Flatness-based hierarchical control of a meshed DC microgrid

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Meshed DC microgrid

Meshed topology provides reliability:

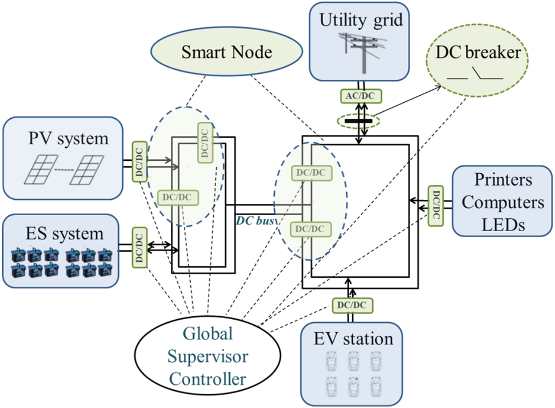
- Power can be provided various sources.
- Power flows through multiple paths among the nodes.

Isolating fault lines ensures:

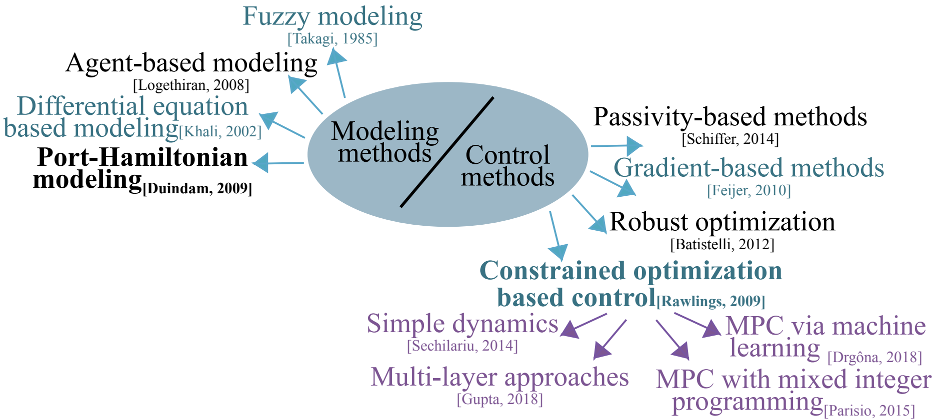
- Continuity and reliability of the power supply.
- Efficiency of the power transmission.

Characteristics

- Strongly nonlinear systems
- Distributed in space
- Multiple timescales
- Hard constraints
- Variable profiles and costs



Modeling and control methods



Hierarchical control via differential flatness and MPC

Flat systems and their trajectories

Consider the continuous nonlinear system

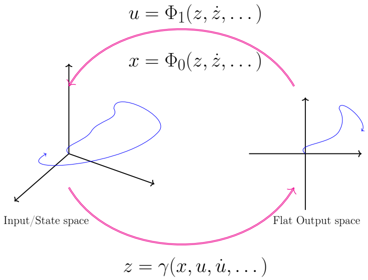
$$\dot{x}(t) = f(x(t), u(t)),$$

it is called differentially flat if there exist $z(t)$ s.t. the states and inputs can be algebraically expressed in terms of $z(t)$ and a finite number of its derivatives:

$$x(t) = \Phi_0(z(t), \dot{z}(t), \cdots, z^{(q)}(t)),$$
$$u(t) = \Phi_1(z(t), \dot{z}(t), \cdots, z^{(q+1)}(t)),$$

where

$$z(t) = \gamma(x(t), u(t), \dot{u}(t), \cdots, u^{(q)}(t))$$



- For any linear and nonlinear flat system, the number of flat outputs equals the number of inputs Lévine (2009), Fliess et al. (1995)
- For linear systems, the flat differentiability is implied by the controllability property Sira-Ramírez and Agrawal (2004)

B-spline curve generation

Considering a collection of *control points*

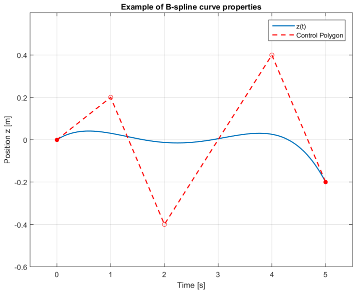
$$\mathbb{P} = \{p_0, p_1, \ldots, p_n\},$$

we rewrite the knot-vector as

$$\mathbb{T} = (\underbrace{\tau_0, \tau_1, \ldots, \tau_{d-1}}_{d \text{ equal knots}}, \underbrace{\tau_d, \tau_{d+1}, \ldots \tau_{n-1}, \tau_n}_{n-d+1 \text{ internal knots}}, \underbrace{\tau_{n+1}, \tau_{n+d}}_{d \text{ equal knots}})$$

and define a *B-spline curve* as a linear combination of the control points and the B-spline basis functions

$$z(t) = \sum_{i=0}^{n} B_{i,d}(t) p_i = \mathbf{P}\mathbf{B}_d(t)$$



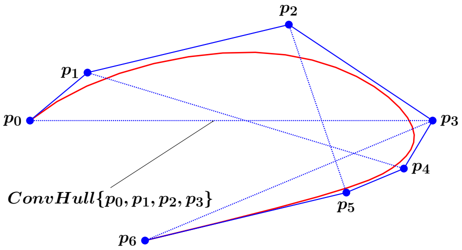
B-spline curve properties

P1) Smoothness and differentiability: $z(t)$ is C^∞ in any $t \notin \mathbb{T}$ and C^{d-1} in any $t \in \mathbb{T}$;

P2) End point interpolation: the control points with a knot of multiplicity $d-1$ coincides with the B-spline curve, thus making it a *clamped B-spline curve*;

P3) Convex hull property: at a time instant $\tau_i < t < \tau_{i+1}$, $z(t)$ depends on the B-splines $B_{i-d+1,d}(t), \ldots, B_{i,d}(t)$; hence, $z(t)$ lies in the convex hull generated by points $p_{i-d+1}, \ldots, p_i$;

P4) Local support and local modification property: the 'r' order derivatives of B-spline basis functions can be expressed as linear combinations of B-splines of the same order (due to relations $\mathbf{B}_d^{(r)}(t) = M_r \mathbf{B}_{d-r}(t)$ and $\mathbf{B}_{d-r}(t) = L_r \mathbf{B}_d(t)$ with matrices M_r, L_r are of appropriate dimensions and content).



Constrained parametrization – I

Let us consider a collection of $N+1$ way-points and time stamps associated to them Prodan (2012):

$$\mathbb{W} = \{w_k\} \text{ and } \mathbb{T}_{\mathbb{W}} = \{t_k\},$$

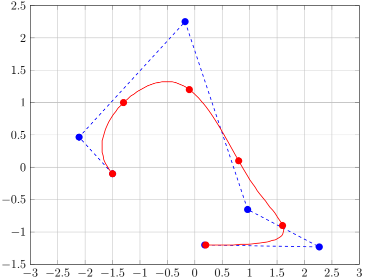
for any $k = 0, \ldots, N$.

