Asymptotic tracking and Unconstrained NCGPC: a comparative study applied to the tracking problem of a turbocharged diesel engine (TDE)

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Summary

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II- Description of TDE

- II-1 Full-order TDE model (dimension 7)
- II-2 Reduced-order TDE model (dimension 3)

III- Asymptotic Tracking and Unconstrained NCGPC

V- Application to the reduced-order TDE model

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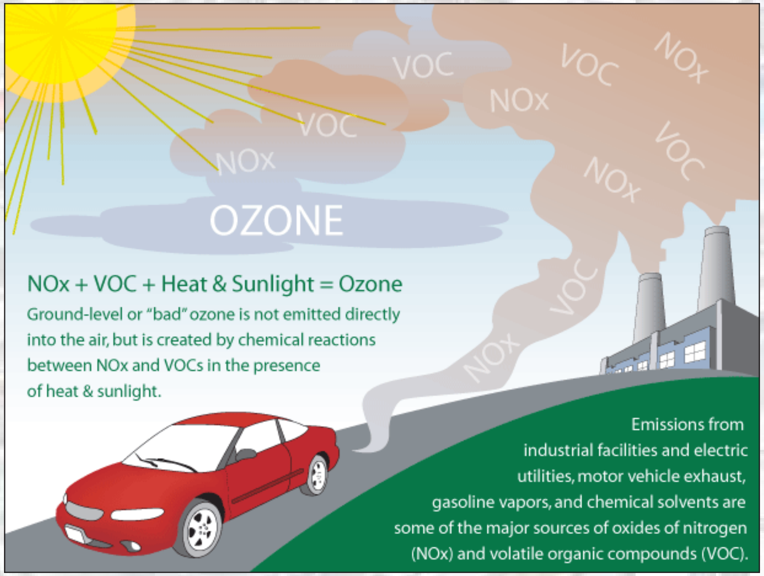
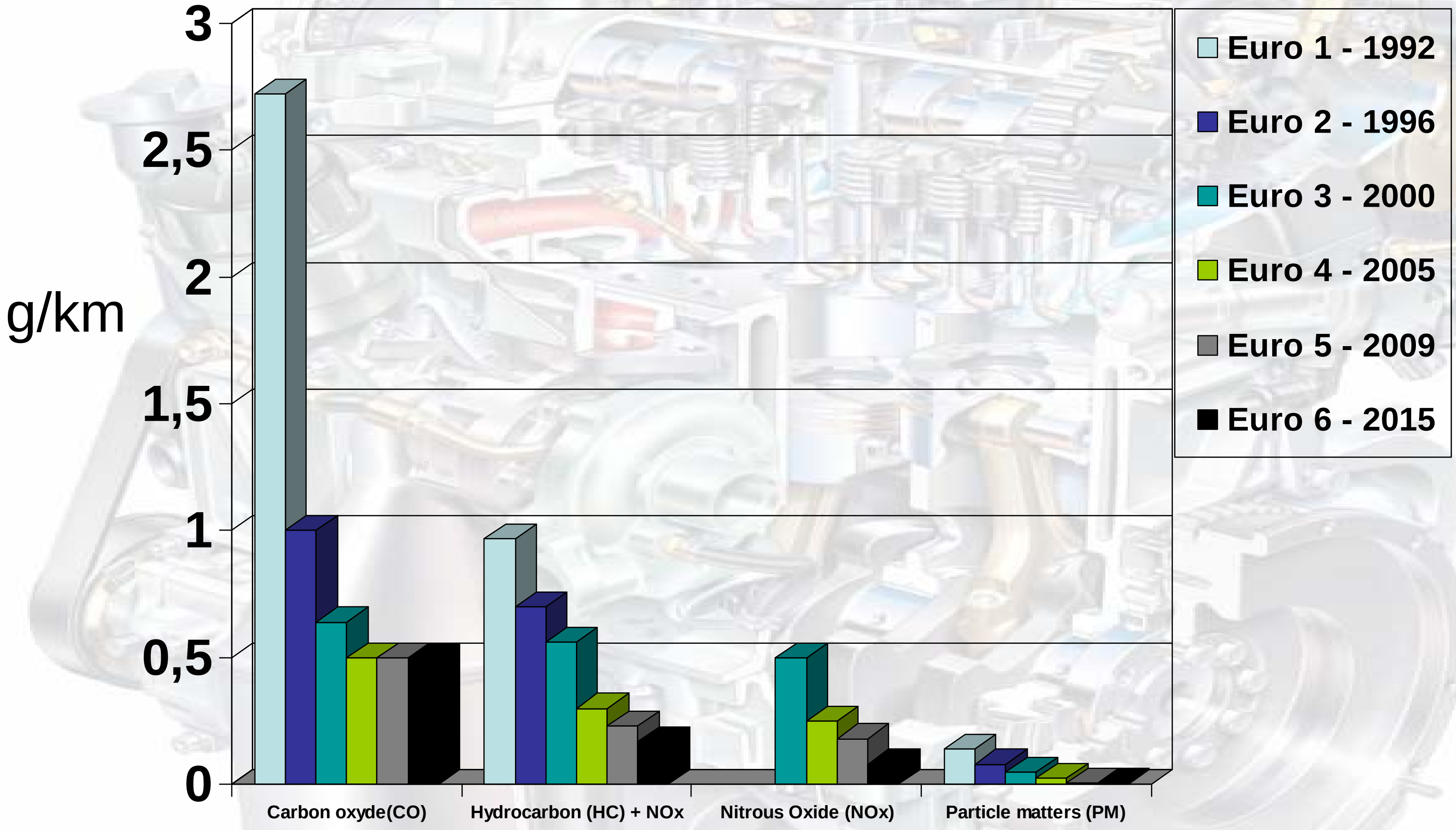
I - Introduction et objectives



