An MPC Approach to Transient Control of Liquid-Propellant Rocket Engines

Sergio PÉREZ ROCA (CNES/ONERA, France)
sergio.perez_roca@onera.fr

Julien MARZAT (ONERA) Marco GALEOTTA (CNES)
Emilien FLAYAC (ONERA) François FARAGO (CNES)
Hélène PIET-LAHANIER (ONERA) Serge LE GONIDEC (AG)
Nicolas LANGLOIS (IRSEEM)
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1. Introduction: problem definition

This article presents:

- A methodology for **controlling** the continuous part of the **transient phases** of Liquid-Propellant Rocket Engines (LPRE):
  - **LPRE are complex thermodynamic systems** → nonlinear model-based approach based on fluid mechanics.
  - **Control goal** in this paper:
    - **End-state tracking** in main engine variables.
    - Verification of **operational constraints** during transient.
  - **Controller**:
    - **Nonlinear preprocessor** for reference generation.
    - **Linear MPC** with robustness considerations.

- Progress within the **research trend of improving** the **control performance and robustness** of these devices.
1. Introduction: context

Global trend for affordable access to space → Reusable launchers + rocket engines

- **Reusability race** after *semi-reusable* Space Shuttle
  - Demanding control robustness requirements: multi-restart and thrust-modulation features. [1]
  - Classical steady-state multivariable and linear control → reduced throttling envelope (70%-120% thrust) → too narrow for reusable engines [1] (30% expected in Prometheus).
1. Introduction: state of the art

- Most common approaches in literature → **linearised models about operating points**, for synthesising **steady-state controllers**:
  - **PID based** [11]. MIMO are decoupled into dominant SISO subsystems.
  - **Off-line optimisation**: [4].
  - Incorporation of:
    - **Nonlinear** techniques [6]
    - **Hybrid** techniques [10].
    - **Robust** techniques [15].
    - **Reconfiguration** control [10].

- **No publications** considering **steady-state and transients**, reviewed in [13].
1. Introduction: objectives

- **Enhance** Liquid-Propellant Rocket Engines (LPRE) control.

- **Main target** → control strategy during their transient phases, currently carried out as sequences of events in open loop.
1. Introduction: objectives

- **Goals:**
  - Final-reference tracking at the end of **start-up transient**, via tuning of continuous controls (valves).
    - Main variables to track: **combustion-chamber pressure** ($p_{CC}$) and **oxidiser-to-fuel mixture ratios** (MR).
  - **Constraints verification** (on states and control).
  - **Robustness** to parameters and **initial-conditions variations** and to perturbations.
  - Constrained enough to consider trajectory tracking unnecessary.

![Graphs showing chamber pressures and mixture ratios](image)
2. Modelling

1st steps: development of simulator and state-space models (in [12])

- T-RETM library: Vulcain simulator.
2. Modelling

- **Engine case study**: retired Vulcain 1 (AG 1996-2009) with biased parameters
  - Gas-generator cycle (GG), LOX/LH2, bi-turbopump.
  - Actuators: 5 continuously-controllable valves (VCH, VCO, VGH, VGO and VGC), 1 binary igniter (\(i_{CC}\)) and 1 binary starter (\(i_{GG}\)).
  - Valve angles \(\alpha\) varied to control chamber pressure \(p_{CC}\) and mixture ratio \(MR = \frac{m_{ox}}{m_{fu}}\).

![VULCAIN 1 simplified flow plan](image)
2. Modelling

Translation into a state-space model

Simulator

Global state-space model

\[ \dot{x} = f_c(x, u_c, u_d); \]

\[ \begin{bmatrix} \dot{\omega}[2 \times 1] \\ \dot{p}[4 \times 1] \\ \dot{m}[6 \times 1] \end{bmatrix} = f_c \begin{bmatrix} \omega[2 \times 1] \\ p[4 \times 1] \\ m[6 \times 1] \end{bmatrix}, \begin{bmatrix} A_{VCH} \\ A_{VGH} \\ A_{VCO} \\ A_{VGO} \\ A_{VGC} \end{bmatrix}, \begin{bmatrix} i_{CC} \\ i_{GG} \end{bmatrix}; \]

12 states
5 continuous inputs
2 discrete inputs

Greater tracking relevance

All equations joined via Maple

\[ x = [\omega_H \ \omega_O \ pCC \ pGG \ pLTH \ pVGC \ m_{LTH} \ m_{VCH} \ m_{VCO} \ m_{VGH} \ m_{VGO} \ m_{VGC}]^T. \]

\[ x_z = [pCC \ m_{VCH} \ m_{VCO} \ m_{VGH} \ m_{VGO}]^T. \]

\[ u = [A_{VCH} \ A_{VCO} \ A_{VGH} \ A_{VGO} \ A_{VGC}]^T. \]
2. Modelling

4 stages of modelling:
- non-dimensional
- all need $\Delta t = 10^{-5}$ s

Simulator
- Full complexity

Complex nonlinear state-space $f_c$
- 12 states
- Cumbersome equations

Simplified nonlinear state-space $f_s$
- 12 states
- More tractable and linearisable

