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Predictive Control of Fast Unstable and Nonminimum-phase Systems

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Outline

- Predictive Control of Fast Unstable Systems
- Cascade Scheme with Feedback Linearization
- Stability Analysis based on Singular Perturbation
- Example - Inverted Pendulum
- Conclusions

Nonlinear Control

- Use of Lyapunov functions: (Khalil 96 …) ➞ Difficult to find
- Back-stepping: (Kokotovic 95…) ➞ Triangular systems
- Predictive control: (Clarke 87, Valluri 97, Mayne 98…)

  Unstable fast dynamics ⇨ Needs fast optimization
- Control by geometric approach: (Isidori 89, Chen 96 …)

  NMP ⇨ Internal dynamics are unstable

Control of fast unstable nonlinear NMP systems?

Predictive Control of Fast Unstable Systems

- Unstable dynamics
  - Necessity to take control action within the interval specified by Nyquist criterion
- Fast dynamics
  - Optimization cannot be completed within that time interval

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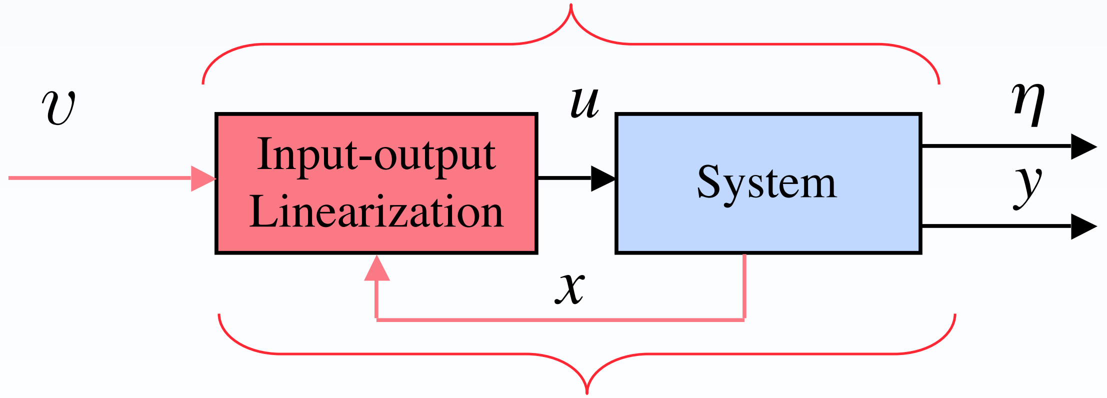
Standard predictive control cannot be applied