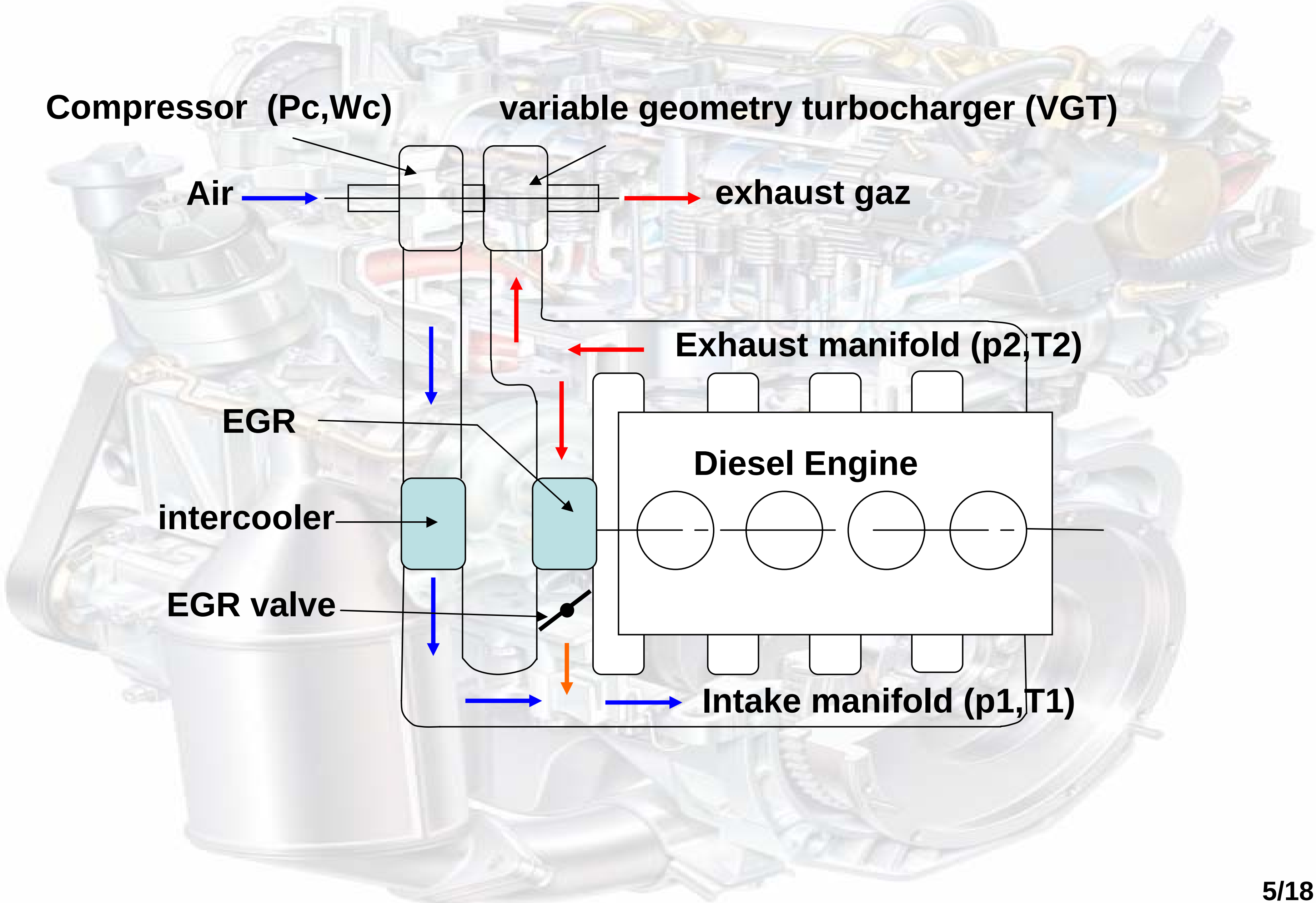
No necessary studies

Necessary studies

I - Introduction et objectives (Euro Standards for Passenger Cars)



