Unconstrained NCGPC with a guaranteed closed-loop stability: case of nonlinear SISO systems with the relative degree greater than four

Marcelin Dabo, Nicolas Langlois, Houcine Chafouk

IRSEEM, Technopôle du Madrillet,

76801 Saint Etienne du Rouvray, France

{dabo,langlois,chafouk}@esigelec.fr

Outline

I- Introduction and objectives

II- Unconstrained NCGPC

III- Unconstrained corrected NCGPC

IV- Application of the control law and the simulation results

V- Conclusion

I - Introduction and objectives

$$\dot{x}(t) = f(x(t)) + g(x(t))u(t)$$
$$y(t) = h(x(t))$$

where $x \in X \subset \mathrm{R}^n$, $y \in Y \subset \mathrm{R}$ and $u \in U \subset \mathrm{R}$.

Goal: Output tracking

Nonlinear Continuous-time Generalized Predictive Control (NCGPC)

II- Unconstrained NCGPC

II- 1 Presentation

where $T \in \mathbf{R}_*^+$

$$J = \frac{1}{2}\int_0^T [\hat{e}(t+\tau)]^2 d\tau \qquad [t, t+T]$$

$$e(t) = y(t) - \omega(t) \quad \Rightarrow \quad \hat{e}(t+\tau) = \hat{y}(t+\tau) - \hat{\omega}(t+\tau)$$

$$y(t) \quad \Rightarrow \quad \hat{y}(t+\tau) \simeq \begin{bmatrix} 1 & \tau & \frac{\tau^2}{2!} & \cdots & \frac{\tau^\rho}{\rho!} \end{bmatrix} \begin{bmatrix} y(t) \\ \dot{y}(t) \\ \vdots \\ y^{(\rho-1)}(t) \\ y^{(\rho)}(t) \end{bmatrix}$$

$$\omega(t) \quad \Rightarrow \quad \hat{\omega}(t+\tau) \simeq \begin{bmatrix} 1 & \tau & \frac{\tau^2}{2!} & \cdots & \frac{\tau^\rho}{\rho!} \end{bmatrix} \begin{bmatrix} \omega(t) \\ \dot{\omega}(t) \\ \vdots \\ \omega^{(\rho-1)}(t) \\ \omega^{(\rho)}(t) \end{bmatrix}$$

II- Unconstrained NCGPC

II- 1 Presentation

$$\begin{cases} y(t) = h(x(t)) \\ \dot{y}(t) = L_f h(x(t)) \\ \vdots \\ y^{(\rho-1)}(t) = L_f^{\rho-1} h(x(t)) \\ y^{(\rho)}(t) = L_f^{\rho} h(x(t)) + L_g L_f^{\rho-1} h(x(t)) u(x(t)) \end{cases}$$

$$\Lambda(\tau) = \left[\begin{array}{ccccc} 1 & \tau & \frac{\tau^2}{2!} & \cdots & \frac{\tau^\rho}{\rho!} \end{array} \right]$$

$$Y(t) = \left[\begin{array}{c} y(t) \\ \dot{y}(t) \\ \vdots \\ y^{(\rho-1)}(t) \\ y^{(\rho)}(t) \end{array} \right] \quad \Omega(t) = \left[\begin{array}{c} \omega(t) \\ \dot{\omega}(t) \\ \vdots \\ \omega^{(\rho-1)}(t) \\ \omega^{(\rho)}(t) \end{array} \right]$$

$$E(t) = Y(t) - \Omega(t)$$

$$\hat{e}(t+\tau) = \Lambda(\tau) E(t)$$

II- Unconstrained NCGPC

II- 1 Presentation

$$J = \frac{1}{2}\int_0^T E^t(t)\Lambda^t(\tau)\Lambda(\tau)E(t)d\tau$$

$$J = \frac{1}{2}E^t(t)\left[\int_0^T \Lambda^t(\tau)\Lambda(\tau)d\tau\right]E(t)$$

$$\frac{\partial J}{\partial u} = 0. \Rightarrow u(x(t)) = \frac{-\sum_{l=0}^{\rho} K_{\rho l}(T,\rho)\left[L_f^l h(x(t)) - \omega^{(l)}(t)\right]}{L_g L_f^{\rho-1} h(x(t))}$$

$$K_{\rho l} = \frac{\rho!}{l!}\frac{(2\rho+1)}{(\rho+l+1)T^{\rho-l}} \qquad 0 \le l \le \rho.$$