Input-output Feedback Linearization

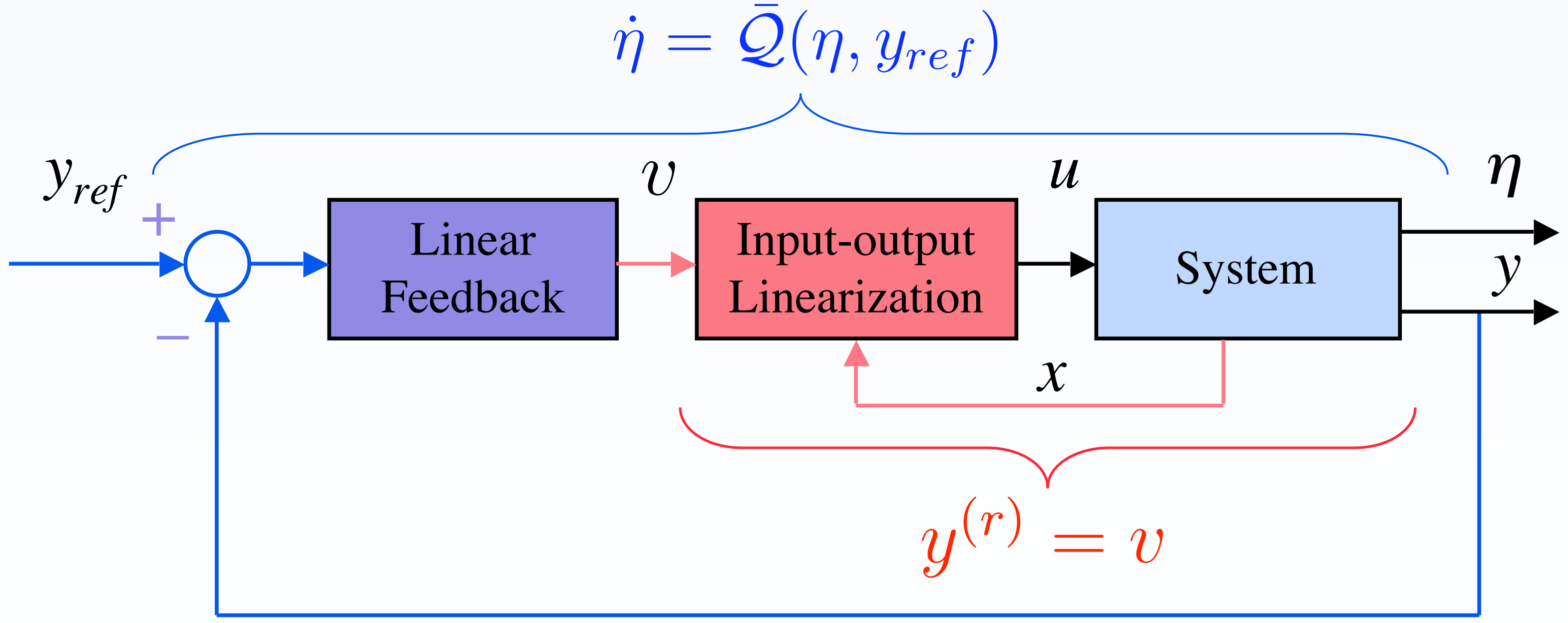
$$\dot{\eta} = \mathcal{Q}(\eta, y, \dot{y}, \cdots, y^{(r-1)}, v)$$



$$y^{(r)} = v$$

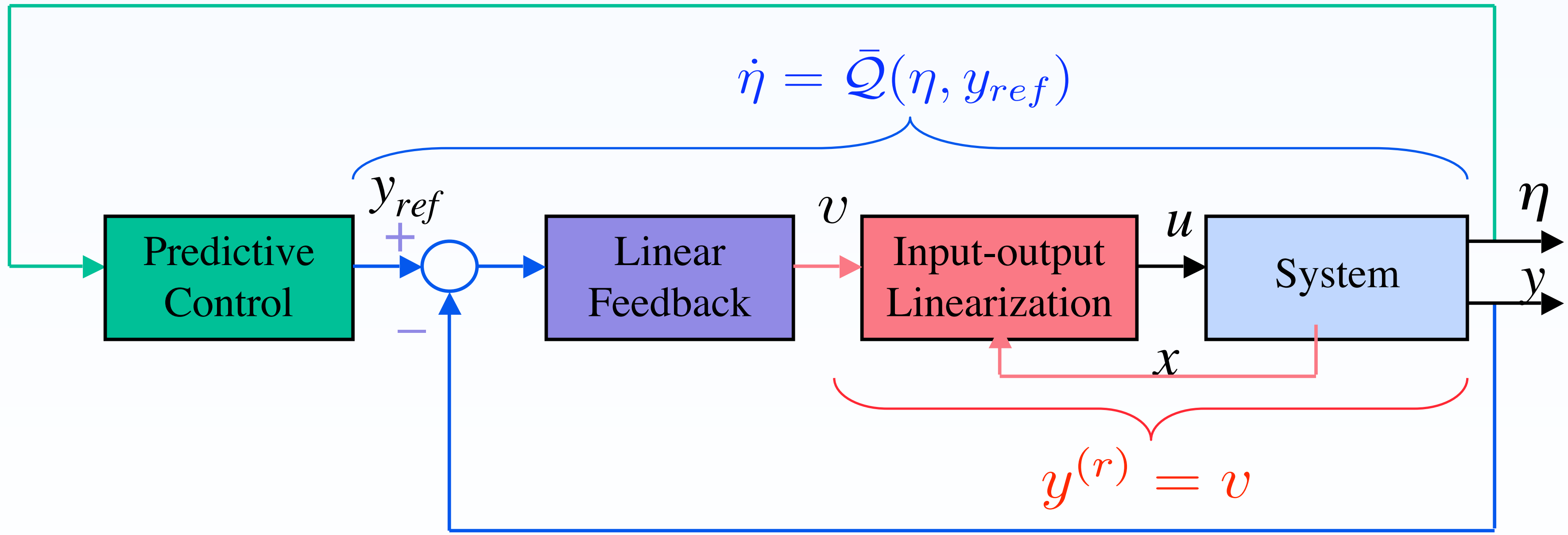
- Linear input-output behaviour (dynamics of y)
- Internal dynamics (η) unstable for nonminimum-phase systems