II - Description of TDE



II-1 Full-order TDE model (dimension 7)

Kolmanovsky *et al.* ([1997])

Intake manifold

$$\dot{m_1} = W_c + W_{egr} - W_e$$

$$\dot{p_1} = \frac{\gamma R}{V_1}(W_c T_c + W_{egr} T_{egr} - W_e T_1)$$

$$\dot{F}_1 = \frac{W_{egr}(F_2 - F_1) - W_c F_1}{m_1}$$

Exhaust manifold

$$\dot{m_2} = W_e - W_{egr} - W_t + W_f$$

$$\dot{p_2} = \frac{\gamma R}{V_2}((W_e + W_f) T_e - W_{egr} T_2 - W_t T_2$$

$$\dot{F}_2 = \frac{W_e[15.6(1 - F_1) + (AF+1)F_1]/(AF-1) - W_e F_2}{m_1}$$

VGT crankshaft

$$\dot{\omega}_{tc} = \frac{1}{J_{tc}\omega_{tc}}(\eta_m P_t - P_c)$$

II-2 Reduced-order TDE model (dimension 3)

Under the hypotheses of Jankovic *et al.* ([2000])

States

$$x = \begin{pmatrix} p_1 \\ p_2 \\ P_c \end{pmatrix}$$

Inputs

$$u_1 = W_{egr}$$

$$u_2 = W_{vgt}$$

MIMO nonlinear system of dimension 3

$$\dot{x} = f(x) + g_1(x)u_1 + g_2(x)u_2$$

$$y = (h_1(x), h_2(x))^t$$

$$f(x) = \begin{bmatrix} k_1 k_c \frac{P_c}{p_1^{\mu} - 1} - k_1 k_e p_1 \\ k_2 k_e p_1 \\ -\frac{P_c}{\tau} \end{bmatrix}$$

$$K_0 = \frac{\eta_m}{\tau} k_t$$

$$g_1(x) = \begin{bmatrix} k_1 \\ -k_2 \\ 0 \end{bmatrix} \text{ and } g_2(x) = \begin{bmatrix} 0 \\ -k_2 \\ K_0\left(1 - p_2^{-\mu}\right) \end{bmatrix}$$