The goal is to construct a flat trajectory which passes through each way-point w_k at the time instant t_k, i.e., find a flat output $z(t)$such that

$$x(t_k) = \Theta(z(t_k), \ldots z^{(r)}(t_k)) = w_k,\ \forall k = 0 \ldots N.$$

$$\tilde{\Theta}(\mathbf{B}_d(t_k), \mathbf{P}) = w_k,\ \forall k = 0 \ldots N,$$

where $\tilde{\Theta}(\mathbf{B}_d(t), \mathbf{P}) = \Theta(\mathbf{P}\mathbf{B}_d(t), \ldots, \mathbf{P}M_r L_r \mathbf{B}_d(t))$ is constructed along property (P5).



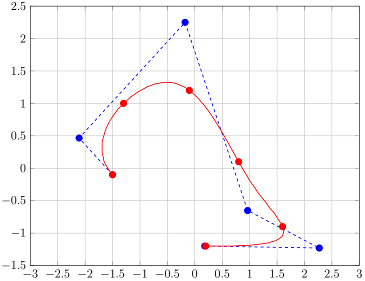
Constrained parametrization – II

Solve an optimization problem De Doná et al. (2009), Suryawan (2012), Stoican et al. (2016):

$$\mathbf{P} = \arg\min_{\mathbf{P}} \int_{t_0}^{t_N} ||\tilde{\Xi}(\mathbf{B}_d(t), \mathbf{P})||_Q \, dt$$
$$\text{s.t. } \tilde{\Theta}(\mathbf{B}_d(t_k), \mathbf{P}) = w_k, \ \forall k = 0 \dots N$$

with Q a positive symmetric matrix.

- The cost $\tilde{\Xi}(\mathbf{B}_d(t), \mathbf{P}) = \Xi(\tilde{\Theta}(\mathbf{B}_d(t), \mathbf{P}), \tilde{\Phi}(\mathbf{B}_d(t), \mathbf{P}))$ can impose any penalization we deem to be necessary (length of the trajectory, input variation, input magnitude, etc).
- In general, such a problem is nonlinear (due to mappings $\tilde{\Theta}(\cdot)$ and $\tilde{\Phi}(\cdot)$) and hence difficult to solve.



Outline

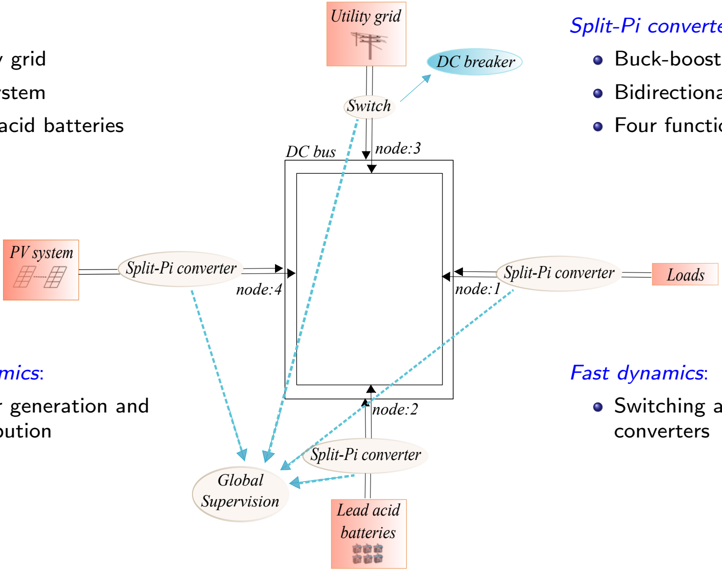
1. Meshed DC microgrid architecture
2. Modeling methodology
3. Hierarchical optimization-based control
4. Comparisons
5. Undergoing work

Outline

Meshed DC microgrid architecture

DERs:

- Utility grid
- PV system
- Lead acid batteries



Split-Pi converters:

- Buck-boost converter
- Bidirectional
- Four functions

Slow dynamics:

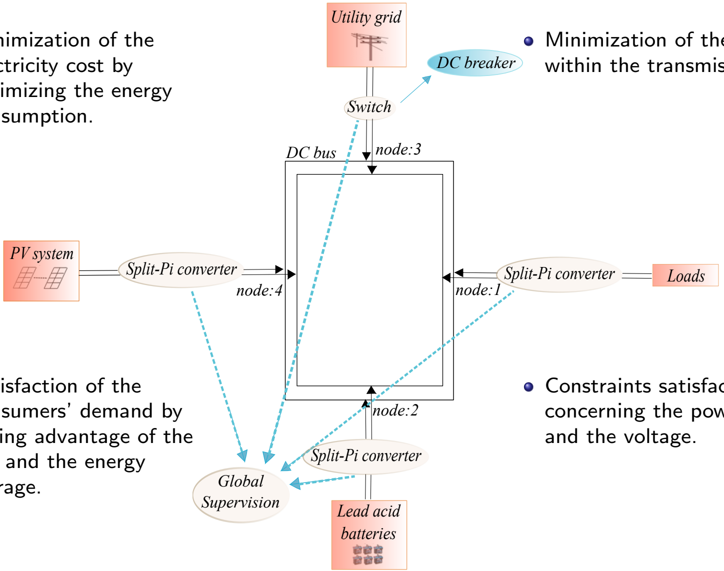
- Power generation and distribution

Fast dynamics:

- Switching activity of the converters

Objectives

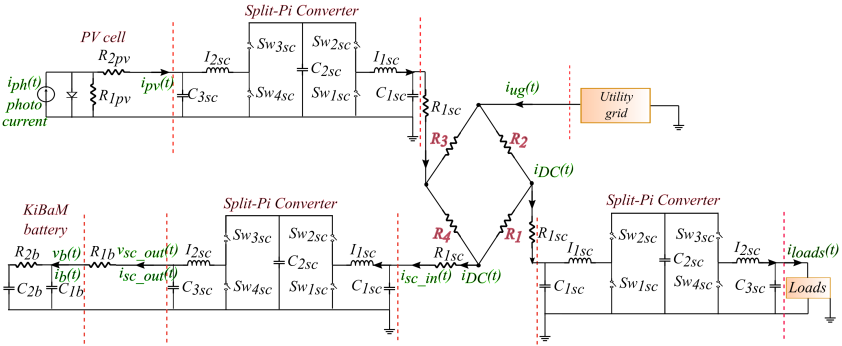
- Minimization of the electricity cost by optimizing the energy consumption.
- Minimization of the power losses within the transmission lines.