Linearised model
- After sequential events
- About end state

\[
\dot{x} = f_c(x, u_c, u_d);
\]
\[
\begin{bmatrix}
\dot{\omega}[2\times1] \\
\dot{p}[4\times1] \\
\dot{m}[6\times1]
\end{bmatrix}
= f_s
\begin{bmatrix}
\omega[2\times1] \\
\alpha_{VCH} \\
\alpha_{VGH} \\
\alpha_{VCO} \\
i_{CC}
\end{bmatrix}
\]
\[
\begin{bmatrix}
\dot{\omega}[2\times1] \\
\dot{p}[4\times1] \\
\dot{m}[6\times1]
\end{bmatrix}
= A
\begin{bmatrix}
\omega[2\times1] \\
\alpha_{VCH} \\
\alpha_{VGH} \\
\alpha_{VCO}
\end{bmatrix}
+ B_c
\begin{bmatrix}
i_{CC}
\end{bmatrix}
\]
3. Controller design

Model-Predictive Control (MPC):

- **Constraints** can be well defined and respected with this family of approaches.
- **Robustness** can also be handled.
- **Drawback**: computational time, due to online computation of control law.

- terminal region constraint $x(t + T_p) \in \mathcal{E}$
- terminal penalty term $E(x(t + T_p))$

\[
\min_{u} J(x(t), u)
\]

subject to:
- $x = f(x, u)$, system dynamics
- $x(t)$ given, “state feedback”
- $u(\tau) \in U$, input constraints
- $x(\tau) \in X$, state constraints
- $x(t + T_p) \in \mathcal{E}$, terminal constraint

**Quasi-infinite horizon**: with terminal-set constraint [3] Chen and Allgöwer. The MPC controller drives the system to an end set, where a simple fictitious controller ensures stability and reachability.

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3. Controller design

- **Reference generator**: reconstruction of whole state from input configuration (4 data).
  - Least squares with complex NLSS $f_c$ at equilibrium (*lsqnonlin* MATLAB).
- **MPC**:
  - **Optimisation NL** under constraints with *IPOPT* [16].
  - $\Delta t = 10^{-2} s$, $N_p = 10\Delta t$, $N_u = 5\Delta t$
3. Controller design

- **Reference generator:** reconstruction of whole state from input command from launcher (4 data).
  - Least squares with complex NLSS $f_c$ at equilibrium (*lsqnonlin* MATLAB).
  - Forced by the unavailability of analytic solution.
  - Verification of static mixture ratios.

\[
\begin{align*}
\dot{x} &= f_c(x_r, u_r) = 0 \quad (\dot{p}_{CC} = 0) \\
\frac{\tilde{m}_{VCO} + \tilde{m}_{VGO}}{\tilde{m}_{VCH} + \tilde{m}_{VGH}} &= MR_{PI,r} \\
\tilde{m}_{VGO} &= MR_{GG,r} \\
\tilde{m}_{VGH} + \tilde{m}_{VGO} &= \tilde{m}_{LTH} + \tilde{m}_{VGC}.
\end{align*}
\]
3. Controller design

- **Standard MPC (with integrator term):**
  - Quadratic cost function, with terminal-cost term:
    \[
    J(x, u, z) = \left( \sum_{k=1}^{N_x} x_k^T Q x_k + \sum_{k=1}^{N_u} u_k^T R u_k + \sum_{k=1}^{N_z} z_k^T S z_k \right) \Delta t + x_{N_p+1}^T P x_{N_p+1}
    \]
    \(Q, R\) optimised via black-box Kriging [8]

- **Robust MPC:**
  - Minimax problem: worst-case scenario for given perturbation \(w\) [9]:
    \[
    \min_{u} \max_{w} \quad J(x, u)
    \]
    \(s.t. \quad x \in X \quad \forall w \in W^n
    \]
    \(u \in U \quad \forall w \in W^n
    \]
  - Minimax optimisation too costly \(\rightarrow\) rewriting as min of scalar \(\gamma \in \mathbb{R}^+\) for given perturbation cases (equivalent epigraph formulation) [5]
  - Smooth convex NLP (nonlinear programme).

- **Constraints:**
  - State and control bounds [7]
  - Linear MR+ actuators \((i_{max})\) inequalities.
  - Linear dynamics.
  - Nonlinear for terminal region [3].
  - Integrator dynamics [14]

Matrices \(A, B\) at reference end point, to converge to \(x_r=0\) in the linear case.