Outputs

$$y_1 = p_1$$

$$y_2 = p_2$$

$$\vdots$$

Set

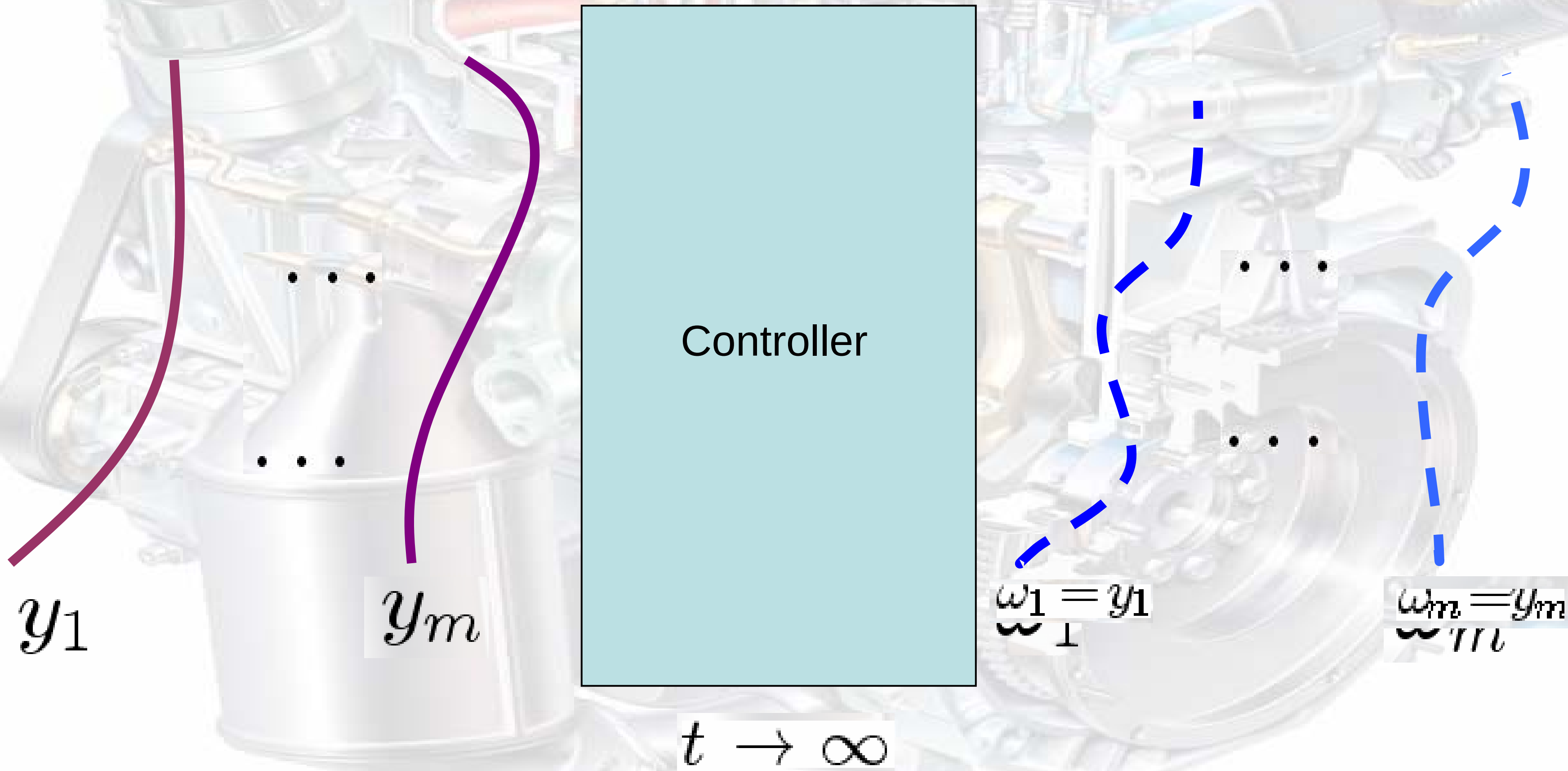
$$\Omega = \{(p_1, p_2, P_c) : 1 < p_1 < p_1^{max}, 1 < p_2 < p_2^{max}, 0 < P_c < P_c^{max}\}$$

III- Asymptotic Tracking and Unconstrained NCGPC

III-1- Asymptotic Tracking

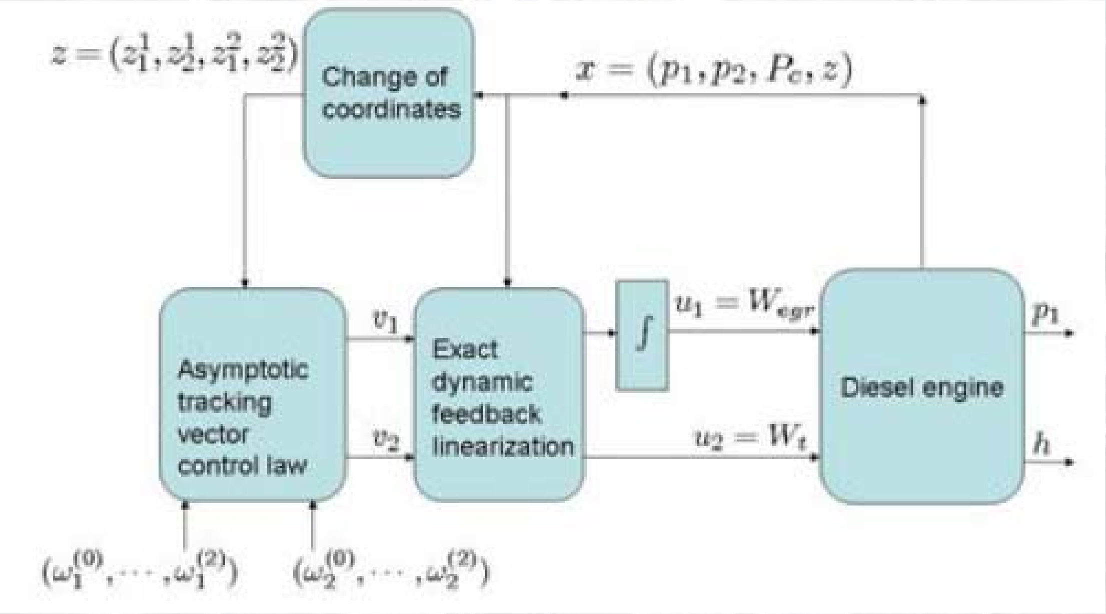
$\dot{x} = f(x) + \sum_{j=1}^{m} g_j(x) u_j$ $\quad x \in R^n,\ y \in R^m$ and $u \in R^m$,

