On the use of a feedback linearization local controller for the terminal region of an NMPC scheme

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Experimental platform for thrust propelled vehicles

Experimental room equipped with the Loco Positioning system

Crazyflie 2.0 with Loco Positioning deck

Two Way Ranging radio message

Input message \( \{T_r, \phi_r, \theta_r, \dot{\psi}_r\} \)

Feedback \( \{\xi, \eta\} \)

\( \times 6 \)

Loco Positioning node

Crazyflie client

Crazyradio PA USB radio dongle

Ionela Prodan (LCIS-Grenoble INP)
Nonlinear Model Predictive Control in continuous-time

Rawlings and Muske (1993); Chen and Allgöwer (1998); Mayne et al. (2000); Mayne (2014)

Solve the open-loop optimal control problem at time $t$ using the measured state $x(t)$ and the prediction horizon $T_p$:

$$
\min_{\bar{u}(\cdot)} \int_t^{t+T_p} \ell(\bar{x}(\tau, t), \bar{u}(\tau, t)) \, d\tau + F(\bar{x}(T_p, t))
$$

subject to:

$$
\begin{cases}
\dot{x} = f(x, u) \text{ (system dynamics)}, \\
x(\tau, t) \in \mathcal{X}, \ u(\tau, t) \in \mathcal{U}, \ \forall \tau \in [t, t + T_p] \text{ (state and input constraints)}, \\
x(t, t) = x(t) \text{ (initial condition)}, \\
x(t + T_p, t) \in \mathcal{X}_f \text{ (terminal constraint set)}.
\end{cases}
$$

Apply to the system at time $\tau \in [t, t + \delta]$ the optimal control action:

$$u_{\text{MPC}}(\tau, t) = \bar{u}^*(\tau, x(t)), \ \forall \tau \in [t, t + \delta],$$

with the sampling time $\delta < T_p$ chosen such that the state measurement is accomplished.

Assumptions:

- The control problem is to stabilize the system around the equilibrium $\{x_e, u_e\}$.
- The stage cost $\ell : \mathcal{X} \times \mathcal{U} \to \mathbb{R}$ satisfies $\ell(x, u) > 0 \ \forall (x, u) \in \mathcal{X} \times \mathcal{U} \setminus \{x_e, u_e\}$ and $\ell(x_e, u_e) = 0$.
- The terminal cost $F : \mathcal{X} \to \mathbb{R}$ satisfies $F(x) > 0 \ \forall x \in \mathcal{X} \setminus \{x_e\}$ and $F(x_e) = 0$. 
Stability conditions of NMPC design with invariant set

The recursive feasibility\(^1\) and the asymptotic (exponential) stability of the closed-loop controlled system are guaranteed if the following conditions are satisfied (Mayne et al. (2000)):

(C1) [State constraints fulfillment in \(\mathcal{X}_f\)] \(\mathcal{X}_f \subseteq \mathcal{X}, \ x_e \in \mathcal{X}_f\).

(C2) [Input constraints fulfillment in \(\mathcal{X}_f\)] There exists a local controller \(u_{\text{loc}}(x)\) such that \(u_{\text{loc}}(x) \in \mathcal{U}, \ \forall x \in \mathcal{X}_f\).

(C3) [Positively invariant terminal set] \(\mathcal{X}_f\) is positively invariant under \(u_{\text{loc}}(x)\).

(C4) [Local Lyapunov function existence] The stage and terminal costs \(\ell(x, u), F(x)\) under \(u_{\text{loc}}(x)\) (i.e., \(\dot{x} = f(x, u_{\text{loc}}(x))\)) satisfy:

\[
\frac{dF(x)}{dt} + \ell(x, u_{\text{loc}}(x)) \leq 0, \ \forall x \in \mathcal{X}_f.
\]

---

\(^1\)feasibility obtained with the assumption of the first successful iteration.
Illustration of recursive feasibility property

\[ x \in \mathbb{R}, \ u \in \mathcal{U} \subset \mathbb{R} \]

\[ x(0) \]

\[ t = 0 \]
\[ T_p = 3\delta \]

\[ x_e \]

\[ u_e \]

\[ 0 \delta \]
\[ 2\delta \]
\[ 3\delta \]
\[ 4\delta \]
\[ 5\delta \]

Time
Various approaches are employed in the literature for stability guarantees:

- **NMPC with terminal equality constraint** (Keerthi and Gilbert (1988); Rawlings and Muske (1993))

  \[ \mathcal{X}_f = \{ x_e \}, \quad F(x) \triangleq 0 \text{ and } u_{loc} = 0. \]

- **Quasi-infinite horizon NMPC** (Chen and Allgöwer (1998))

  \[ \mathcal{X}_f \] is an ellipsoidal invariant set under linear feedback controller.

- **NMPC with polytopic invariant set** (Cannon, Deshmukh, and Kouvaritakis (2003))

  \[ \mathcal{X}_f \] is a polytopic invariant terminal set under linear feedback controller \( u_{loc} \), applied for input-affine nonlinear system.

- **NMPC design employing a feedback linearization law** (Simon, Lofberg, and Glad (2013))

  A feedback linearization law is applied to linearize the considered nonlinear system. Then, an NMPC controller is designed for the resulted linear system under the varying input constraints.

**Can we enlarge the stabilizing terminal set in NMPC by using a nonlinear local controller?**

If so, under what limitations and with what performances?
State-of-the-art on NMPC stability with terminal invariant set

Various approaches are employed in the literature for stability guarantees:

- **NMPC with terminal equality constraint** \((\text{Keerthi and Gilbert (1988); Rawlings and Muske (1993)})\)
  
  \[ \mathcal{X}_f = \{ x_e \}, \quad F(x) \triangleq 0 \text{ and } u_{loc} = 0. \]

- **Quasi-infinite horizon NMPC** \((\text{Chen and Allgöwer (1998)})\)
  
  \(\mathcal{X}_f\) is an ellipsoidal invariant set under linear feedback controller.

- **NMPC with polytopic invariant set** \((\text{Cannon, Deshmukh, and Kouvaritakis (2003)})\)
  
  \(\mathcal{X}_f\) is a polytopic invariant terminal set under linear feedback controller \(u_{loc}\), applied for input-affine nonliner system.

- **NMPC design employing a feedback linearization law** \((\text{Simon, Lofberg, and Glad (2013)})\)
  
  A feedback linearization law is applied to linearize the considered nonlinear system. Then, an NMPC controller is designed for the resulted linear system under the varying input constraints.