\(w_{i,k}\) selected according to eigenvectors of \(A_d\) [2,17]
\(I = \{1,2,3\}\)
4. Analysis of results

- Results of robust MPC for a set perturbed cases.

**Combustion-chamber pressure tracking**

- **MR constraints** relaxed at beginning, due to initial conditions outside feasible area.

**Mixture-ratios tracking and constraints**

<table>
<thead>
<tr>
<th>Indicator</th>
<th>( p_{CC} ) tracking error (%)</th>
<th>MR tracking errors (CC, GG, PI) (%)</th>
<th>Overshoot (% in ( p_{CC} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operating point</td>
<td>OL</td>
<td>CL</td>
<td>OL</td>
</tr>
<tr>
<td>70%</td>
<td>2.8</td>
<td>0.26</td>
<td>2.58; 1.31; 2.84</td>
</tr>
<tr>
<td>100%</td>
<td>0.25</td>
<td>0.26</td>
<td>0.17; 1.39 1.43</td>
</tr>
<tr>
<td>120%</td>
<td>0.34</td>
<td>0.67</td>
<td>3.18; 1.23; 3.41</td>
</tr>
</tbody>
</table>
4. Analysis of results

- PID and LQR achieve tracking, but MPC also ensures constraints verification:
5. Conclusion

- Evolving **operating requirements** of pump-fed rocket engines, currently related to **reusability scenarios** → force the **improvement of their control algorithm**, in OL during transients.

- **Control of continuous phase of LPRE transients:**
  - **Full-state reference generation** via complex nonlinear state-space model.
  - **Linear MPC with integral action:** end-state tracking while verifying constraints.
    - Robustness considerations for a given set of perturbation scenarios.

- **Perspectives:**
  - Investigation of other ways of posing robustness globally.
  - Trajectory generation and tracking.
  - Extensive validation and sensitivity study.
Thank you for your attention.
Any questions or suggestions?
References


State-space model

\[ \dot{x} := f(x, u_c, u_d); \]

\[ \dot{x} := \begin{bmatrix} \dot{\omega}_H \; \dot{\omega}_O \; \dot{p}_{CC} \; \dot{p}_{GG} \; \dot{p}_{LTH} \; \dot{p}_{VGC} \; \dot{m}_{LTH} \; \dot{m}_{VCH} \; \dot{m}_{VCO} \; \dot{m}_{VGH} \; \dot{m}_{VGO} \; \dot{m}_{VGC} \end{bmatrix}^T; \]

\[ x := \begin{bmatrix} \omega_H \; \omega_O \; p_{CC} \; p_{GG} \; p_{LTH} \; p_{VGC} \; m_{LTH} \; m_{VCH} \; m_{VCO} \; m_{VGH} \; m_{VGO} \; m_{VGC} \end{bmatrix}^T; \]

\[ u_c := [AVCH, AVCO, AVGH, AVGO, AVGC]^T; \text{#equivalent to } \alpha_i \]

\[ u_d := [i_{CC} \; i_{GG}]^T; \]

\[ \begin{align*}
\dot{\omega}_H &= \frac{\omega_H}{\omega_{Heq}}; & \dot{\omega}_O &= \frac{\omega_O}{\omega_{Heq}}; & \dot{p}_{CC} &= \frac{p_{CC}}{p_{CCeq}}; & \dot{p}_{GG} &= \frac{p_{GG}}{p_{CCeq}}; & \dot{p}_{LTH} &= \frac{p_{LTH}}{p_{CCeq}}; & \dot{p}_{VGC} &= \frac{p_{VGC}}{p_{CCeq}}; \\
\dot{m}_{LTH} &= \frac{\dot{m}_{LTH}}{m_{VHeq}}; & \dot{m}_{VCH} &= \frac{\dot{m}_{VCH}}{m_{VHeq}}; & \dot{m}_{VCO} &= \frac{\dot{m}_{VCO}}{m_{VHeq}}; & \dot{m}_{VGH} &= \frac{\dot{m}_{VGH}}{m_{VHeq}}; & \dot{m}_{VGO} &= \frac{\dot{m}_{VGO}}{m_{VHeq}}; & \dot{m}_{VGC} &= \frac{\dot{m}_{VGC}}{m_{VHeq}}; \\
i_{CC} &= 1; & i_{GG} &= 1; & t_{eq} &= 1; \\
\end{align*} \]

State constraints

\[ \omega_H \in (0, 40) \text{ krpm}; \quad \omega_O \in (0, 15) \text{ krpm}; \]

\[ MR_{CC} = \frac{\dot{m}_{VCO}}{m_{VCH}} \in [0.3, 7]; \quad MR_{GG} = \frac{\dot{m}_{VGO}}{m_{VGH}} \in [0.3, 1.4]; \]

\[ \dot{p}_{GG} \leq 40 \text{ bar} \]