$y = (h_1(x), \ldots, h_m(x))^t,$ $\quad \omega = (\omega_1, \cdots, \omega_m)^t.$



III-1- Asymptotic Tracking

$$u(x) = A(x)^{-1}\left[\begin{pmatrix} v_1 \\ \vdots \\ v_m \end{pmatrix} - \begin{pmatrix} L_f^{\rho_1} h_1(x) \\ \vdots \\ L_f^{\rho_m} h_m(x) \end{pmatrix}\right]$$



$$v_i = \omega_i^{(\rho_i)} - \sum_{k=1}^{\rho_i} c_{i(k-1)}(z_k^i - \omega_i^{(k-1)})$$

$$z_k^i = L_f^{k-1} h_i$$

$$s^{\rho_i} + c_{i(\rho_i-1)} s^{\rho_i-1} + \cdots + c_{i1} s + c_{i0} = 0$$

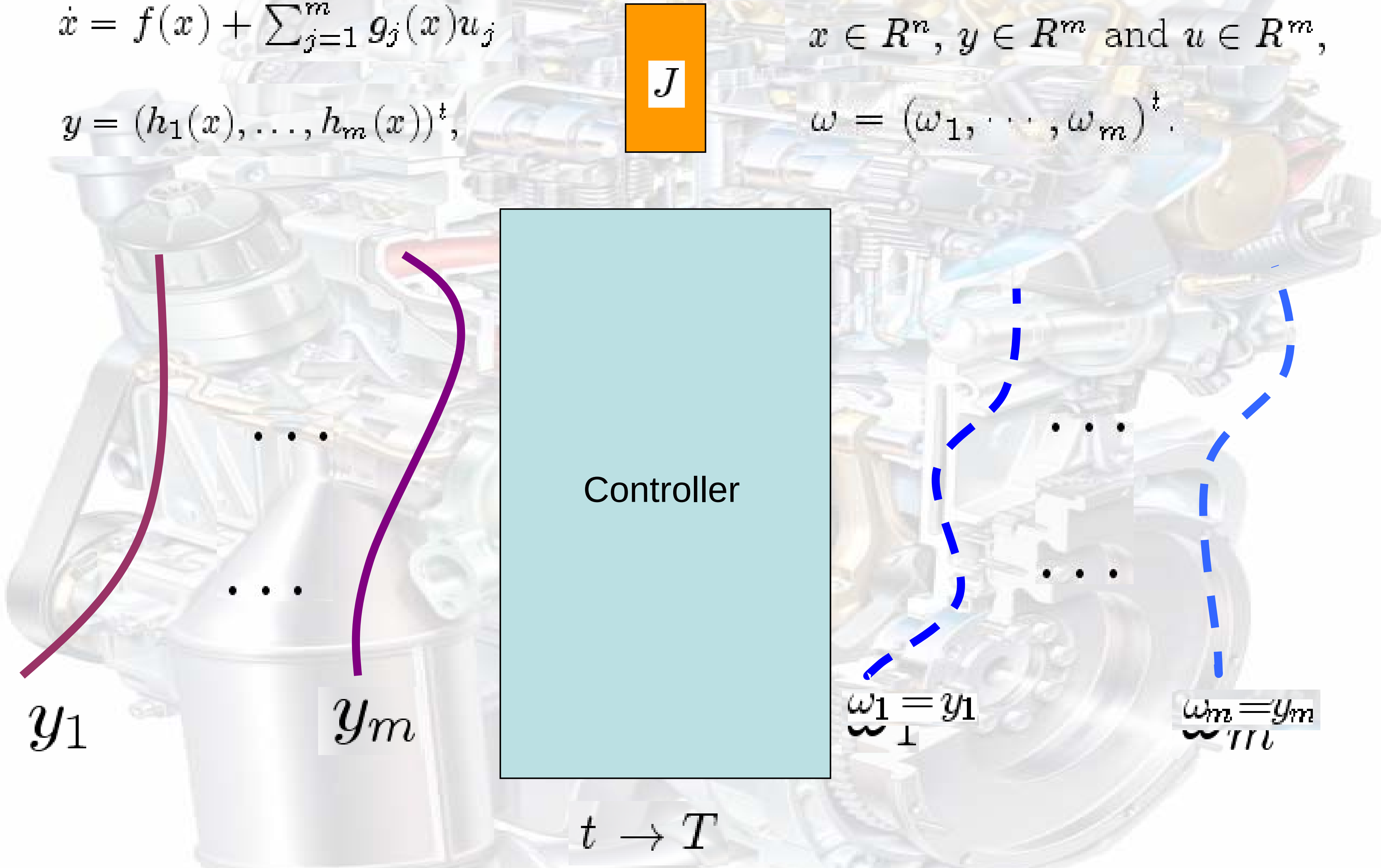
III-2 Unconstrained NCGPC

$$\dot{x} = f(x) + \sum_{j=1}^{m} g_j(x) u_j$$

$$y = (h_1(x), \ldots, h_m(x))^t,$$

$x \in R^n$, $y \in R^m$ and $u \in R^m$,

$$\omega = (\omega_1, \cdots, \omega_m)^t.$$

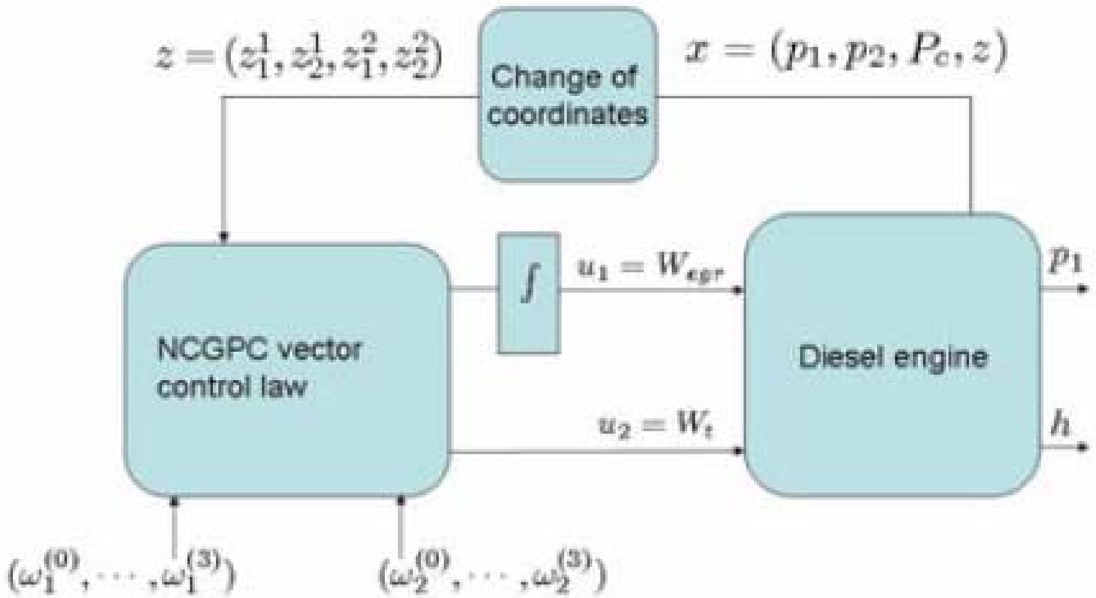


III-2- Unconstrained NCGPC