- **NMPC design with invariance induced by a computed-torque control law** \((\text{Nguyen, Prodan, and Lefèvre (2019b)})\)

- **NMPC design for quadcopter system with invariance induced by a feedback linearization control law** \((\text{Nguyen, Prodan, and Lefèvre (2019a)})\)
  
  \(\mathcal{X}_f\) is an ellipsoidal invariant set under nonlinear feedback linearization controller \(u_{loc}\).
Highlights

- NMPC particularized for systems which admit a feedback linearization law which linearizes the controlled system into some double integrator systems.

- Construction of an ellipsoid, invariant and constraint admissible set under the feedback linearization law.

- Upper bound of the feedback linearization law described in terms of the state within the invariant set obtained by using Taylor’s theorem and worst-case guarantees.

- The invariant set is employed as the terminal constraint set to guarantee the (nominal) closed-loop stability and recursive feasibility of the NMPC scheme.

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Outline

1. On the use of a computed-torque control law in an NMPC scheme
2. A stabilizing NMPC design for thrust-propelled vehicles dynamics
3. Conclusions and future developments
Outline

1. On the use of a computed-torque control law in an NMPC scheme
   - System admitting a computed-torque control law
   - Input constraints satisfaction
   - Positive invariant set construction
   - Bound of the weighted norm of the input
   - NMPC design with guaranteed stability
   - Simulation results

2. A stabilizing NMPC design for thrust-propelled vehicles dynamics

3. Conclusions and future developments
On the use of a computed-torque control law in an NMPC scheme

System admitting a computed-torque control law

Systems modeling

Dynamics of a system which admits a CTC law:

\[ M(q)\ddot{q} + N(\dot{q}, q) = u, \]

with the state \( q = [q_1, ..., q_n]^\top \in \mathbb{R}^n \) and the actuator input \( u \in \mathbb{R}^n \).

\( M(q) \in \mathbb{R}^{n \times n} \) is a symmetric positive definite inertia matrix, \( N(\dot{q}, q) \in \mathbb{R}^n \) is the vector of the nonlinear terms (Coriolis forces, centrifugal forces).

Examples:

- Inverted pendulum:

  \[ mL^2 \ddot{q} + mgL \cos(q) = \tau. \]

- 3D rigid body rotation dynamics:

  \[
  \Omega = W(\eta)\dot{\eta}, \\
  J\dot{\Omega} + \Omega \times (J\Omega) = \tau, \\
  \Rightarrow J W(\eta) \ddot{\eta} + J \frac{dW(\eta)}{dt} \dot{\eta} + \Omega \times (J\Omega) = \tau.
  \]

\[ M(\eta) N(\dot{\eta}, \eta) \]
Systems modeling

Dynamics of a system which admits a CTC law:

\[ M(q) \ddot{q} + N(\dot{q}, q) = u, \]

with the state \( q = [q_1, \ldots, q_n]^T \in \mathbb{R}^n \) and the actuator input \( u \in \mathbb{R}^n \).

\( M(q) \in \mathbb{R}^{n \times n} \) is a symmetric positive definite inertia matrix,

\( N(\dot{q}, q) \in \mathbb{R}^n \) is the vector of the nonlinear terms (Coriolis forces, centrifugal forces).

State-space representation, i.e., \( \dot{x} = f(x, u) \):

\[
\begin{bmatrix}
    \dot{q} \\
    \dot{\dot{q}}
\end{bmatrix} = \begin{bmatrix}
    I_n & 0 \\
    0 & M(q)^{-1}
\end{bmatrix} \begin{bmatrix}
    \dot{q} \\
    -N(\dot{q}, q) + u
\end{bmatrix}.
\]

Equilibrium point:

\( x_e = 0 \) and \( u_e = 0 \).

System constraints:

\( u \in \mathcal{U} = \{ u \in \mathbb{R}^n \mid -u_{\text{max}} \leq u \leq u_{\text{max}} \} \),

where \( u_{\text{max}} \) consists of all the positive maximal values of the actuator inputs.

\( x \in \mathcal{X} \),

where \( \mathcal{X} \) is a convex set in \( \mathbb{R}^{2n} \) containing \( x_e \).
Computed-torque control law

For the system $\mathbf{M}(q)\ddot{q} + \mathbf{N}(\dot{q}, q) = \mathbf{u}$, the computed-torque control law (Craig (2018)) is given by:

$$
\mathbf{u}_b(x, \nu) = \mathbf{M}(q)\nu + \mathbf{N}(\dot{q}, q),
$$

with $\nu \in \mathbb{R}^n$ the virtual control input. Note that, $\mathbf{u}_b(x_e, \nu_e) = \mathbf{u}_e = 0$.

If $\mathbf{u}_b(x, \nu) \in \mathcal{U}$, it transforms the system into the linear system:

$$
\dot{x} = Ax + B\nu,
$$

with $A \in \mathbb{R}^{2n \times 2n}$ and $B \in \mathbb{R}^{2n \times n}$ given by:

$$
A = \begin{bmatrix} 0_n & I_n \\ 0_n & 0_n \end{bmatrix}, \quad B = \begin{bmatrix} 0_n \\ I_n \end{bmatrix}.
$$
**Application of Taylor’s theorem**

Considering a ball parameterized by a radius \( \varepsilon \in \mathbb{R}^+ \) as:

\[
C(\varepsilon) = \{ (x, v) | \|x\|^2 + \|v\|^2 \leq \varepsilon^2 \}.
\]

For any \( \varepsilon > 0 \), applying Taylor’s theorem (Folland (1990)) to the CTC law \( u_b(x, v) \) for all \( (x, v) \in C(\varepsilon) \), we have that:

\[
u_b(x, v) = u_b(x_e, v_e) + x J x + v J v + R_\varepsilon(x, v),
\]

with the two Jacobians
\[
x J = \left. \frac{\partial u_b}{\partial x} \right|_{(x, v) = (x_e, v_e)} \quad \text{and} \quad v J = \left. \frac{\partial u_b}{\partial v} \right|_{(x, v) = (x_e, v_e)}.
\]

The remainder vector \( R_\varepsilon \in \mathbb{R}^n \) is bounded from Taylor’s inequality as follows:

\[
|R_\varepsilon(x, v)| \leq \frac{M_\varepsilon}{2!} (\|x\|^2 + \|v\|^2),
\]

with \( M_\varepsilon \triangleq [M_{\varepsilon,1}, \ldots, M_{\varepsilon,n}] \) having each element, \( M_{\varepsilon,i} \in \mathbb{R}^+ \) \( (i \in \{1, \ldots, n\}) \) defined as:

\[
M_{\varepsilon,i} = \max_{\|x\|^2 + \|v\|^2 \leq \varepsilon^2} |H(u_{b,i}(x, v))|,
\]

where \( u_{b,i}(x, v) \) is the \( i^{th} \) element of the vector function \( u_b(x, v) \). \( H(\cdot) \) is the Hessian matrix of a scalar-valued function containing all of its second-order partial derivatives (Meyer (2000)).
Application of the Cauchy-Schwarz inequality