- Satisfaction of the consumers' demand by taking advantage of the PV and the energy storage.
- Constraints satisfaction concerning the power, the current and the voltage.

RLC circuit network

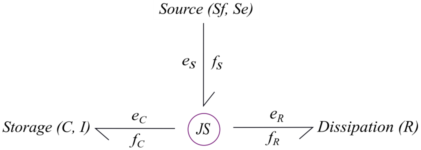
The meshed DC microgrid system will be globally represented as an RLC electrical circuit for proceeding to the modeling part.



Outline

DC microgrid modeling

Bond Graph representation Schiffer et al. (2016):



Port-Hamiltonian formulation van der Schaft et al. (2014):

$$\dot{x}(t) = [J(t) - R]Qx(t) - Gu(t),$$
$$y(t) = G^T Qx(t) + Du(t),$$

with $x \in \mathbb{R}^n$ the state vector, $u \in \mathbb{R}^m$ the input vector, $y \in \mathbb{R}^p$ the output vector, $J(t) \in \mathbb{R}^{n\times n}$ a skew-symmetric matrix, $R \in \mathbb{R}^{n\times n}$ contains the dissipating elements, $G \in \mathbb{R}^{n\times m}$ the input matrix, $Q \in \mathbb{R}^{n\times n}$ contains the circuit parameters and $D \in \mathbb{R}^{m\times m}$ depends on the relation among the port variables.

Total energy of the system (Hamiltonian): $H(x(t)) = \frac{1}{2}x(t)^T Qx(t)$.

Advantages of the PH dynamical formulation:

The power exchange, the dissipation and the energy storage are given explicitly.

DC microgrid modeling

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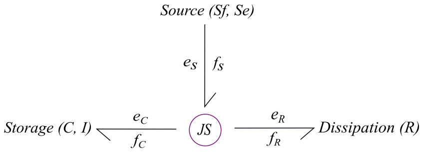

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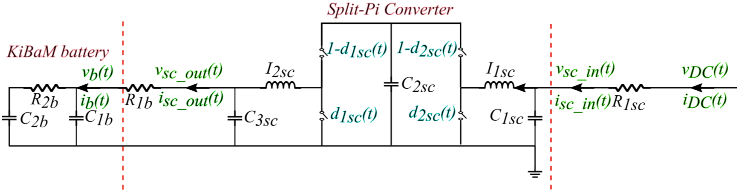
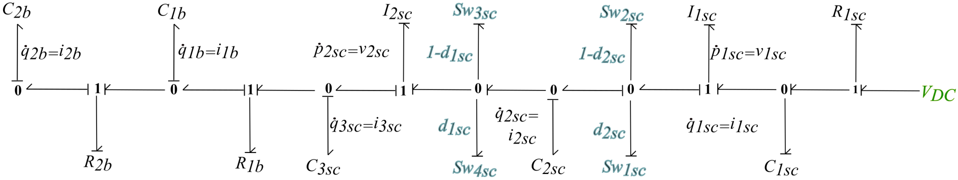
Total energy of the system (Hamiltonian): $H(x(t)) = \frac{1}{2}x(t)^T Qx(t)$.

Example for the RLC circuit:

- the total energy is: $H(x(t)) = \frac{1}{2}\frac{p(t)^2}{I} + \frac{1}{2}\frac{q(t)^2}{C}$;
- the state vector is: $x(t) = [p(t)\ \ q(t)]^\top$ (magnetic flux of the inductors and charge of the capacitors);
- the current-voltage relations are: $p(t) = I \cdot i_I(t)$, $q(t) = C \cdot v_C(t)$.

Detailed presentation of the Energy Storage Unit





Port-Hamiltonian model of the Split-Pi converter and the battery

Consider the dynamical nonlinear model:

$$\begin{cases} \dot{x}_{es}(t) = [J_{es}(t) - R_{es}]Q_{es}x_{es}(t) + \begin{bmatrix} 0 & 0 & \frac{-1}{R_{1sc}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}^T u_{es}(t) \\ i_{DC}(t) = \begin{bmatrix} 0 & 0 & \frac{-1}{R_{1sc}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix} Q_{es}x_{es}(t) + \begin{bmatrix} \frac{1}{R_{1sc}} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} -v_{DC}(t) \\ -i_{R_{1b}} \end{bmatrix} \end{cases}$$

with $J_{es}(t) = \begin{bmatrix} 0 & 0 & 1 & -(1-d_{1sc}(t)) & 0 & 0 & 0 \\ 0 & 0 & 0 & (1-d_{2sc}(t)) & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1-d_{1sc}(t) & -(1-d_{2sc}(t)) & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$,

$R_{es} = diag(diag(0,\ 0,\ \frac{1}{R_{1sc}},\ 0,\ 0),\ \begin{bmatrix} \frac{1}{R_{1b}} + \frac{1}{R_{2b}} & -\frac{1}{R_{2b}} \\ -\frac{1}{R_{2b}} & \frac{1}{R_{2b}} \end{bmatrix}) \in \mathbb{R}^{7\times 7}$,

the state vector $x_{es}(t) = \begin{bmatrix} p_{1sc} & p_{2sc} & q_{1sc} & q_{2sc} & q_{3sc} & q_{1b} & q_{2b} \end{bmatrix}^T \in \mathbb{R}^7$,

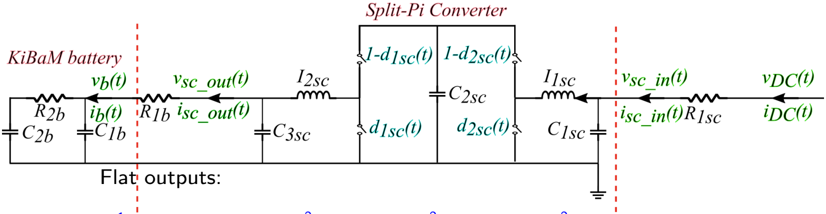
the input vector $u_{es}(t) = \begin{bmatrix} -v_{DC} & -i_{R_{1b}} \end{bmatrix}^T \in \mathbb{R}^2$

the output vector $y_{es} = \begin{bmatrix} i_{DC} & v_{R_{1b}} \end{bmatrix}^T \in \mathbb{R}^2$,

the circuit parameters' matrix $Q_{es} = diag(\frac{1}{l_{1sc}}, \frac{1}{l_{1sc}}, \frac{1}{C_{1sc}}, \frac{1}{C_{2sc}}, \frac{1}{C_{3sc}}, \frac{1}{C_{1b}}, \frac{1}{C_{2b}}) \in \mathbb{R}^{7\times 7}$

and the duty cycles: $d_{1sc}(t), d_{2sc}(t)$.