$$u(x(t),t) = (A(x(t)))^{-1} \begin{bmatrix} L_1(B) \\ \vdots \\ L_{[(m-1)(r_i+1)+1]}(B) \end{bmatrix} [W(t) - \bar{Y}(t)]$$

$$J_i = \frac{1}{2}\int_0^{T_i}[\hat{e}_i(t+\tau_i)]^2 d\tau_i$$

$$[W - \bar{Y}] = \begin{bmatrix} \omega_1^{(0)} - L_f^0 h_1(x) \\ \vdots \\ \omega_1^{(l_1)} - L_f^{(l_1)} h_1(x) \\ \vdots \\ \omega_m^{(0)} - L_f^0 h_m(x) \\ \vdots \\ \omega_m^{(l_m)} - L_f^{(l_m)} h_m(x) \end{bmatrix}$$



III-3- Control laws comparison

$$u(x(t),t) = (A(x(t)))^{-1}\left[\begin{pmatrix} v_1(t) \\ \vdots \\ v_m(t) \end{pmatrix} - \begin{pmatrix} L_f^{\rho_1} h_1(x(t)) \\ \vdots \\ L_f^{\rho_m} h_m(x(t)) \end{pmatrix}\right]$$

$$u(x(t),t) = (A(x(t)))^{-1}\begin{bmatrix} L_1(B) \\ \vdots \\ L_{[(m-1)(r_i+1)+1]}(B) \end{bmatrix}[W(t) - \bar{Y}(t)]$$

$$v_i = \omega_i^{(\rho_i)} - \sum_{k=1}^{\rho_i} c_{i(k-1)}(z_k^i - \omega_i^{(k-1)})$$