Linear Feedback



- Linear dynamics are arbitrarily fast:
- $y = y_{ref}$
- Internal dynamics depend only on y_{ref}

Cascade Control Scheme



- Predictive control of the unstable internal dynamics
  - Stabilization of η via adjustment of y_{ref}
  - Two-time-scale system (under the assumption of slow internal dynamics)

Stability Analysis

- Exponential stability of the linear I-O part
  - $[K_1,\ldots, K_r]$ is Hurwitz and ε the tuning parameter for linear controller

$$v = \frac{K_1}{\epsilon^r}(y_{ref} - y) - \sum_{i=1}^{r-1} \frac{K_{i+1}}{\epsilon^{(r-i)}} y^{(i)}$$

- Exponential stability of the predictive controller
  - Terminal cost should be a local Lyapunov function

$$J = \frac{1}{2}\eta(t+T)^T P\,\eta(t+T) + \frac{1}{2}\int_t^{t+T} \eta(\tau)^T Q\,\eta(\tau) + R\,y_{ref}^2(\tau)\ \ d\tau$$

$$\eta^T P \bar{Q}(\eta, y_{ref}^{\S}) + \eta^T Q \eta + R\ {y_{ref}^{\S}}^2 \leq 0$$

Stability Results

- Linear controller exponentially stabilizes linear I-O part
- Predictive controller exponentially stabilizes internal dynamics

⬇

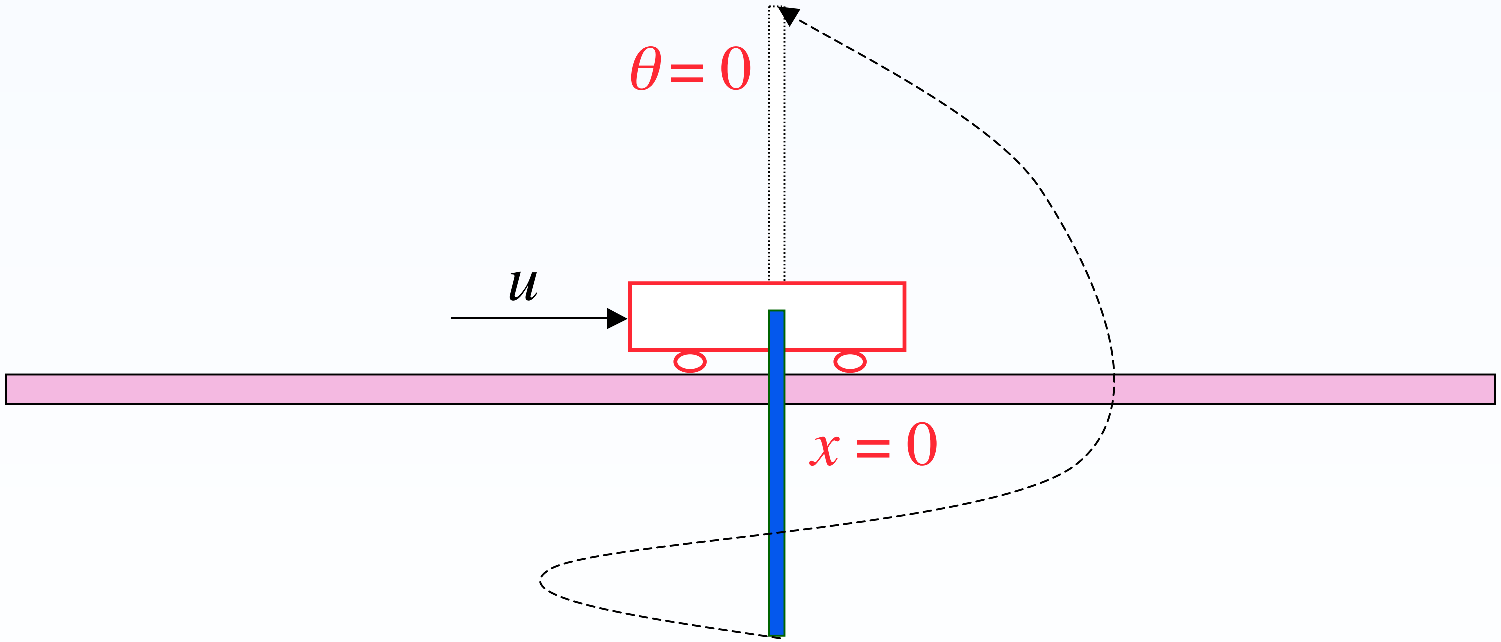
The cascade system is exponentially stable
for a high enough gain for the linear part