Flat outputs of the Energy Storage Unit



Flat outputs:

$$\begin{cases} z_1(t) &= \frac{1}{l_{1sc}} \frac{p_{1sc}(t)^2}{2} + \frac{1}{l_{2sc}} \frac{p_{2sc}(t)^2}{2} + \frac{1}{C_{2sc}} \frac{q_{2sc}(t)^2}{2} \\ z_2(t) &= q_{3sc}(t) + q_{1b}(t) \\ z_3(t) &= q_{2b}(t) \\ z_4(t) &= q_{2sc}(t) \end{cases}$$

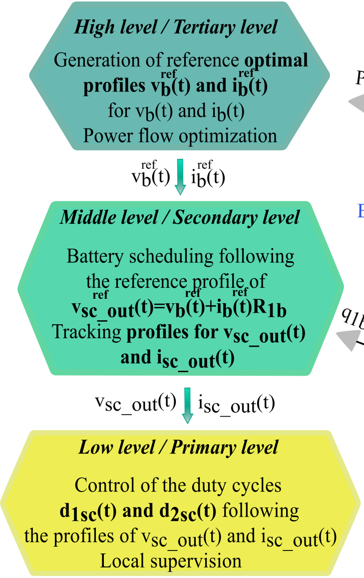
Zafeiratou, I., D. Nguyen, I. Prodan, L. Lefevre et L. Pietrac, Flatness-based hierarchical control of a meshed dc microgrid, in Proceedings of the 6th IFAC Conference on Nonlinear Model Predictive Control (NMPC18), Madison, Wisconsin, USA, 19-21 August 2018, pp. 3340, 19-21 August 2018.

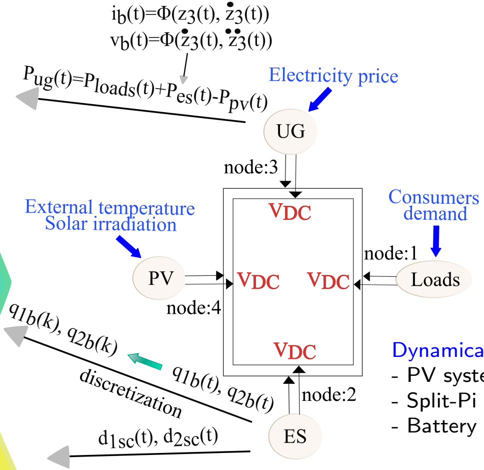
Zafeiratou, I., I. Prodan, L. Lefvre et L. Pitrac, Dynamical modelling of a DC microgrid using a port-hamiltonian formalism, IFAC-PapersOnLine, Proceedings of the 9th Vienna International Conference on Mathematical Modelling, t. 51, no 2, pp. 469474, 2018. 2018

Prodan, I. et E. Zio, A model predictive control for reliable microgrid energy management, International Journal of Electrical Power and Energy Systems, t. 61, no 1, pp. 399409, 2014. 2014, ISSN : 0142-0615.

Outline

Hierarchical control



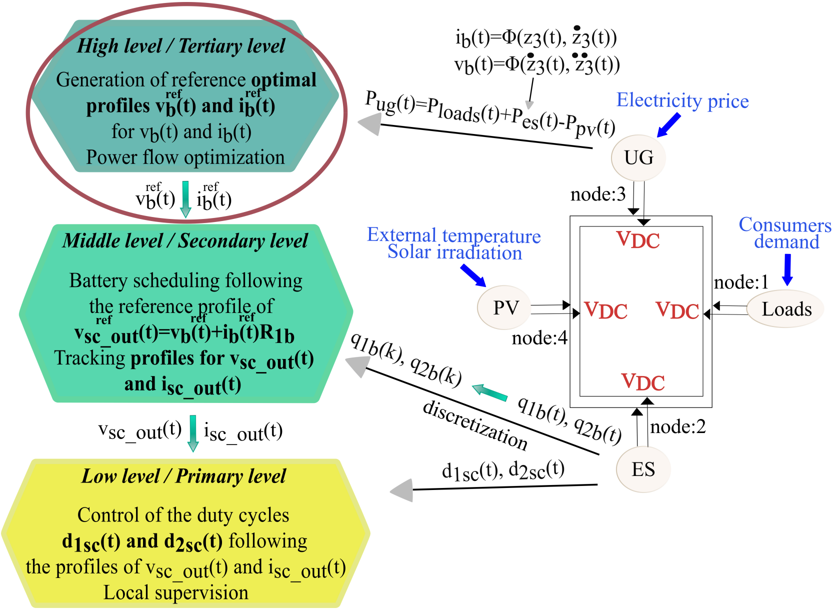


Dynamical models:
- PV system
- Split-Pi converter
- Battery

Reference profiles:
- External temperature
- Solar irradiation
- Load demand
- Electricity price

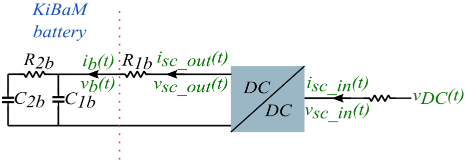
Control variables:
- Duty cycles of the Split-Pi converters
- Voltages at the nodes (approximately 400 V)

Hierarchical control



High level control

Goal: Generate optimal profiles for the battery charges and discharges while minimizing the electricity purchase from the utility grid.



Solution: Use the flat representation of the Split-Pi/Battery system and generate optimal reference profiles for the battery current, i_b, and voltage, v_b, while ensuring continuous-time constraints validation through B-splines parametrization.

Flat outputs of the system Split-Pi/Battery:

$$\begin{cases} z_1(t) &= \frac{1}{l_{1sc}}\frac{p_{1sc}(t)^2}{2} + \frac{1}{l_{2sc}}\frac{p_{2sc}(t)^2}{2} + \frac{1}{C_{2sc}}\frac{q_{2sc}(t)^2}{2}, \\ z_2(t) &= q_{3sc}(t) + q_{1b}(t), \\ z_3(t) &= q_{2b}(t), \\ z_4(t) &= q_{2sc}(t). \end{cases}$$

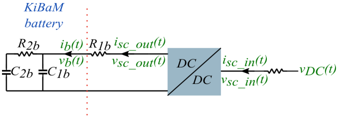
$$\begin{cases} i_b(t) &= C_{1b}R_{2b}\ddot{z}_3(t) + (\frac{C_{1b}}{C_{2b}} + 1)\dot{z}_3(t), \\ v_b(t) &= R_{2b}\dot{z}_3(t) + \frac{1}{C_{2b}}z_3(t). \end{cases}$$