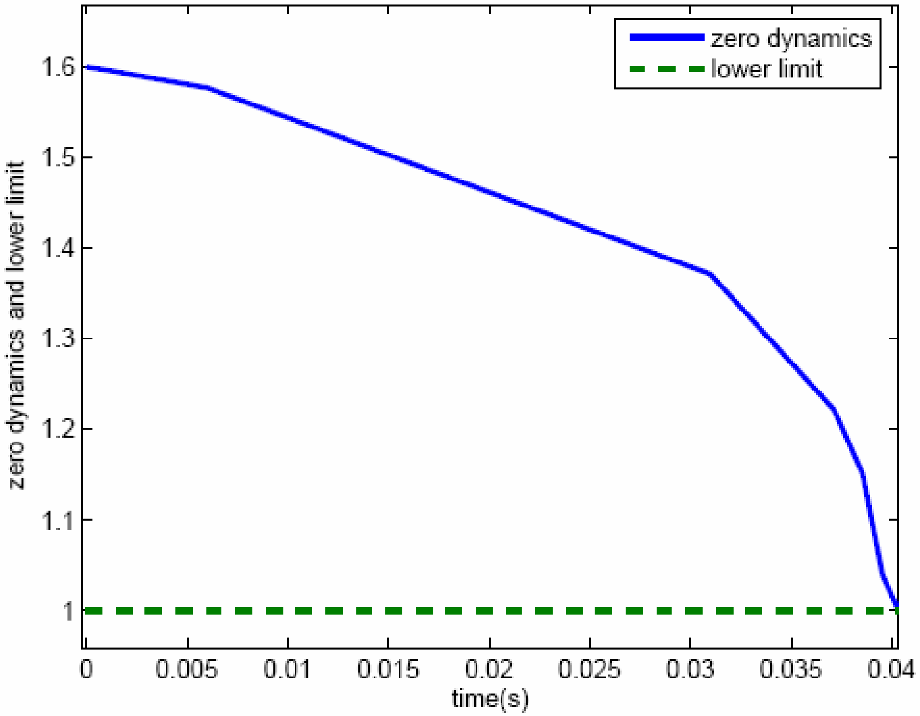
$$z_k^i = L_f^{k-1} h_i$$

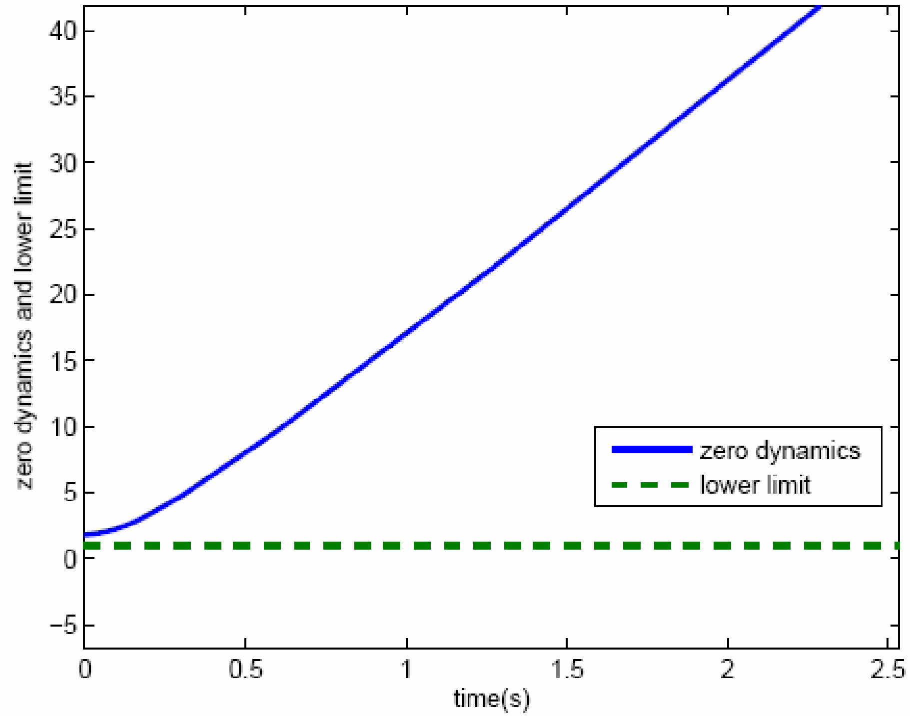
IV- Application to the reduced-order TDE model (details)

$$Y_{ori} = \begin{bmatrix} AFR \\ EGR \end{bmatrix} \Rightarrow y = \begin{bmatrix} p_1 \\ W_c \end{bmatrix} \Rightarrow (\rho_1, \rho_2) = (1, 1)$$