For any $\varepsilon > 0$, we have that:

$$ u_b(x, v) = x Jx + v Jv + R_\varepsilon(x, v), \text{ with } |R_\varepsilon(x, v)| \leq \frac{M_\varepsilon}{2!} (\|x\|^2 + \|v\|^2). $$

Applying the Cauchy-Schwarz inequality (i.e., $|ax + by| \leq \sqrt{(a^2 + b^2)(x^2 + y^2)}$) to each element $u_{b,i}(x, v) (i \in \{1, \ldots, n\})$, it is straightforward to obtain:

$$ |u_{b,i}(x, v)| \leq C_i \sqrt{\|x\|^2 + \|v\|^2 + \frac{M_\varepsilon,i}{2} (\|x\|^2 + \|v\|^2)}, $$

where $C_i \in \mathbb{R}^+$ ($i \in \{1, \ldots, n\}$), is defined as:

$$ C_i = \sqrt{\|x J_i\|^2 + \|v J_i\|^2}, $$

with $x J_i$ and $v J_i$ the $i^{th}$ rows of $x J$ and $v J$ matrices. Thus, by choosing $\varepsilon_{\max} \in \mathbb{R}^+$, such that:

$$ C\varepsilon_{\max} + \frac{M_{\varepsilon_{\max}}}{2} \varepsilon_{\max}^2 \leq u_{\max}, $$

with $C \triangleq [C_1, \ldots, C_n]^{\top}$, we have that:

$$ |u_b(x, v)| \leq u_{\max}, \forall (x, v) \in C(\varepsilon_{\max}). $$
Proof of concept example

Find $\varepsilon_{\text{max}}$ such that $|u_b(x, v)| \leq u_{\text{max}}$ for all $(x, v) \in \mathcal{C}(\varepsilon_{\text{max}})$: $x^2 + v^2 \leq \varepsilon_{\text{max}}^2$ with

$$u_b(x, v) = \begin{bmatrix} v + \sin(x) \\ \exp(x)v \end{bmatrix} \quad \text{and} \quad u_{\text{max}} = \begin{bmatrix} 1 \\ 2 \end{bmatrix},$$
Proof of concept example

Find $\varepsilon_{\text{max}}$ such that $|u_b(x, v)| \leq u_{\text{max}}$ for all $(x, v) \in C(\varepsilon_{\text{max}}) : x^2 + v^2 \leq \varepsilon^2_{\text{max}}$ with

$$u_b(x, v) = \begin{bmatrix} v + \sin(x) \\ \exp(x)v \end{bmatrix} \quad \text{and} \quad u_{\text{max}} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad (u_b(x, v) = M(q)v + N(q, q)).$$
Proof of concept example

Find \( \varepsilon_{\text{max}} \) such that \( |u_b(x, v)| \leq u_{\text{max}} \) for all \( (x, v) \in C(\varepsilon_{\text{max}}) : x^2 + v^2 \leq \varepsilon_{\text{max}}^2 \) with

\[
|u_b(x, v)| = \begin{bmatrix} \cos(x) \\ v \\ \exp(x) v \end{bmatrix}
\]

\[
\text{and} \quad u_{\text{max}} = \begin{bmatrix} 1 \\ 2 \end{bmatrix},
\]

1) Taylor’s series:

\[
u_b(x, v) = \begin{bmatrix} \cos(x_e) \\ v_e \exp(x_e) \end{bmatrix} x + \begin{bmatrix} 1 \\ \exp(x_e) \end{bmatrix} v + R_\varepsilon(x, v).
\]
Proof of concept example

Find $\varepsilon_{\text{max}}$ such that $|u_b(x, v)| \leq u_{\text{max}}$ for all $(x, v) \in C(\varepsilon_{\text{max}}): x^2 + v^2 \leq \varepsilon_{\text{max}}^2$ with

$$u_b(x, v) = \begin{bmatrix} v + \sin(x) \\ \exp(x)v \end{bmatrix} \quad \text{and} \quad u_{\text{max}} = \begin{bmatrix} 1 \\ 2 \end{bmatrix},$$

1) Taylor’s series:

$$u_b(x, v) = \cos(x) \begin{bmatrix} \cos(x) \\ v \exp(x) \end{bmatrix} x + \begin{bmatrix} 1 \\ \exp(x) \end{bmatrix} v + R_\varepsilon(x, v).$$

2) Taylor’s inequality:

$$|R_\varepsilon(x, v)| \leq \frac{1}{2} \begin{bmatrix} M_{\varepsilon,1} \\ M_{\varepsilon,2} \end{bmatrix} (x^2 + v^2).$$

with

$$M_{\varepsilon,1} = \max_{x^2 + v^2 \leq \varepsilon^2} H(u_b,1(x,v)) \quad \text{and} \quad M_{\varepsilon,2} = \max_{x^2 + v^2 \leq \varepsilon^2} H(u_b,2(x,v)).$$

$$H(u_b,1(x,v)) = \begin{bmatrix} (\partial^2 u_b,1)/(\partial x^2) & (\partial^2 u_b,1)/(\partial x \partial v) \\ (\partial^2 u_b,1)/(\partial v \partial x) & (\partial^2 u_b,1)/(\partial v^2) \end{bmatrix} = \max_{x^2 + v^2 \leq \varepsilon^2} \begin{bmatrix} -\sin(x) & 0 \\ 0 & 0 \end{bmatrix},$$

$$H(u_b,2(x,v)) = \begin{bmatrix} (\partial^2 u_b,2)/(\partial x^2) & (\partial^2 u_b,2)/(\partial x \partial v) \\ (\partial^2 u_b,2)/(\partial v \partial x) & (\partial^2 u_b,2)/(\partial v^2) \end{bmatrix} = \max_{x^2 + v^2 \leq \varepsilon^2} \begin{bmatrix} v \exp x & \exp x \\ \exp x & 0 \end{bmatrix}.$$
Proof of concept example