High level control

Obtain optimal profiles i_b, v_b by solving the continuous-time optimization problem:

$$\min_{i_b(t), v_b(t)} \int_{t_0}^{t_f} e(t)(\underbrace{P_{es}(t)}_{i_b(t)v_b(t)} + P_{loads}(t) - P_{pv}(t))dt,$$

$$\text{subject to} \begin{cases} \text{power balancing equation:} & P_{es}(t) + P_{loads}(t) - P_{pv}(t) = P_{ug}, \\ \text{battery voltage constraint:} & v_b^{min,h} \le v_b(t) \le v_b^{max,h}, \\ \text{battery current constraint:} & i_b^{min,h} \le i_b(t) \le i_b^{max,h}, \\ \text{charge of the capacitor limits:} & q_{2b}^{min,h} \le q_{2b}(t) \le q_{2b}^{max,h}, \\ \text{external grid power limits:} & P_{ug}^{min,h} \le P_{ug}(t) \le P_{ug}^{max,h}, \\ \text{battery dynamical model:} & \end{cases}$$



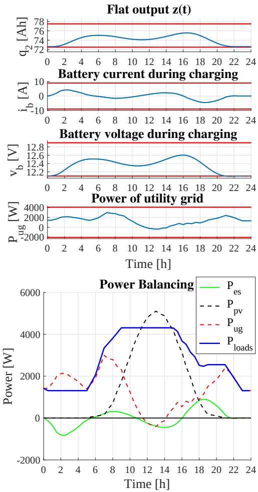
Main idea:

- rewrite the problem in terms of the flat output $z(t) = [z_1(t)\ z_2(t)\ z_3(t)\ z_4(t)]^T$;
- the flat output $z(t)$ is projected over N B-splines of order d: $z(t) = \sum_{i=1}^{N} p_i \cdot b_{i,d}(t) = \mathcal{P}\mathcal{B}_d(t)$;
- ultimately, the problem is rewritten in terms of the control points.

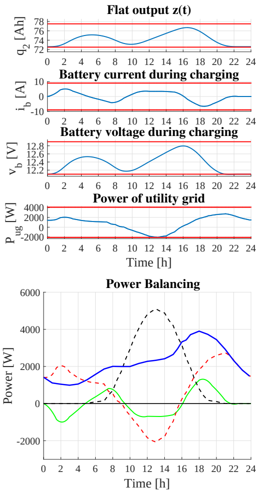
High level - simulation results

- use a set of DS-100 PV modules by collecting real external temperature and irradiation data of a whole day in June;
- consider a collection of AGM 12-165 model of lead acid batteries;
- consider a variable electricity price, commercial and domestic consumers' demand profiles.

Commercial load profile



Domestic load profile



High level - simulation results

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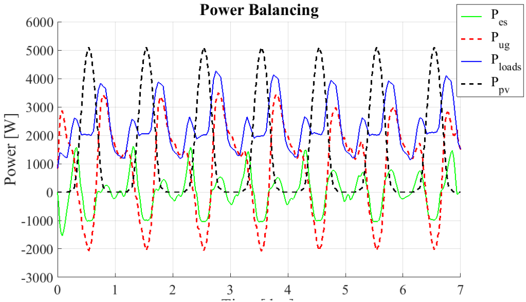
Table: Simulation results for different no. of control points N

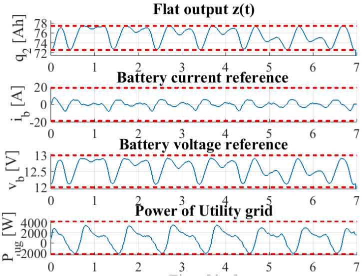
Load profile	N	Electricity cost [euros]	Computation time [s]	ES discharges
	18	4.090	146	2
	27	4.029	257	3
Commercial	36	3.614	514	7
	45	3.447	854	9
	54	3.226	1322	11
	18	2.534	667	2
	27	2.577	779	2
Domestic	36	2.265	1013	7
	45	2.074	1230	8
	54	1.869	1230	9

The cost without using the battery is equal to 4.173 and 2.644 euros respectively.

High level - simulation results

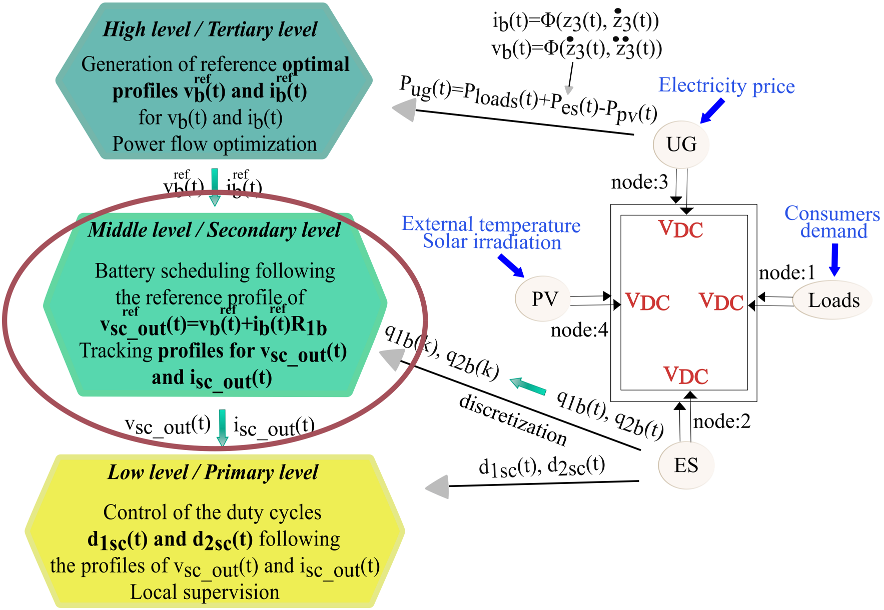
Power balancing and reference trajectories profiles generation for one week (domestic load profile):





Load profile	d	N	Electricity cost [euros]	Computation time [h]	ES discharges
Domestic	4	199	19.018	3	16

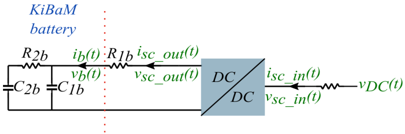
Hierarchical control



Middle level control

Goal: Solve a tracking reference problem according to the reference trajectories obtained in the high level for the voltage and the current of the battery.