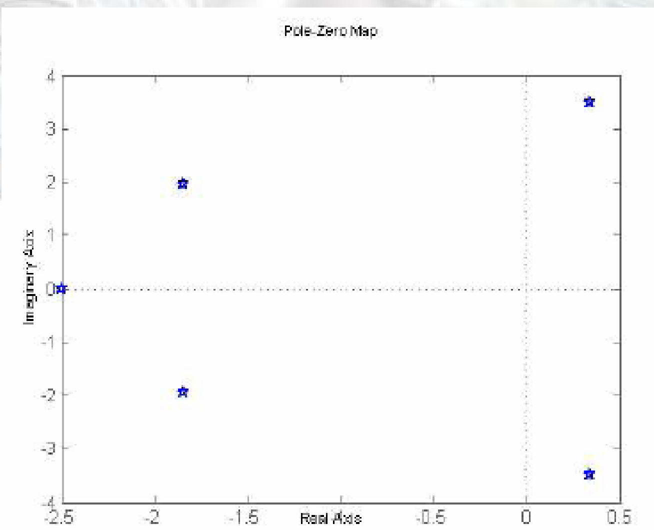
II- Unconstrained NCGPC

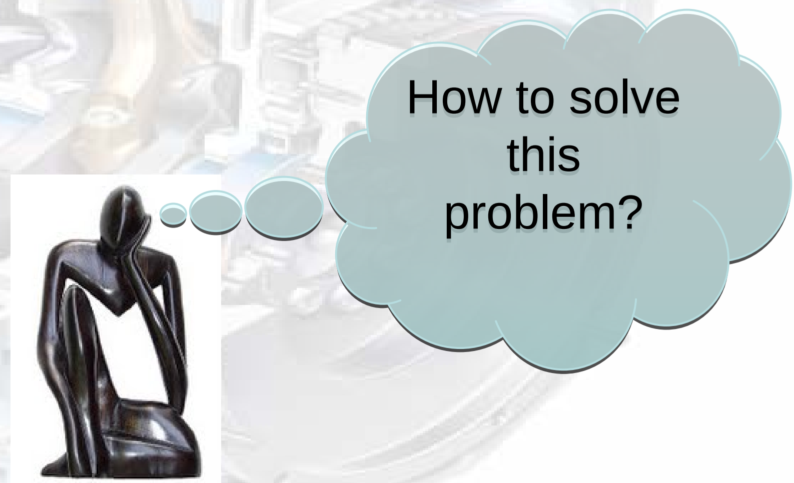
II- 2 Properties

$$u(x(t)) = \frac{-\sum_{l=0}^{\rho} K_{\rho l}(T,\rho)\left[L_f^l h(x(t)) - \omega^{(l)}(t)\right]}{L_g L_f^{\rho-1} h(x(t))}$$

Guaranteed stability for systems with a relative degree less than four.

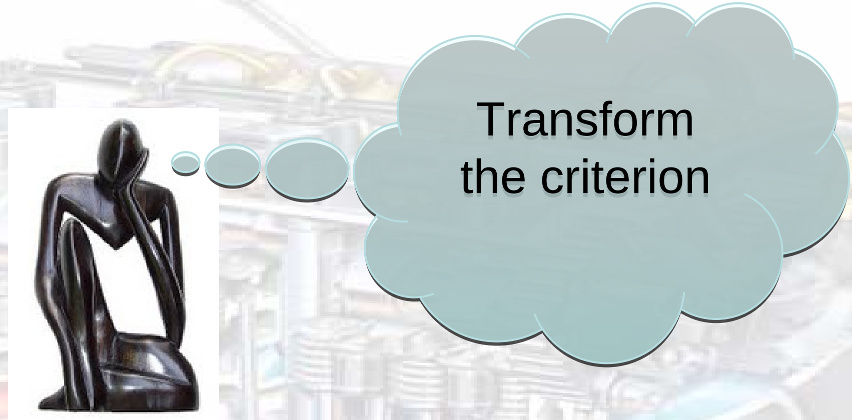
Instability for systems with a relative degree greater than four.