Find $\varepsilon_{\text{max}}$ such that $|u_b(x, v)| \leq u_{\text{max}}$ for all $(x, v) \in C(\varepsilon_{\text{max}}): x^2 + v^2 \leq \varepsilon_{\text{max}}^2$ with

$$u_b(x, v) = \begin{bmatrix} v + \sin(x) \\ \exp(x) v \end{bmatrix} \quad \text{and} \quad u_{\text{max}} = \begin{bmatrix} 1 \\ 2 \end{bmatrix},$$

1) Taylor’s series:

$$u_b(x, v) = \begin{bmatrix} \cos(x_e) \\ v_e \exp(x_e) \end{bmatrix} x + \begin{bmatrix} 1 \\ \exp(x_e) \end{bmatrix} v + R_\varepsilon(x, v).$$

2) Taylor’s inequality:

$$|R_\varepsilon(x, v)| \leq \frac{1}{2} \begin{bmatrix} M_{\varepsilon,1} \\ M_{\varepsilon,2} \end{bmatrix} (x^2 + v^2).$$

3) Cauchy-Schwarz inequality:

$$|u_b(x, v)| \leq \begin{bmatrix} \sqrt{\cos^2(x_e) + 1} \\ \sqrt{v_e^2 \exp(2x_e) + \exp(2x_e)} \end{bmatrix} \begin{bmatrix} \sqrt{x^2 + v^2} + \frac{1}{2} \begin{bmatrix} M_{\varepsilon,1} \\ M_{\varepsilon,2} \end{bmatrix} (x^2 + v^2).
Proof of concept example

Find $\varepsilon_{\text{max}}$ such that $|u_b(x, v)| \leq u_{\text{max}}$ for all $(x, v) \in C(\varepsilon_{\text{max}}) : x^2 + v^2 \leq \varepsilon_{\text{max}}^2$ with

$$u_b(x, v) = \begin{bmatrix} v + \sin(x) \\ \exp(x)v \end{bmatrix} \text{ and } u_{\text{max}} = \begin{bmatrix} 1 \\ 2 \end{bmatrix},$$

1) Taylor’s series:

$$u_b(x, v) = \begin{bmatrix} \cos(x_e) \\ v_e \exp(x_e) \end{bmatrix} x + \begin{bmatrix} 1 \\ \exp(x_e) \end{bmatrix} v + R_\varepsilon(x, v).$$

2) Taylor’s inequality:

$$|R_\varepsilon(x, v)| \leq \frac{1}{2} \begin{bmatrix} M_{\varepsilon,1} \\ M_{\varepsilon,2} \end{bmatrix} (x^2 + v^2).$$

3) Cauchy-Schwarz inequality:

$$|u_b(x, v)| \leq \sqrt{\cos^2(x_e) + 1} \sqrt{v^2 \exp(2x_e) + \exp(2x_e)} \sqrt{x^2 + v^2} + \frac{1}{2} \begin{bmatrix} M_{\varepsilon,1} \\ M_{\varepsilon,2} \end{bmatrix} (x^2 + v^2).$$

4) Find $\varepsilon_{\text{max}}$ such that:

$$\begin{bmatrix} \sqrt{2} \\ 1 \end{bmatrix} \varepsilon_{\text{max}} + \max_{x^2 + v^2 \leq \varepsilon_{\text{max}}^2} \left\{|v \exp(x)|, |\exp(x)|\right\} \varepsilon_{\text{max}}^2 \leq \begin{bmatrix} 1 \\ 2 \end{bmatrix} \Rightarrow \varepsilon_{\text{max}} = 0.578 \text{ with } LHS = \begin{bmatrix} 0.99 \\ 1.17 \end{bmatrix}.$$
Reminder (stability conditions of NMPC design with invariant set)!

The recursive feasibility and the asymptotic (exponential) stability of the closed-loop controlled system are guaranteed if the following conditions are satisfied (Mayne et al. (2000)):

(C1) [State constraints fulfillment in $X_f$] $X_f \subseteq X, \; x_e \in X_f$.

(C2) [Input constraints fulfillment in $X_f$] There exists a local controller $u_{loc}(x)$ such that $u_{loc}(x) \in U, \; \forall x \in X_f$.

(C3) [Positively invariant terminal set] $X_f$ is positively invariant under $u_{loc}(x)$.

(C4) [Local Lyapunov function existence] The stage and terminal costs $\ell(x, u), F(x)$ under $u_{loc}(x)$ (i.e., $\dot{x} = f(x, u_{loc}(x))$) satisfy:

$$\frac{dF(x)}{dt} + \ell(x, u_{loc}(x)) \leq 0, \; \forall x \in X_f.$$

---

feasibility obtained with the assumption of the first successful iteration.
Reminder (stability conditions of NMPC design with invariant set)!

The recursive feasibility\(^3\) and the asymptotic (exponential) stability of the closed-loop controlled system are guaranteed if the following conditions are satisfied (Mayne et al. (2000)):

1. **(C1) State constraints fulfillment in \(\mathcal{X}_f\)**: \(\mathcal{X}_f \subseteq \mathcal{X}, x_e \in \mathcal{X}_f\).

2. **(C2) Input constraints fulfillment in \(\mathcal{X}_f\)**: There exists a local controller \(u_{\text{loc}}(x)\) such that \(u_{\text{loc}}(x) \in \mathcal{U}, \forall x \in \mathcal{X}_f\).

3. **(C3) Positively invariant terminal set** \(\mathcal{X}_f\) is positively invariant under \(u_{\text{loc}}(x)\).

4. **(C4) Local Lyapunov function existence** The stage and terminal costs \(\ell(x, u), F(x)\) under \(u_{\text{loc}}(x)\) (i.e., \(\dot{x} = f(x, u_{\text{loc}}(x))\)) satisfy:

\[
\frac{dF(x)}{dt} + \ell(x, u_{\text{loc}}(x)) \leq 0, \ \forall x \in \mathcal{X}_f.
\]

\(^3\)feasibility obtained with the assumption of the first successful iteration.
Lemma 1 (Nguyen et al. (2019b))

Let us construct the matrix $K$ as:

$$K = \begin{bmatrix} \text{diag}(K_{p1}, \ldots, K_{pn}) & \text{diag}(K_{d1}, \ldots, K_{dn}) \end{bmatrix},$$

where the $2n$ control gains $K_{p1}, \ldots, K_{pn}$ and $K_{d1}, \ldots, K_{dn}$ are chosen such that:

\[
\begin{align*}
K_{pi} &< 0, \quad K_{di} < 0, \\
4K_{di}^2 &> -K_{pi}(K_{pi} + 1)^2 - K_{pi} - \frac{(K_{pi} + 1)^2}{K_{pi}},
\end{align*}
\]

with $i \in \{1, \ldots, n\}$ and define the set $\mathcal{X}_f$ as:

$$\mathcal{X}_f = \{x \in \mathbb{R}^n \mid x^\top \left( I_{2n} + K^\top K \right) x \leq \varepsilon^2 \},$$

where $\varepsilon$ is chosen such that:

\[
\begin{align*}
\varepsilon &\leq \varepsilon_{\max}, \\
\mathcal{X}_f &\subseteq \mathcal{X},
\end{align*}
\]