Solution: Consider an MPC based controller for tracking the Split-Pi converter output voltage $v_{sc\_out}^{ref}$.



$$\min_{\tilde{u}(k)} \sum_{i=k}^{k+N_p-1} (\tilde{y}(i) - \tilde{y}^{ref}(i))^\top Q_{\tilde{y}}(\tilde{y}(i) - \tilde{y}^{ref}(i)) + (\tilde{u}(i) - \tilde{u}^{ref}(i))^\top R_{\tilde{u}} \left(\tilde{u}(i) - \tilde{u}^{ref}(i)\right)$$

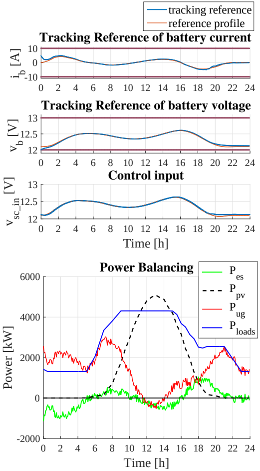
$$\text{subject to} \begin{cases} \text{battery voltage constraint:} & v_b^{min,m} \leq v_b(k) \leq v_b^{max,m}, \\ \text{battery current constraint:} & i_b^{min,m} \leq i_b(k) \leq i_b^{max,m}, \\ \text{charge of the capacitor limits:} & q_{2b}^{min,m} \leq q_{2b}(k) \leq q_{2b}^{max,m}, \\ \text{external grid power limits:} & P_{ug}^{min,m} \leq P_{ug}(k) \leq P_{ug}^{max,m}, \\ \text{battery dynamical model:} & \begin{cases} \tilde{x}(k+1) & = A\tilde{x}(k) + B\tilde{u}(k), \\ \tilde{y}(k) & = C\tilde{x}(k) + D\tilde{u}(k), \end{cases} \end{cases}$$

with $\tilde{x}(k) = \begin{bmatrix} q_{1b}(k) & q_{2b}(k) \end{bmatrix}^\top$, $\tilde{u}(k) = v_{sc\_in}(k)$, $\tilde{y}(k) = \begin{bmatrix} i_b(k) & v_b(k) \end{bmatrix}^\top$, $i_b(k) = i_{sc}(k)$ and $v_b(k) = \frac{q_{1b}(k)}{C_{1b}}$.

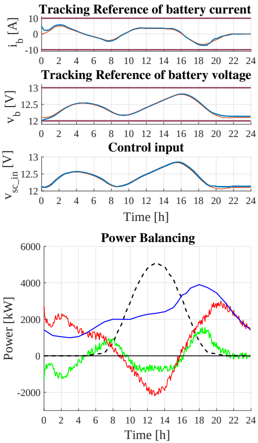
Middle level - simulation results

Simulations under perturbation of 5% of the difference between the minimum and the maximum value of $v_{sc\_out}^{ref}$.
Parameters: $N_p = 10$, $T_s = 300$ [s], $Q_y = diag(1, 1)$, $R_u = 800$.

Commercial load profile



Domestic load profile



Middle level - Tracking error dynamics

Discrete-time model of the system in R^2:

$$\tilde{x}_w(k+1) = A\tilde{x}_w(k) + B\tilde{u}_w(k) + \tilde{w}(k),$$
$$w \in W \subset \mathbb{R}^2$$

Nominal model of the system:

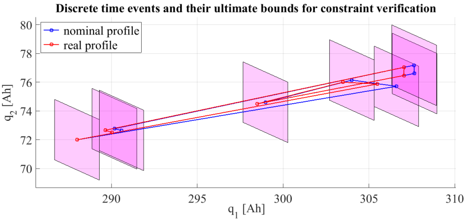
$$\tilde{x}(k+1) = A\tilde{x}(k) + B\tilde{u}(k)$$

Construct a RPI set $\mathcal{S}$ for the dynamics:

$\tilde{z}(k+1) = (A+BK)\tilde{z}(k) + \tilde{w}(k)$,

with $\tilde{z}(k) = \tilde{x}_w(k) - \tilde{x}(k) \in \mathcal{S}$

and $\tilde{u}_w(k) = \tilde{u}(k) + K\tilde{z}(k)$ a stabilizing controller



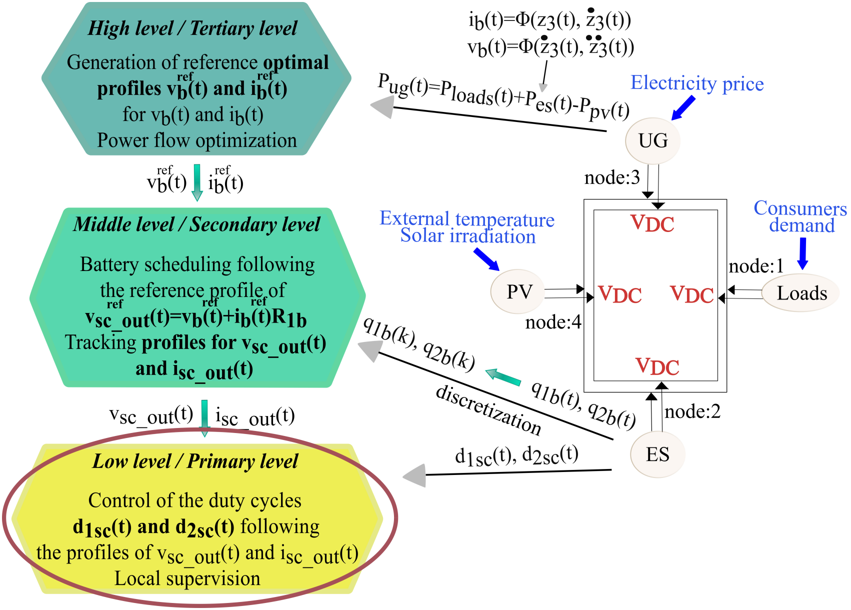
In our case:

$$\mathcal{S} \triangleq \left\{ \begin{bmatrix} -622.7\ [Ah] \\ -202.6\ [Ah] \end{bmatrix} \leqslant \tilde{z}(k) \leqslant \begin{bmatrix} 622.7\ [Ah] \\ 202.6\ [Ah] \end{bmatrix} \right\}$$

with $K = \begin{bmatrix} -0.688 \cdot 10^{-4} & -0.210 \cdot 10^{-4} \end{bmatrix}$

Tighter constraints are considered at the high level which account for the error dynamics at the middle level.

Hierarchical control



Low level - explicit solution for the duty cycles

For the Split-PI/Battery system we have:

$$\frac{v_{sc\_out}(t)}{v_{sc\_in}(t)} = 1 - d_{2sc}(t)$$
$$v_{sc\_in}(t) = v_{DC}(t) - i_{DC}(t)R_{1sc}$$
$$v_{sc\_out}(t) = v_b(t) + i_b(t)R_{1b}$$

From the power conservation equation in the RLC circuit:

$$i_{DC}(t)v_{DC}(t) = i_b(t)v_{sc\_out}(t)$$

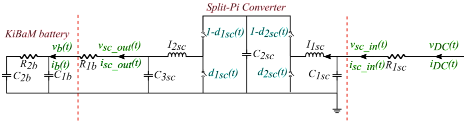
We conclude in the following relation:

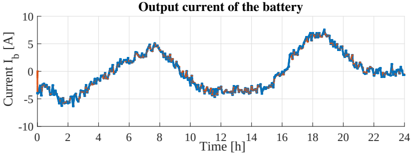
$$d_{2sc}(t) = 1 - \frac{v_{DC}(t) - c}{2(v_{sc\_out}(t) - v_b(t))}$$

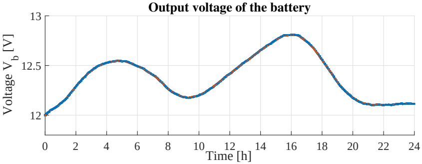
$$c = \sqrt{v_{DC}^2(t) - 4(v_{sc\_out}(t) - v_b(t))(v_{sc\_out})}$$

The i_b and v_b are the references we need to track.