Stability results are derived as in singularly-perturbed systems

Features of the Cascade Scheme

- Two-time-scale separation is artificially created
  - The Higher the gain (linear part), the better the time scale separation
- Stabilizability of the internal dynamics is assumed
  - Not always true, if time-scale separation against the physics
- Internal dynamics is slower than the I-O dynamics
  - Predictive control can be done at a lower frequency

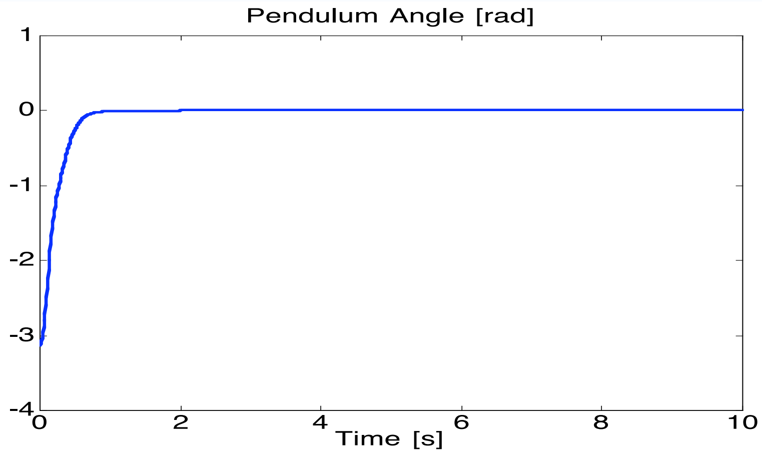
Inverted Pendulum on a Cart

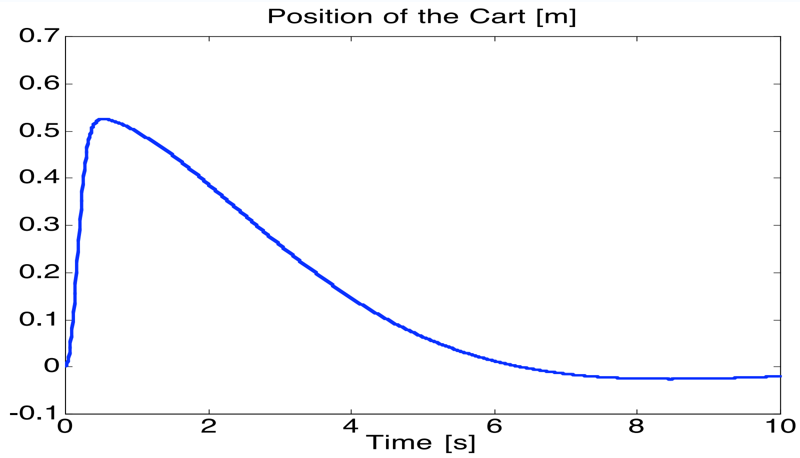


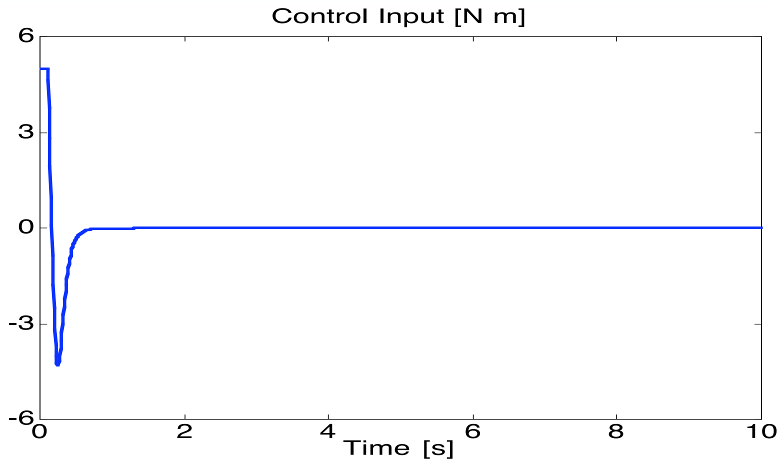
- Objective: Swing up and regulate at $\theta = 0, x = 0$
- For the cascade scheme
  - Output: Pendulum angle, Internal dynamics: Cart position

Standard Predictive Control

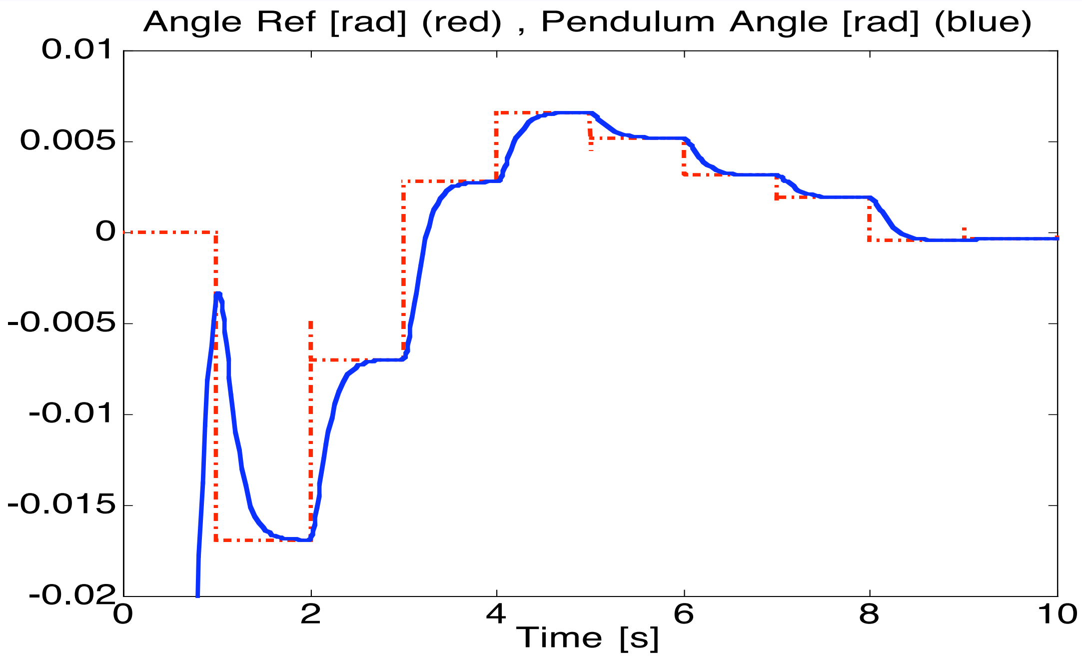
Optimization interval = 0.01 s - Prediction horizon = 0.05 s