Then, we have that:

(i) $x \in \mathcal{X}_f$ imposes that the control action $u_b(x, Kx) \in \mathcal{U}$ (i.e., the CTC law $u_b(x, v)$ employing the virtual input design $v = Kx$ satisfies the input constraints);

(ii) the set $\mathcal{X}_f$ is positively invariant for the controlled system $\dot{x} = f(x, u_b(x, Kx)).$
Bound of the weighted norm of the input

Lemma 2 (Nguyen et al. (2019b))

If \( x \in X_f \), there exists \( R^* \in \mathbb{R}^{2n \times 2n} \) such that:

\[
\| u_b(x, Kx) \|^2_R \leq \| x \|^2_{R^*},
\]

with \( R \triangleq \text{diag}\{R_1, \ldots, R_n\} \) semi-positive definite matrix and \( K \) the control gain matrix.

Proof.

Employing the diagonal matrix \( R \) leads to:

\[
\| u_b(x, Kx) \|^2_R = \sum_{i=1}^{n} R_i u_{b,i}^2(x, Kx).
\]

Previous results on bound of \( u_{b,i}^2(x, v) \) with \( i \in \{1, \ldots, n\} \):

\[
u_{b,i}^2(x, Kx) \leq \left( C_i + \frac{M_{e,i}}{2} \varepsilon \right)^2 (\| x \|^2 + \| Kx \|^2) = \left( C_i + \frac{M_{e,i}}{2} \varepsilon \right)^2 x^\top (I_{2n} + K^\top K)x.\]

Hence, Lemma 2 is validated with the matrix \( R^* \in \mathbb{R}^{2n \times 2n} \) chosen such that:

\[
R^* - \sum_{i=1}^{n} R_i \left( C_i + \frac{M_{e,i}}{2} \varepsilon \right)^2 (I_{2n} + K^\top K) \succeq 0.
\]
Reminder (stability conditions of NMPC design with invariant set)!

The recursive feasibility\textsuperscript{4} and the asymptotic (exponential) stability of the closed-loop controlled system are guaranteed if the following conditions are satisfied (Mayne et al. (2000)):

\begin{enumerate}[(C1)]
\item [State constraints fulfillment in $\mathcal{X}_f$] $\mathcal{X}_f \subseteq \mathcal{X}$, $x_e \in \mathcal{X}_f$.
\item [Input constraints fulfillment in $\mathcal{X}_f$] There exists a local controller $u_{\text{loc}}(x)$ such that $u_{\text{loc}}(x) \in \mathcal{U}$, $\forall x \in \mathcal{X}_f$.
\item [Positively invariant terminal set] $\mathcal{X}_f$ is positively invariant under $u_{\text{loc}}(x)$.
\item [Local Lyapunov function existence] The stage and terminal costs $\ell(x,u), F(x)$ under $u_{\text{loc}}(x)$ (i.e., $\dot{x} = f(x, u_{\text{loc}}(x))$) satisfy:
\[ \frac{dF(x)}{dt} + \ell (x, u_{\text{loc}}(x)) \leq 0, \: \forall x \in \mathcal{X}_f. \]
\end{enumerate}

\textsuperscript{4}feasibility obtained with the assumption of the first successful iteration.
Reminder (stability conditions of NMPC design with invariant set)!

The recursive feasibility\(^4\) and the asymptotic (exponential) stability of the closed-loop controlled system are guaranteed if the following conditions are satisfied (Mayne et al. (2000)):

1. **(C1) [State constraints fulfillment in \(\mathcal{X}_f\)]** \(\mathcal{X}_f \subseteq \mathcal{X}, \ x_e \in \mathcal{X}_f\).

2. **(C2) [Input constraints fulfillment in \(\mathcal{X}_f\)]** There exists a local controller \(u_{\text{loc}}(x)\) such that \(u_{\text{loc}}(x) \in \mathcal{U}, \ \forall x \in \mathcal{X}_f\).

3. **(C3) [Positively invariant terminal set]** \(\mathcal{X}_f\) is positively invariant under \(u_{\text{loc}}(x)\).

4. **(C4) [Local Lyapunov function existence]** The stage and terminal costs \(\ell(x, u), F(x)\) under \(u_{\text{loc}}(x)\) (i.e., \(\dot{x} = f(x, u_{\text{loc}}(x))\)) satisfy:

\[
\frac{dF(x)}{dt} + \ell(x, u_{\text{loc}}(x)) \leq 0, \ \forall x \in \mathcal{X}_f.
\]

---

\(^4\)feasibility obtained with the assumption of the first successful iteration.
Recursive feasibility and stability guarantees

- Stage cost design:
  \[ \ell(x, u) = \|x\|^2_Q + \|u\|^2_R, \]
  with \( Q \in \mathbb{R}^{2n \times 2n} \) positive definite and \( R \triangleq \text{diag}\{R_1, \ldots, R_n\} \) semi-positive definite matrices.

- Terminal cost design:
  \[ F(x) = \|x\|^2_P, \]
  with \( P \in \mathbb{R}^{2n \times 2n} \) positive definite matrix defined hereinafter.

**Lemma 3 (Nguyen et al. (2019b))**

*Let us consider the matrix \( P \succ 0 \) as the unique solution of the Lyapunov equation given by:*

\[ A_K^T P + PA_K + Q + R^* = 0, \]

*with the Routh-Hurwitz matrix \( A_K = A + BK \) of the linear system resulted from the CTC controller \( u_b(x, Kx) \). Then, the nominal closed-loop system controlled by the NMPC controller with the terminal region \( \mathcal{X}_f = \{ x^T (I_{2n} + K^T K)x \leq \varepsilon^2 \} \) achieves recursive feasibility and asymptotic stability.*
Satisfaction of four NMPC design conditions

[Proof sketch of Lemma 3] The asymptotic stability is proven by the satisfaction of the four NMPC design conditions with the local controller $u_{\text{loc}}(x)$ chosen as the CTC law $u_b(x, Kx)$.