[UK Patent, 2005] Power converter and method for power conversion. GB2376357 B. Date of publication: 04.05.2005





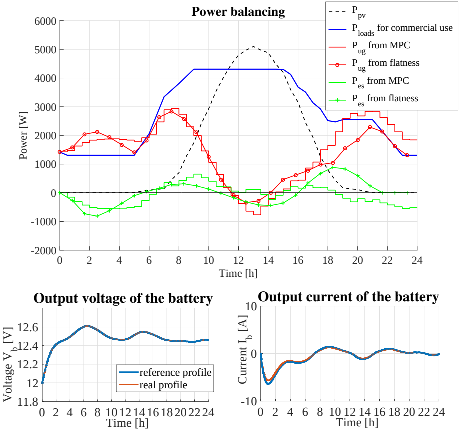


Outline

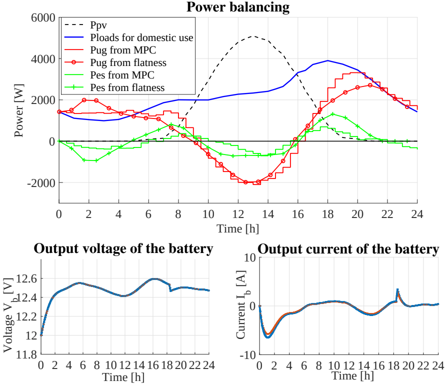
Comparison optimal profiles generation with MPC

Simulations parameters: $N_p = 24$ h, $T_s = 1800$ s.

Commercial load



Domestic load



Comparison optimal profiles generation with MPC

Simulation results for different N_p and T_s for optimal profile generation with MPC

Load	Prediction horizon N_p [h]	Sampling time T_s [s]	Electricity cost [euros]	Calculation time [s]
Commercial	48	1800	4.416	2853
	24	1800	4.657	1384
	24	1200	4.491	1670
	24	600	4.319	1862
	10	1800	4.322	1475
	10	1200	4.281	2075
Domestic	24	1800	2.912	1868
	24	1200	2.815	2062
	10	1800	2.894	1533
	10	1200	2.889	1491
	10	600	2.774	1530

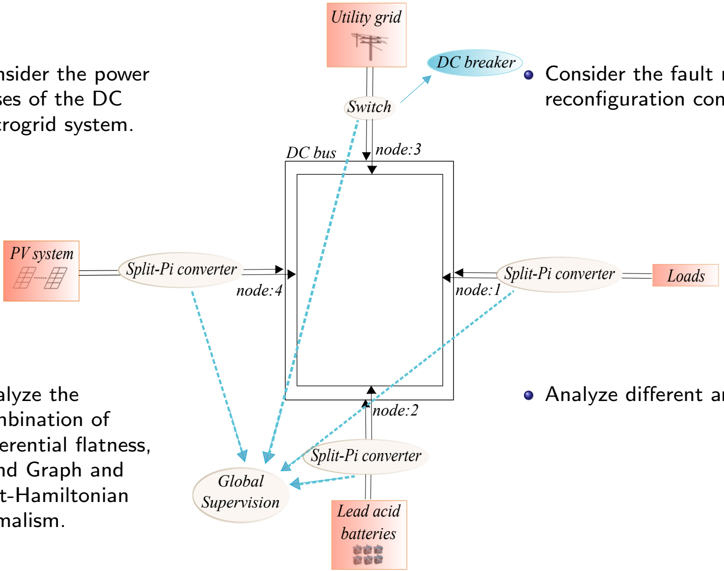
Advantages on using differential flatness for optimal profiles generation:

- no discretization - profile generation in continuous time (results not affected by discretization approximations or under-sampling)
- profile generation in its entirety for the full simulation horizon (not piece by piece as in a discrete optimization problem with MPC)
- total economic cost and required computational resources are lower than with MPC

Outline

Undergoing work

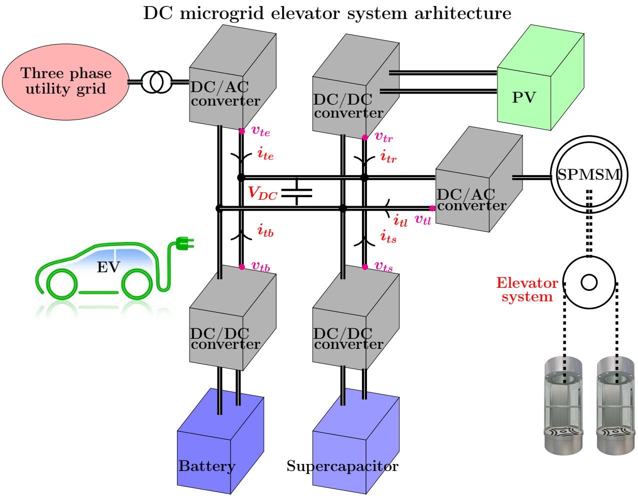
- Consider the power losses of the DC microgrid system.
- Consider the fault mitigation and reconfiguration control problem.



- Analyze the combination of differential flatness, Bond Graph and port-Hamiltonian formalism.
- Analyze different architectures.

Undergoing work

Analyze different architectures



[3] Pham, T., I. Prodan, D. Genon-Catalot et L. Lefevre: *Dissipated energy minimization for an electro-mechanical elevator of a dc microgrid*, in Proceedings of the 25th IEEE European Control Conference (ECC18), Limassol, Cyprus, 3-6 July 2017, pp. 17-24.

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Florin Stoican, Vlad-Mihai Ivanusca, Ionela Prodan, and Dan Popescu. Obstacle avoidance via b-spline parameterizations of flat trajectories. In *Proceedings of the 24th Mediterranean Conference on Control and Automation*, pages 1002–1007, Athens, Greece, 2016.

