}\right]^T;$$

$$u_c := \left[\widetilde{AVCH}, \widetilde{AVCO}, \widetilde{AVGH}, \widetilde{AVGO}, \widetilde{AVGC}\right]^T; \#equivalent\ to\ \alpha_i$$

$$u_d := \left[i_{CC}, i_{GG}\right]^T;$$

$$\widetilde{\omega_H} = \frac{\omega_H}{\omega_{Heq}}; \widetilde{\omega_O} = \frac{\omega_O}{\omega_{Heq}}; \widetilde{p_{CC}} = \frac{p_{CC}}{p_{CCeq}}; \widetilde{p_{GG}} = \frac{p_{GG}}{p_{CCeq}}; \widetilde{p_{LTH}} = \frac{p_{LTH}}{p_{CCeq}}; \widetilde{p_{VGC}} = \frac{p_{VGC}}{p_{CCeq}};$$

$$\widetilde{\dot{m}_{LTH}} = \frac{\dot{m}_{LTH}}{\dot{m}_{VGHeq}}; \widetilde{\dot{m}_{VCH}} = \frac{\dot{m}_{VCH}}{\dot{m}_{VCHeq}}; \widetilde{\dot{m}_{VCO}} = \frac{\dot{m}_{VCO}}{\dot{m}_{VCHeq}}; \widetilde{\dot{m}_{VGH}} = \frac{\dot{m}_{VGH}}{\dot{m}_{VGHeq}}; \widetilde{\dot{m}_{VGO}} = \frac{\dot{m}_{VGO}}{\dot{m}_{VGHeq}}; \widetilde{\dot{m}_{VGC}} = \frac{\dot{m}_{VGC}}{\dot{m}_{VGHeq}};$$

$$i_{CC} := 1 : i_{GG} := 1 : t_{eq} := 1 :$$

State constraints

$$\omega_H \in (0, 40]\ krpm;\ \omega_O \in (0, 15]\ krpm;$$

$$MR_{CC} = \frac{\dot{m}_{VCO}}{\dot{m}_{VCH}} \in [0.3, 7]; MR_{GG} = \frac{\dot{m}_{VGO}}{\dot{m}_{VGH}} \in [0.3, 1.4];$$

$$\dot{p}_{GG} \leq 40\ bar$$

Control constraints

$$i_i \in \{0, 1\};\ A_i \in [0, A_{\max}]^\circ; \dot{A}_i \leq \dot{A}_{i,\max}$$

Backup slides: Vulcain NL model

f(x) NL | Ax L | f(x)u NL affine | f(x,u) NL non-affine | k

$\dot{\omega}_H$;

$$-\frac{kom_{H2_{fp}}\, x_5 x_1}{\sqrt{T_{GG}}} + kom_{H4_{fp}}\, x_5 + \left(-kom_{H3_{fp}}\, x_8 - kom_{H3_{fp}}\, x_{10}\right) x_1 + kom_{H5_{fp}}\, x_8^2 + 2\, kom_{H5_{fp}}\, x_8 x_{10}$$

$\dot{\omega}_O$;

$$-kom_{O1_{fp}}\, x_2^2 - \frac{kom_{O2_{fp}}\, x_6 x_2}{\sqrt{T_{GG}}} + kom_{O4_{fp}}\, x_6 + \left(-kom_{O3_{fp}}\, x_9 - kom_{O3_{fp}}\, x_{11}\right) x_2 + kom_{O5_{fp}}\, x_9^2 + 2\, kom_{O5_{fp}}\, x_9 x_{11}$$

$\dot{p}_{CC}$;

$$\left(\left(-kp_{CC1_{fp}} + kp_{CC2_{fp}}\right) T_{CC} + kp_{CC4_{fp}} + kp_{CC5_{fp}}\right) x_8 + \left(\left(-kp_{CC1_{fp}} + kp_{CC2_{fp}}\right) T_{CC} + kp_{CC4_{fp}} + kp_{CC5_{fp}}\right) x_9 - kp_{CC3_{fp}}\, x_3 \sqrt{T_{CC}}$$

$\dot{p}_{GG}$;

$$\left(\left(kp_{GG2_{fp}} - kp_{GG1_{fp}}\right) T_{GG} + kp_{GG6_{fp}} + kp_{GG5_{fp}}\right) x_{10} + \left(\left(kp_{GG2_{fp}} - kp_{GG1_{fp}}\right) T_{GG} + kp_{GG6_{fp}} + kp_{GG5_{fp}}\right) x_{11} - T_{GG}\left(x_7 + x_{12}\right) kp_{GG4_{fp}} + \left(kp_{GG6_{fp}} + kp_{GG2_{fp}}\, T_{GG}\right) w_t$$

$\dot{p}_{LTH}$;

$$kp_{LTH1_{fp}}\, x_7 T_{GG} - kp_{LTH2_{fp}}\, x_5 \sqrt{T_{GG}}$$

$\dot{p}_{VGC}$;

$$kp_{VGC1_{fp}}\, x_{12} T_{GG} - kp_{VGC2_{fp}}\, x_6 \sqrt{T_{GG}}$$

$\ddot{m}_{LTH}$;

$$\frac{x_4 - x_5}{InePipe_{LTH_0}} - \frac{km_{LTH_{fp}}\, T_{GG} x_7^2}{x_4}$$

Backup slides: Vulcain NL model

f(x) NL

Ax L

f(x)u NL affine

f(x,u) NL non-affine

k

$\ddot{m}_{VCH}$;

$$\frac{\left(km_{VCH3_{fp}}\, x_1^2 - km_{VCH4_{fp}}\, x_8^2 - km_{VCH5_{fp}}\, x_{10}\, x_8 - km_{VCH6_{fp}}\, x_3 + km_{VCH7_{fp}}\right) u_1^2 - km_{VCH8_{fp}}\, x_8^2}{\left(km_{VCH1_{fp}}\, u_1 + km_{VCH2_{fp}}\right) u_1}$$

$\ddot{m}_{VCO}$;

$$\left(km_{VCO1_{fp}}\, x_2^2 - km_{VCO2_{fp}}\, x_9\, x_2 - km_{VCO3_{fp}}\, x_9^2 - km_{VCO4_{fp}}\, x_3 + km_{VCO5_{fp}}\right) u_2 - \frac{km_{VCO6_{fp}}\, x_9^2}{u_2}$$

$\ddot{m}_{VGH}$;

$$\left(km_{VGH1_{fp}}\, x_1^2 - km_{VGH2_{fp}}\, x_8^2 - km_{VGH3_{fp}}\, x_{10}\, x_8 - km_{VGH4_{fp}}\, x_{10}^2 - km_{VGH5_{fp}}\, x_4 + km_{VGH6_{fp}}\right) u_3 - \frac{km_{VGH7_{fp}}\, x_{10}^2}{u_3}$$

$\ddot{m}_{VGO}$;

$$\left(km_{VGO1_{fp}}\, x_2^2 - km_{VGO2_{fp}}\, x_9\, x_2 - km_{VGO3_{fp}}\, x_9^2 - km_{VGO4_{fp}}\, x_{11}^2 - km_{VGO5_{fp}}\, x_4 + km_{VGO6_{fp}}\right) u_4 - \frac{km_{VGO7_{fp}}\, x_{11}^2}{u_4}$$

$\ddot{m}_{VGC}$;

$$\left(x_4 - x_6 - \frac{kVGC_p\, R_{outGG_{fp}}\, T_{GG}\, x_{12}^2}{2\, x_4\, u_5^2}\right) u_5$$