III- Unconstrained corrected NCGPC

III- 1 Presentation



$$J_v = J + J_c \quad \Rightarrow \quad J_v = \frac{1}{2}\int_0^T [\hat{e}(t+\tau)]^2 d\tau + Gvu$$

$$G = -\Pi_{ss} D$$

$$u(x(t)) = \frac{-\sum_{l=0}^{\rho} K_{\rho l}\left[L_f^l h(x(t)) - \omega^{(l)}(t)\right]}{L_g L_f^{\rho-1} h(x(t))} + \frac{v}{L_g L_f^{\rho-1} h(x(t))}$$

$$v = -FZ = -\sum_{l=0}^{\rho-1} F_l\left[L_f^l h(x) - \omega^{(l)}\right]$$

III- Unconstrained corrected NCGPC

III- 1 Theorem

Theorem 3.1: *Consider a nonlinear system of the form (1) verifying assumptions A1 to A4, with a relative degree greater than four. A closed-loop stability for such a system can be achieved, in a new space of coordinates, with a control law of the form*

$$u(x) = \frac{-\sum_{l=0}^{\rho} K_l \left[L_f^l h(x) - \omega^{(l)}\right]}{L_g L_f^{\rho-1} h(x)} + \frac{v}{L_g L_f^{\rho-1} h(x)},$$

resulting from the minimization of the following criterion

$$J_v = \frac{1}{2} \int_0^T [\hat{e}(t+\tau)]^2 d\tau + Gvu$$

where Gvu is a linear term of correction with respect to control law u, with $G = -\Pi_{ss} D$.

IV- Application of the control law and the simulation results

IV- 1 Application of the control law

$$Z = \begin{bmatrix} z_1 \\ \vdots \\ z_\rho \end{bmatrix} = \begin{bmatrix} y - \omega \\ \vdots \\ y^{(\rho-1)} - \omega^{(\rho-1)} \end{bmatrix}$$

$$\begin{cases} \dot{z}_1 & = & z_2 \\ & \vdots & \\ \dot{z}_\rho & = & L_f^\rho h - \omega^{(\rho)} + u L_g L_f^{\rho-1} h \end{cases}$$

$$u(x) = -\frac{L_f^\rho h(x) - \omega^{(\rho)}}{L_g L_f^{\rho-1} h(x)} - \frac{\sum_{l=0}^{\rho-1} (K_{\rho_l} + F_l) \left[L_f^l h(x) - \omega^{(l)} \right]}{L_g L_f^{\rho-1} h(x)}$$

$$\dot{Z} = AZ$$

IV- Application of the control law and the simulation results

IV- 1 Simulations results

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 + x_1^2 \\ \dot{x}_3 = x_4 - x_2 \\ \dot{x}_4 = x_5 - x_1 \\ \dot{x}_5 = x_2 - 3u \\ \dot{x}_6 = x_5 - x_6 \end{cases} \quad \text{and} \quad y = h(x) = x_1$$

$$\dot{x}_6 = -x_6$$

$$\begin{cases} y = h(x) = x_1 \\ \dot{y} = L_f h(x) = \dot{x}_1 = x_2 \\ \ddot{y} = L_f^2 h(x) = \dot{x}_2 = x_3 + x_1^2 \\ y^{(3)} = L_f^3 h(x) = 2x_1 x_2 + x_4 - x_2 \\ y^{(4)} = L_f^4 h(x) = 2x_2^2 + (2x_1 - 1)(x_3 + x_1^2) \\ \qquad + x_5 - x_1 \\ y^{(5)} = L_f^5 h(x) + u L_g L_f^4 h(x) = (2x_3 + 2x_1^2 \\ \qquad + 2(2x_1 - 1)x_1 - 1)x_2 + 4x_2(x_3 + x_1^2) \\ \qquad + (2x_1 - 1)(x_4 - x_2) + x_2 + 3u \end{cases}$$