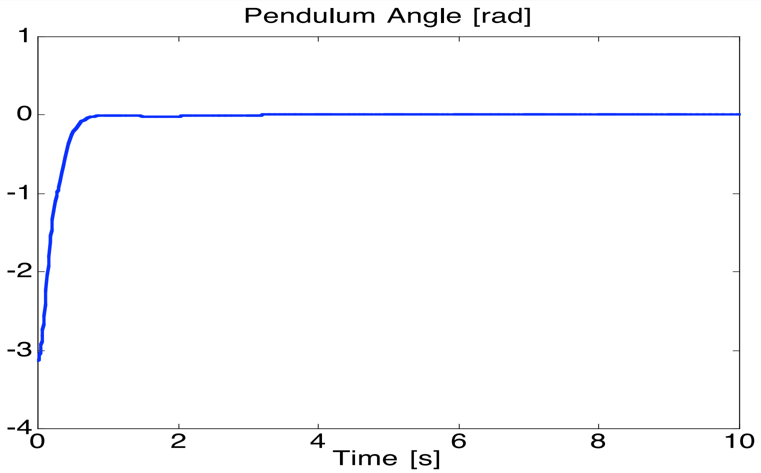
Cascade Control Scheme

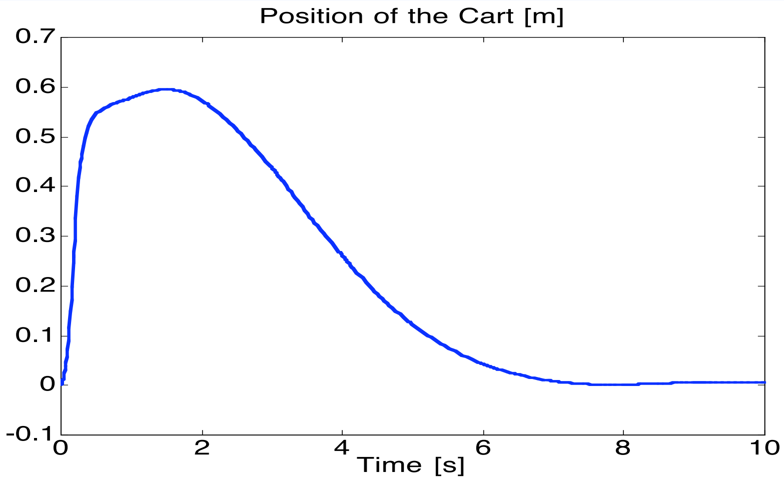


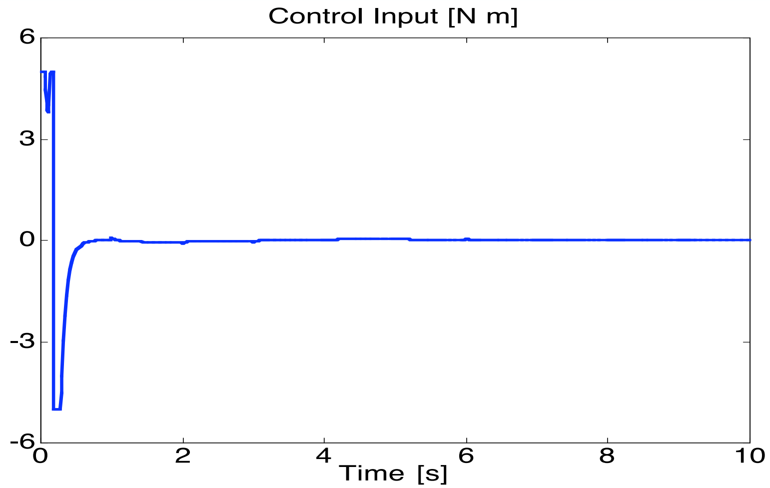
- Predictive control gives the reference for pendulum angle
  - In order to push the cart position ad velocity to zero
- Linearization + Linear feedback tracks the reference

Cascade Control Scheme

Optimization interval = 1 s - Prediction horizon = 3 s







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Conclusions

- Cascade control scheme that combines :
  - I-O linearization + Linear feedback: pre-stabilizer
  - Predictive control: handles unstable internal dynamics
- Stability analysis
  - Based on the singular perturbation theory
- Implementation aspects
  - Easy to tune
  - Re-optimization frequency can be reduced