Control constraints

\[ i_i \in \{0, 1\}; \quad A_i \in [0, A_{i, \text{max}}]^\circ; \quad \dot{A}_i \leq \dot{A}_{i, \text{max}} \]
Backup slides: Vulcain NL model

\[ \dot{\omega}_H = -\frac{\kappa_{\text{om}_{H2}} x_5 x_1}{\sqrt{T_{GG}}} + \kappa_{\text{om}_{H4}} x_5 + \left( -\kappa_{\text{om}_{H3}} x_8 - \kappa_{\text{om}_{H3}} x_10 \right) x_1 + \kappa_{\text{om}_{H5}} x_8^2 + 2 \kappa_{\text{om}_{H5}} x_8 x_10 \]

\[ \dot{\omega}_O = -\kappa_{\text{om}_{O1}} x_2^2 - \frac{\kappa_{\text{om}_{O2}} x_6 x_2}{\sqrt{T_{GG}}} + \kappa_{\text{om}_{O4}} x_6 + \left( -\kappa_{\text{om}_{O3}} x_9 - \kappa_{\text{om}_{O3}} x_{11} \right) x_2 + \kappa_{\text{om}_{O5}} x_9^2 + 2 \kappa_{\text{om}_{O5}} x_9 x_{11} \]

\[ \dot{p}_{CC} = \left( -k_{p\text{CC1}} + k_{p\text{CC2}} \right) T_{CC} + k_{p\text{CC4}} + k_{p\text{CC5}} \right) x_8 + \left( -k_{p\text{CC1}} + k_{p\text{CC2}} \right) T_{CC} + k_{p\text{CC4}} + k_{p\text{CC5}} \right) x_9 - k_{p\text{CC3}} x_3 \sqrt{T_{CC}} \]

\[ \dot{p}_{GG} = \left( k_{p\text{GG2}} - k_{p\text{GG1}} \right) T_{GG} + k_{p\text{GG6}} + k_{p\text{GG5}} \right) x_{10} + \left( k_{p\text{GG2}} - k_{p\text{GG1}} \right) T_{GG} + k_{p\text{GG6}} + k_{p\text{GG5}} \right) x_{11} - T_{GG} \left( x_7 + x_{12} \right) k_{p\text{GG4}} + \left( k_{p\text{GG6}} + k_{p\text{GG2}} \right) T_{GG} \right) w_1 \]

\[ \dot{p}_{LTH} = k_{p\text{LTH}} x_7 T_{GG} - k_{p\text{LTH}} x_5 \sqrt{T_{GG}} \]

\[ \dot{p}_{VGC} = k_{p\text{VGC}} x_9 T_{GG} - k_{p\text{VGC2}} x_6 \sqrt{T_{GG}} \]

\[ \dot{m}_{LTH} = \frac{x_4 - x_5}{\text{InEP}_{LTH}} - \frac{k_{m\text{LTH}} T_{GG} x_4^2}{x_4} \]
Backup slides: Vulcain NL model

\[ \ddot{m}_{VCH'} = \frac{\left( k_{m_{VCH1}} \frac{x_1^2 - k_{m_{VCH4}}}{f_p} x_8 - k_{m_{VCH5}} \right) u_1^2 - k_{m_{VCH8}} \frac{x_8}{f_p} }{u_1} \]

\[ \ddot{m}_{VCO'} = \frac{\left( k_{m_{VCO1}} \frac{x_2^2 - k_{m_{VCO2}}}{f_p} x_9 - k_{m_{VCO3}} \right) u_2^2 - k_{m_{VCO6}} \frac{x_9}{f_p} }{u_2} \]

\[ \ddot{m}_{VGH'} = \frac{\left( k_{m_{VGH1}} \frac{x_1^2 - k_{m_{VGH2}}}{f_p} x_8 - k_{m_{VGH3}} \right) u_3^2 - k_{m_{VGH7}} \frac{x_{10}}{f_p} }{u_3} \]

\[ \ddot{m}_{VGO'} = \frac{\left( k_{m_{VGO1}} \frac{x_2^2 - k_{m_{VGO2}}}{f_p} x_9 - k_{m_{VGO3}} \right) u_4^2 - k_{m_{VGO7}} \frac{x_{11}}{f_p} }{u_4} \]

\[ \ddot{m}_{VGC'} = \frac{\left( k_{VGC} \frac{R_{outGG}}{f_p} - T_{GG}^2 \frac{g^2}{2x_4 u_5^2} \right) u_5}{u_5} \]