IV- Application of the control law and the simulation results

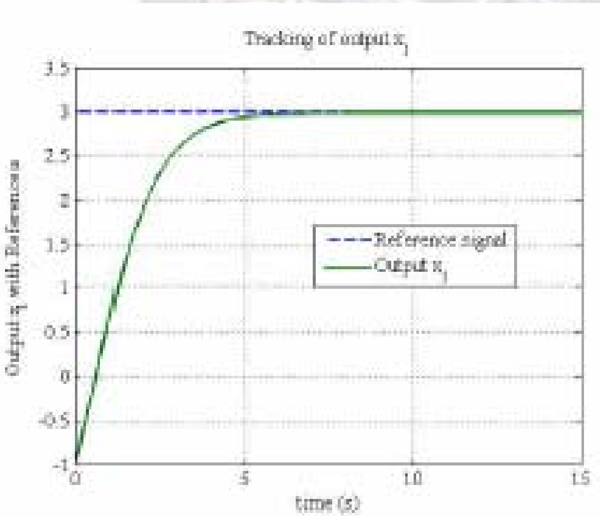
IV- 1 Simulations results

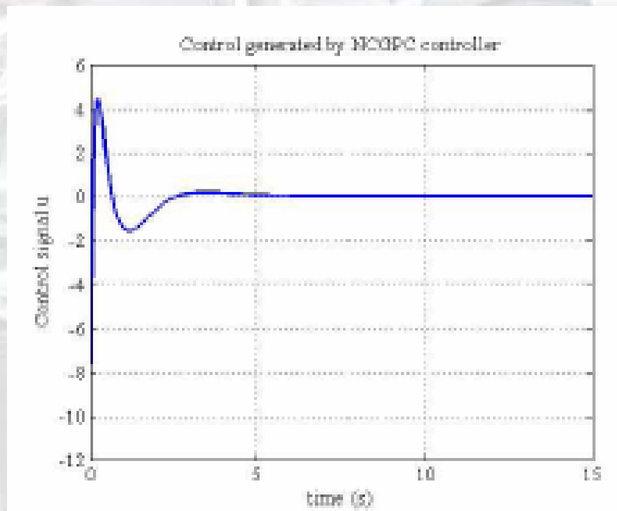
$$K \cong [\ 2.2 \quad 0.19 \quad 82.10^{-4} \quad 2.10^{-4} \quad 0 \quad 1.10^{-7}\ ] * 10^7$$

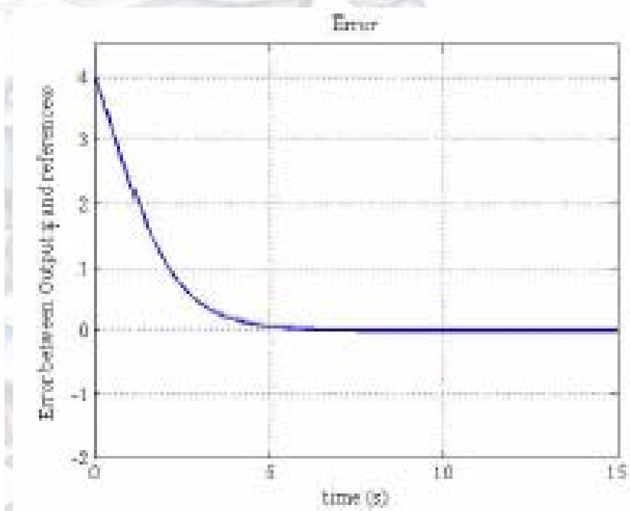
$$T = 0.1 \text{ second}$$

$$P = [\ -2 \quad -1 \quad -3 \quad -4 \quad -6\ ]$$

$$F \cong [\ -2.2 \quad -0.19 \quad -82.10^{-4} \quad -2.10^{-4} \quad 0\ ] * 10^7$$







V- Conclusions

Output tracking of a nonlinear SISO system

- **Correction of the criterion to solve problem of instability for systems with arelative degree greater than four**

Jerejef

Thank you for attention