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Port-Hamiltonian representation of the KiBaM battery

The storage system is represented by a two tank Kinetic Battery Model (KiBaM).

$$\begin{cases} \dot{x}_b(t) = [J_b(t) - R_b] Q_b x_b(t) + \begin{bmatrix} -\frac{1}{R_{1b}} \\ 0 \end{bmatrix} (-v_{sc\_out}(t)) \\ i_{sc\_out}(t) = \begin{bmatrix} -\frac{1}{R_{1b}} & 0 \end{bmatrix} Q_b x_b(t) + \begin{bmatrix} -\frac{1}{R_{1b}} \end{bmatrix} (-v_{sc\_out}(t)) \end{cases}$$

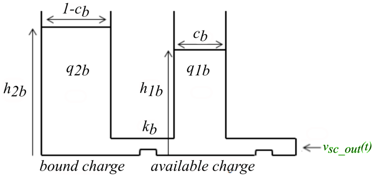
with $J_b(t) = 0,\ R_b = \begin{bmatrix} \frac{1}{R_{1b}} + \frac{1}{R_{2b}} & -\frac{1}{R_{2b}} \\ -\frac{1}{R_{2b}} & \frac{1}{R_{2b}} \end{bmatrix} \in R^{2\times 2}$,

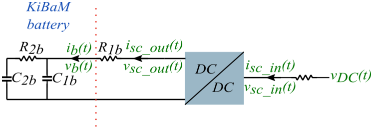
the state vector $x_b(t) = \begin{bmatrix} q_{1b} & q_{2b} \end{bmatrix}^T \in R^{2\times 1}$,

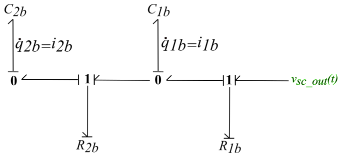
the input vector $u_b(t) = \begin{bmatrix} -v_{sc\_out} \end{bmatrix}^T \in R$,

the output vector $y_b(t) = \begin{bmatrix} i_{sc\_out} \end{bmatrix}^T \in R$,

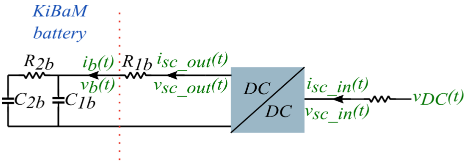
the circuit parameters matrix $Q_b = diag(\frac{1}{C_{1b}}, \frac{1}{C_{2b}}) \in R^{2\times 2}$







Flat representation of the KiBaM



The Battery model:

- linear system
- controllable and flat
- one input and one flat output

Battery state-space representation:

$$\dot{q}_{1b}(t) = -\left(\frac{1}{R_{1b}} + \frac{1}{R_{2b}}\right)\frac{q_{1b}(t)}{C_{1b}} + \frac{1}{R_{2b}}\frac{q_{2b}(t)}{C_{2b}} + \frac{1}{R_{1b}}v_{sc\_out}(t)$$

$$\dot{q}_{2b}(t) = \frac{1}{R_{2b}}\frac{q_{1b}(t)}{C_{1b}} - \frac{1}{R_{2b}}\frac{q_{2b}(t)}{C_{2b}}$$

Flat output of the Battery:

$$z(t) = q_{2b}(t)$$

Battery flat representation:

$$q_{1b}(t) = R_{2b}C_{1b}\dot{z}(t) + \frac{C_{1b}}{C_{2b}}z(t)$$

$$q_{2b}(t) = z(t)$$

$$v_{sc\_out}(t) = \Big(R_{2b}C_{1b}\ddot{z}(t) + \frac{C_{1b}}{C_{2b}}\dot{z}(t) + \Big(\frac{1}{R_{1b}} + \frac{1}{R_{2b}}\Big)R_{2b}\dot{z}(t) + \frac{1}{C_{2b}}z(t) - \frac{1}{R_{2b}C_{2b}}\Big)R_{1b}$$

Taking into account the relations within the RLC circuit components:

$$i_b(t) = i_{R2b}(t) + i_{1b}(t) = i_{2b}(t) + \dot{q}_{1b} = \dot{q}_{1b}(t) + \dot{q}_{2b}(t)$$

$$v_b(t) = v_{1b}(t) = \frac{q_{1b}(t)}{C_{1b}}$$

Flat representation of the battery current and voltage:

$$\begin{cases} i_b(t) = C_{1b}R_{2b}\ddot{z}(t) + \left(\frac{C_{1b}}{C_{2b}} + 1\right)\dot{z}(t) \\ v_b(t) = R_{2b}\dot{z}(t) + \frac{1}{C_{2b}}z(t). \end{cases}$$

B-spline parametrization

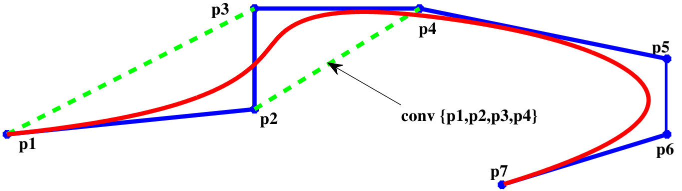
[Boor, 1978]

Flat output parametrization:

$$\mathbf{z}(t) = \sum_{i=0}^{N-1} \mathbf{p}_i \lambda_{i,d}(t) = \mathbf{P}\Lambda_d(t).$$

B-splines:

$$\lambda_{i,1}(t) = \begin{cases} 1, & \tau_i \leq t < \tau_{i+1}, \\ 0, & \text{otherwise}, \end{cases}$$
$$\lambda_{i,d}(t) = \alpha_1(t)\lambda_{i,d-1}(t) + \alpha_2(t)\lambda_{i+1,d-1}(t).$$



$\mathbf{z}(t) \in \mathbb{R}^2$

$\mathbf{z}(t)$	: time-dependent vector
$\mathbf{p}_i$	: control point
d	: order of B-splines
$\lambda_{i,d}(t)$	: i^{th} B-spline of order d
N	: number of B-splines
τ_i	: i^{th} knot

Advantages:

- local convexity property (the spline curve lies in a union of convex regions),
- d-degree smoothness and easy computations of the derivatives.

High level control problem written in function of B-splines curves and control points

We describe the B-splines as it follows:

$$\mathcal{B}_d^{(r)}(t) = M_{d,d-r}\mathcal{B}_{d-r}(t) = M_{d,d-r}S_{k,d-r,d}\mathcal{B}_d(t), \quad \forall t \in [\tau_k, \tau_{k+1}). \tag{1}$$

We obtain the following for the battery's output current and voltage:

$$\begin{aligned} v_b(t) &= \sum_{i=1}^{N}\left[\frac{1}{C_{2b}}p_i + R_{2b}\left(\mathcal{P}M_{d,d-1}S_{k,d-1,d}\right)_i\right]\mathcal{B}_{i,d}(t), \ \ \forall t \in [\tau_k, \tau_{k+1}) \\ i_b(t) &= \sum_{i=1}^{N}\left[\left(1 + \frac{C_{1b}}{C_{2b}}\right)\left(\mathcal{P}M_{d,d-1}S_{k,d-1,d}\right)_i + C_{1b}R_{2b}\cdot\left(\mathcal{P}M_{d,d-2}S_{k,d-2,d}\right)_i\right]\mathcal{B}_{i,d}(t). \end{aligned} \tag{2}$$