- **C1**: $x_e \in \mathcal{X}_f = \{x^\top (I_{2n} + K^\top K)x \leq \varepsilon^2\}$ is trivial and $\mathcal{X}_f \subseteq \mathcal{X}$ is by tuning $\varepsilon$.

- **C2**: $u_{\text{loc}}(x) \equiv u_b(x, Kx) \in U$, $\forall x \in \mathcal{X}_f$ since $\mathcal{X}_f$ is constraint admissible due to $\varepsilon \leq \varepsilon_{\text{max}}$.

- **C3**: $\mathcal{X}_f$ is positively invariant under $u_{\text{loc}}(x)$ due to Lemma 1.

- **C4**: for all $x \in \mathcal{X}_f$, we have that:

$$\frac{d}{dt} \left( \|x\|^2_P \right) + \|x\|_Q^2 + \|u_{\text{loc}}(x)\|^2_R \leq x^\top \left( A_K^\top P + PA_K + Q + R^* \right) x = 0,$$

for which, $\frac{d}{dt} \left( \|x\|^2_P \right) = x^\top (A_K^\top P + PA_K) x$ is resulted from applying the CTC law $u_b(x, Kx)$, thus, obtaining the stable linear system. $\|u_{\text{loc}}(x)\|^2_R \leq \|x\|^2_{R^*}$ previously introduced. $A_K^\top P + PA_K + Q + R^* = 0$ is by choosing $P$ as its solution.
Design procedure of the NMPC controller for stabilizing the system possessing the CTC law:

1) Choose the weighting matrices $Q \in \mathbb{R}^{2n \times 2n}$ and $R = \text{diag}\{R_1, \ldots, R_n\}$ of the stage cost.

2) Estimate the prediction horizon $T_p$ based on the computational constraint of the platform (e.g., the processing speed requirement).

3) Find the largest possible $\varepsilon_{\text{max}}$ satisfying $C\varepsilon_{\text{max}} + \frac{M\varepsilon_{\text{max}}}{2} \leq \varepsilon_{\text{max}}^2$.

4) Define the control gain matrix $K$ which stabilizes the resulted linear system and guarantees the invariance property. Then, tune $\varepsilon \leq \varepsilon_{\text{max}}$ in order to obtain the terminal region $X_f$:

$$X_f = \{x \in \mathbb{R}^n | x^\top (I_{2n} + K^\top K)x \leq \varepsilon^2 \} \subseteq \mathcal{X}.$$

5) Define the matrix $R^*$ such that $\|u_b(x, Kx)\|_R^2 \leq \|x\|_R^2$ for all $x \in X_f$. Then, solve the Lyapunov equation $A_K^\top P + PA_K + Q + R^* = 0$ for the terminal weighting matrix $P$. 

Ionela Prodan (LCIS-Grenoble INP)
Inverted pendulum system on a cart

Srinivasan et al. (2009)

The angular dynamics of the system is

\[ M(q)\ddot{q} + N(\dot{q}, q) = u \]

with \( q \in \mathbb{R} \), the angle between the vertical line and the pendulum. The nonlinear terms \( M(q) \) and \( N(q, \dot{q}) \) are given by:

\[ M(q) = \mu \cos q - \frac{mJ}{\mu \cos q}, \quad N(q, \dot{q}) = mg - \mu \dot{q}^2 \sin q, \]

with \( m = 0.3235, \mu = 1.3625 \times 10^{-3} \) and \( J = 1.5265 \times 10^{-4} \) the physical parameters of the system, \( g = 9.81 \) the gravity and \( u \in \mathbb{R} \) the force applied to the cart.

- Input constraint: \( |u| \leq u_{max} \) with \( u_{max} = 0.6 \).
- State constraints: \( |q| \leq 0.16, \ |\dot{q}| \leq 0.3 \).
- Equilibrium point: \( (x_e, u_e) = (0, 0) \).
- Initial state: \( x_0 = [0.15 \ 0]^T \).
Simulation scenarios and tuning parameters

- Scenario 1: stabilizing the inverted pendulum using CTC within the positive invariant set $\mathcal{X}^1_f$.
- Scenario 2: using the proposed NMPC design with the terminal region $\mathcal{X}^2_f$.
- Scenario 3: using the quasi-infinite horizon NMPC controller with the terminal region $\Omega_\alpha$.

**Table:** Design parameters for stabilizing the pendulum

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Common parameters</strong></td>
<td><strong>Sampling time $\delta$</strong></td>
<td>0.1 sec</td>
</tr>
<tr>
<td></td>
<td><strong>Weighting matrices $Q$, $R$</strong></td>
<td>$\text{diag}{5, 1}$, 1</td>
</tr>
<tr>
<td></td>
<td><strong>Jacobian matrices $xJ$, $vJ$</strong></td>
<td>$[3.1735 \ 0]$, $-0.0349$</td>
</tr>
<tr>
<td></td>
<td>$C$</td>
<td>3.1737</td>
</tr>
<tr>
<td></td>
<td>$M_\varepsilon$</td>
<td>1.1882</td>
</tr>
<tr>
<td></td>
<td>$\varepsilon = \varepsilon_{\text{max}}$</td>
<td>0.1793</td>
</tr>
<tr>
<td><strong>Scenario 1</strong></td>
<td>Control gain matrix $K_1$</td>
<td>$[-0.5 \ -0.55]$</td>
</tr>
<tr>
<td><strong>Scenario 2</strong></td>
<td>Prediction horizon $T_2$</td>
<td>0.4 sec</td>
</tr>
<tr>
<td></td>
<td>Control gain matrix $K_2$</td>
<td>$[-1 \ -0.6]$</td>
</tr>
<tr>
<td></td>
<td>$R^*$</td>
<td>$[21.52 \ 6.46; \ 6.46 \ 14.63]$</td>
</tr>
<tr>
<td></td>
<td>Terminal weighting matrix $P$</td>
<td>$[36.63 \ 13.26; \ 13.26 \ 35.13]$</td>
</tr>
<tr>
<td><strong>Scenario 3</strong></td>
<td>Prediction horizon $T_3$</td>
<td>0.6 sec</td>
</tr>
<tr>
<td></td>
<td>$K_q$, $\kappa$</td>
<td>$[7.0557 \ 1.2216]$, 3</td>
</tr>
<tr>
<td></td>
<td>Terminal weighting matrix $P_q$</td>
<td>$[29.90 \ 1.05; \ 1.05 \ 0.072]$</td>
</tr>
<tr>
<td></td>
<td>$\alpha$</td>
<td>0.003</td>
</tr>
</tbody>
</table>
Simulation results

Figure: Terminal regions $\mathcal{X}_{f_1}$, $\mathcal{X}_{f_2}$ and $\Omega_\alpha$ and state trajectories under different scenarios.

Figure: Convergence of states under different scenarios.

Figure: Values of the input $u$ under different scenarios.

Figure: Computing time for the two Scenarios 2 and 3.

Outline

1. On the use of a computed-torque control law in an NMPC scheme

2. A stabilizing NMPC design for thrust-propelled vehicles dynamics
   - Thrust-propelled translation dynamics
   - Feedback linearization law and input constraints satisfaction
   - Construction of the constraint admissible invariant set
   - NMPC design for thrust-propelled vehicles
   - Simulation results

3. Conclusions and future developments
Thrust-propelled translation dynamics

Thrust-propelled translation dynamics (Mellinger and Kumar (2011); Nguyen et al. (2017, 2019a)):

\[ \ddot{\xi} = \overrightarrow{g} + R \overrightarrow{T}, \]

with \( \xi \triangleq (x, y, z)^\top \) the position of the vehicle, \( \overrightarrow{g} \triangleq (0, 0, -g)^\top \) the gravity and \( \overrightarrow{T} \triangleq (0, 0, T)^\top \) the normalized thrust force. \( R \) is the rotation matrix of the roll-pitch yaw XYZ (\( \phi, \theta, \psi \)) rotating sequence.

Two Way Ranging radio message
Input message \( \{T_r, \phi_r, \theta_r, \psi_r\} \)
Feedback \( \{\xi, \eta\} \)

Experimental room equipped with the Loco Positioning system

Thrust-propelled translation dynamics

Thrust-propelled translation dynamics (Mellinger and Kumar (2011); Nguyen et al. (2017, 2019a)):

\[ \ddot{\xi} = \ddot{g} + R \dot{T}, \]

with \( \xi \triangleq (x, y, z)^\top \) the position of the vehicle, \( \ddot{g} \triangleq (0, 0, -g)^\top \) the gravity and \( \dot{T} \triangleq (0, 0, T)^\top \) the normalized thrust force. \( R \) is the rotation matrix of the roll-pitch yaw XYZ (\( \phi, \theta, \psi \)) rotating sequence.

State-space representation:

\[ \dot{x} = f(x, u) = Ax + h_\psi(u), \]

with \( A = \begin{bmatrix} 0_{3 \times 3} & I_3 \\ 0_{3 \times 3} & 0_{3 \times 3} \end{bmatrix} \), \( h_\psi(u) = \begin{bmatrix} 0_{3 \times 1} \\ T(\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi) \\ T(\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi) \\ -g + T \cos \phi \cos \theta \end{bmatrix}, \)

where \( x \triangleq (\xi^\top, \dot{\xi}^\top)^\top \) and \( u \triangleq (T, \phi, \theta)^\top \) (i.e., thrust, roll and pitch angles).

The yaw angle \( \psi \) is an assumed known constant variable affecting the system.

The equilibrium point is fixed at \( x_e = 0 \) and \( u_e = [g \ 0 \ 0]^\top \).

The input \( u \) is constrained as \( u \in \mathcal{U} = \{ u \in \mathbb{R}^3 \mid 0 \leq T \leq T_{\text{limit}}, |\phi| \leq \epsilon_c, |\theta| \leq \epsilon_c \} \),

with \( T_{\text{limit}} > g \) the thrust limit and \( \epsilon_c \in (0, \pi/2) \) the desired maximum value of the angles.
Feedback linearization law

Mellinger and Kumar (2011); Formentin and Lovera (2011); Nguyen et al. (2017)

Feedback linearization law \( u_b(u, \psi) : \mathbb{R}^3 \times \mathbb{R} \rightarrow \mathbb{R}^3 \) with \( u_b \triangleq [T_b \ \phi_b \ \theta_b]^\top \) and the virtual input vector \( u \triangleq [u_x \ u_y \ u_z]^\top \):

\[
T_b = \sqrt{u_x^2 + u_y^2 + (u_z + g)^2},
\]

\[
\phi_b = \arcsin \left( \frac{u_x \sin \psi - u_y \cos \psi}{\sqrt{u_x^2 + u_y^2 + (u_z + g)^2}} \right),
\]

\[
\theta_b = \arctan \left( \frac{u_x \cos \psi + u_y \sin \psi}{u_z + g} \right).
\]

If \( u_b \in \mathcal{U} \), \( u_b \) linearizes the nonlinear system \( \dot{x} = Ax + h_\psi(u) \) into the linear stabilizable system:

\[
\dot{x} = A_x + h_\psi(u_b(u, \psi)) \iff \dot{x} = Ax + Bu \quad \text{(i.e.,)} \quad \begin{cases}
\dot{x} = v_x, & \dot{v}_x = u_x, \\
\dot{y} = v_y, & \dot{v}_y = u_y, \\
\dot{z} = v_z, & \dot{v}_z = u_z.
\end{cases}
\]

with \( B = [0_{3 \times 3} \ 1_3]^\top \).
Input constraint satisfaction with feedback linearization control

Input constraints

\[ u \in U = \{ u \in \mathbb{R}^3 | 0 \leq T \leq T_{\text{limit}}, |\phi| \leq \epsilon_c, |\theta| \leq \epsilon_c \}, \ T_{\text{limit}} < g, \ \epsilon_c \in (0, \pi/2) \].

Lemma 4

*Nguyen et al. (2018).* By choosing three positive constants \( U_x, U_y \) and \( U_z \) such that:

\[ U_z < g, \quad U_x^2 + U_y^2 \leq (-U_z + g)^2 \tan^2 \epsilon_c, \quad \sqrt{U_x^2 + U_y^2 + (U_z + g)^2} \leq T_{\text{limit}}, \]

we have that, if \(|u_x| \leq U_x, |u_y| \leq U_y \) and \(|u_z| \leq U_z\), then \( u_b(u, \psi) \in U, \ \forall \psi \in [-\pi, \pi] \).

Only bounds on the virtual inputs are required to ensure the input constraints satisfaction, \( u_b(u, \psi) \in U \).

---

Constraint admissible invariant set

Proposition 5

For any positive definite matrix $M \in \mathbb{R}^{6 \times 6}$, we obtain the positive definite matrix $P \in \mathbb{R}^{6 \times 6}$ as the unique solution of the Lyapunov equation:

$$A_{cl}^\top P + PA_{cl} = -M,$$

with $A_{cl} = A + BK$. Let us define

$$\delta = \lambda_{\min}(P)r^2, \quad \text{with} \quad r^2 = \min_{q \in \{x, y, z\}} \left\{ \frac{U_q^2}{K_{1q}^2 + K_{2q}^2} \right\}.$$

with $U_x, U_y, U_z$ the virtual input limit. Then, considering the set $\mathcal{X}_f$ defined as follows:

$$\mathcal{X}_f = \{ \mathbf{x} \in \mathbb{R}^6 \mid \|\mathbf{x}\|_P^2 \leq \delta \},$$

we have that, under the feedback linearization controller $u_b(K\mathbf{x}, \psi)$, $\mathcal{X}_f$ is an input constraint admissible invariant set.

Sketch of the proof:

- $A_{cl}$ is a stable matrix which ensures the existence of the positive definite matrix $P$ as the unique solution of the Lyapunov equation for any positive definite matrix $M$.
- $\|\mathbf{x}\|_P^2 \leq \delta \Rightarrow \|\mathbf{x}\|^2 \leq r^2 \Rightarrow |u_q| \leq U_q, \forall q \in \{x, y, z\} \Rightarrow u_b(K\mathbf{x}, \psi) \in \mathcal{U}$ by Lemma 4.
- $\frac{d}{dt} \|\mathbf{x}\|_P^2 = \mathbf{x}^\top (A_{cl}^\top P + PA_{cl})\mathbf{x} = -\mathbf{x}^\top M\mathbf{x} < 0 \Rightarrow$ positive invariant.
Summary of the NMPC design for thrust-propelled vehicle

Design procedure of the NMPC controller for stabilizing the thrust-propelled system:

1) Choose the positive definite matrices $Q \in \mathbb{R}^{6 \times 6}$ and $R \in \mathbb{R}^{3 \times 3}$ (semi-positive) to formulate the stage cost.

2) Estimate the prediction horizon $T_p$ based on the computational constraint of the platform (e.g., the processing speed requirement).

3) Choose the saturation limits $U_x, U_y$ and $U_z$ satisfying the required conditions for ensuring input constraints satisfaction.

4) Choose the control gains $K_{1q}, K_{2q}$ with $q \in \{x, y, z\}$ which stabilize the resulted linear system.

5) Find the matrix $Q^*$, then, define the matrix $M \geq Q^*$ and solve the Lyapunov equation for $P$.

6) Find $\delta$ to obtain the terminal region $\mathcal{X}_f = \{x \in \mathbb{R}^6 \mid \|x\|_P^2 \leq \delta\}$. 
Simulation scenarios and tuning parameters

Crazyflie 2.0 nano-quadcopter: thrust limit $T_{\text{limit}} = 2g$ and maximum angle values $\epsilon_c = 10^\circ$.

Fixing the NMPC sampling time $\delta = 0.1$ seconds, two scenarios are considered as follows:

**Scenario 1**: Stabilizing the thrust-propelled translation dynamics with $\psi = 0$ using the proposed NMPC controller.

<table>
<thead>
<tr>
<th>Table: Parameters of the proposed NMPC controller.</th>
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</thead>
<tbody>
<tr>
<td>$Q, R$</td>
</tr>
<tr>
<td>$U_x, U_y, U_z$</td>
</tr>
<tr>
<td>$K_{1q} = K_{2q}$, $q \in {x, y, z}$</td>
</tr>
<tr>
<td>$M$ ($M \preceq Q^*$)</td>
</tr>
<tr>
<td>$P$</td>
</tr>
<tr>
<td>$\delta$</td>
</tr>
<tr>
<td>$T_p$</td>
</tr>
</tbody>
</table>
Simulation scenarios and tuning parameters

Crazyflie 2.0 nano-quadcopter: thrust limit \( T_{\text{limit}} = 2g \) and maximum angle values \( \epsilon_c = 10^\circ \).

Fixing the NMPC sampling time \( \delta = 0.1 \) seconds, two scenarios are considered as follows:

**Scenario 1:** Stabilizing the thrust-propelled translation dynamics with \( \psi = 0 \) using the proposed NMPC controller.

**Scenario 2:** Stabilizing thrust-propelled translation dynamics with \( \psi = 0 \) using the quasi-infinite horizon NMPC controller (Chen and Allgöwer (1998)) with the ellipsoidal terminal region \( \Omega_\alpha = \{ \mathbf{x} \in \mathbb{R}^6 | \mathbf{x}^\top P_{qM} \mathbf{x} \leq \alpha \} \).

---

**Table:** Parameters of the quasi-infinite horizon NMPC controller.

<table>
<thead>
<tr>
<th></th>
<th>Values</th>
</tr>
</thead>
</table>
| \( K_{qM} \) | \[
\begin{bmatrix}
0 & 0 & -1 & 0 & 0 & -1 \\
0 & 1/g & 0 & 0 & 1/g & 0 \\
-1/g & 0 & 0 & -1/g & 0 & 0 \\
\end{bmatrix}
\]                                      |
| \( \kappa \) | 0.4                                        |
| \( P_{qM} \) | \[
\begin{bmatrix}
\text{diag}\{69.8, 69.8, 72.8\} & \text{diag}\{32.9, 32.9, 33.6\} \\
\text{diag}\{32.9, 32.9, 33.6\} & \text{diag}\{63.2, 63.2, 66.8\} \\
\end{bmatrix}
\]                                      |
| \( \alpha \) | 0.17                                       |
| \( T_p \)   | 1.5 seconds                                |
Simulation results

Figure: Terminal regions $\mathcal{X}_f$ and $\Omega_\alpha$ and state trajectories under different scenarios.

Figure: Values of the input $u$ under different scenarios.

Figure: Convergence of states under different scenarios.

Figure: Computing time under different scenarios.


Outline

1. On the use of a computed-torque control law in an NMPC scheme
2. A stabilizing NMPC design for thrust-propelled vehicles dynamics
3. Conclusions and future developments
Conclusions and future developments

Conclusions:

- Terminal region design in an NMPC (Nonlinear Model Predictive Control) scheme via CTC (Computed-Torque Control) and feedback linearization.
- Analyze the use of a CTC (Computed-Torque Control) law in an NMPC (Nonlinear Model Predictive Control) scheme to stabilize a particular type of systems (Nguyen et al. (2019b)).
- Analyze to the use of a standard feedback linearization controller in an NMPC (Nonlinear Model Predictive Control) scheme to stabilize the thrust-propelled vehicles (Nguyen et al. (2019a)).
- Provide simulations and comparisons with quasi-infinite horizon NMPC.

Future developments:

- Robustness under model mismatches and bounded disturbances.
- Polyhedral terminal regions.
References


John Craig. *Introduction to robotics*. 2018.


References II