Not accessible for measurements

$$\dot{p}_2 = k_2 W_{cd}\left[1 - \frac{(p_{1d}^{\mu}-1)}{\eta_m k_t k_c (1-p_2^{-\mu})}\right]$$





Non-minimum phase system

IV- Application to the reduced-order TDE model (details)

Change the vector output under the constraint $L_{g_2}h = 0$

$$\tilde{y} = \begin{bmatrix} p_1 \\ h(x) = P_c + \frac{K_0}{k_2}\left[p_2 - \frac{1}{1-\mu}p_2^{1-\mu}\right] \end{bmatrix} \Rightarrow \tilde{A}(x) = \begin{bmatrix} k_1 & 0 \\ -\frac{\mu k_c k_1 P_c p_1^{\mu-1}}{(p_1^{\mu}-1)^2} & 0 \end{bmatrix}$$

⇒ **Non-invertible matrix** ⇒ **Undefined vector relative degree**

⇓

$D = span\{g_1, g_2\}$ **D is not involutive**

$$A^e(x) = \begin{bmatrix} k_1 & k_1 k_c K_0 \frac{1-p_2^{-\mu}}{p_1^{\mu}-1} \\ K_0(p_2^{-1} - 1) & \mu k_2 K_0 (z - k_e p_1) p_2^{-\mu-1} + \frac{K_0}{\tau}(p_2^{-\mu} - 1) \end{bmatrix}$$

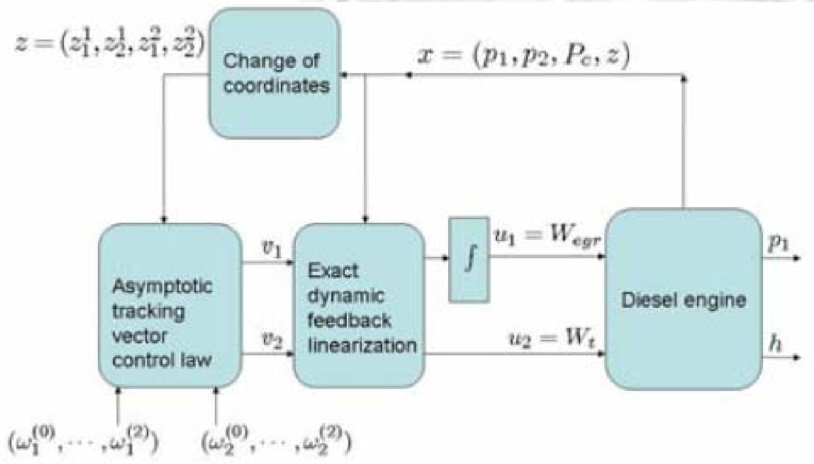
⇒
- $(\rho_1^e, \rho_2^e) = (2, 2)$
- **Well-defined vector relative degree**
- **Trivial zero dynamics**

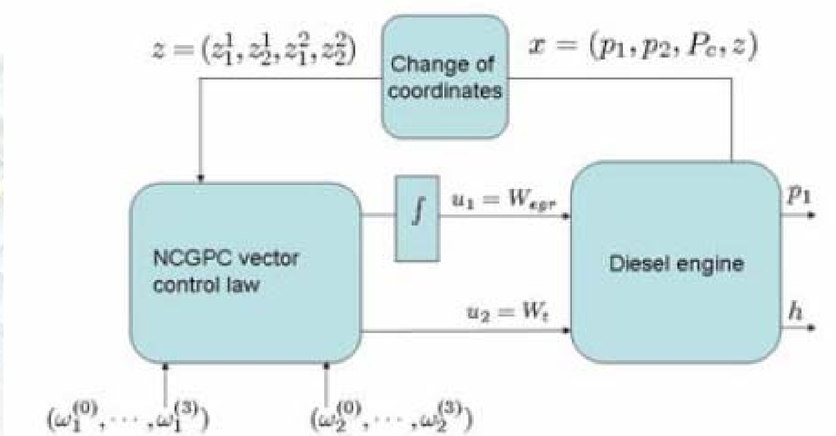
⇒ **System without zero dynamics**

IV- Application to the reduced-order TDE model (résumé-Algo)

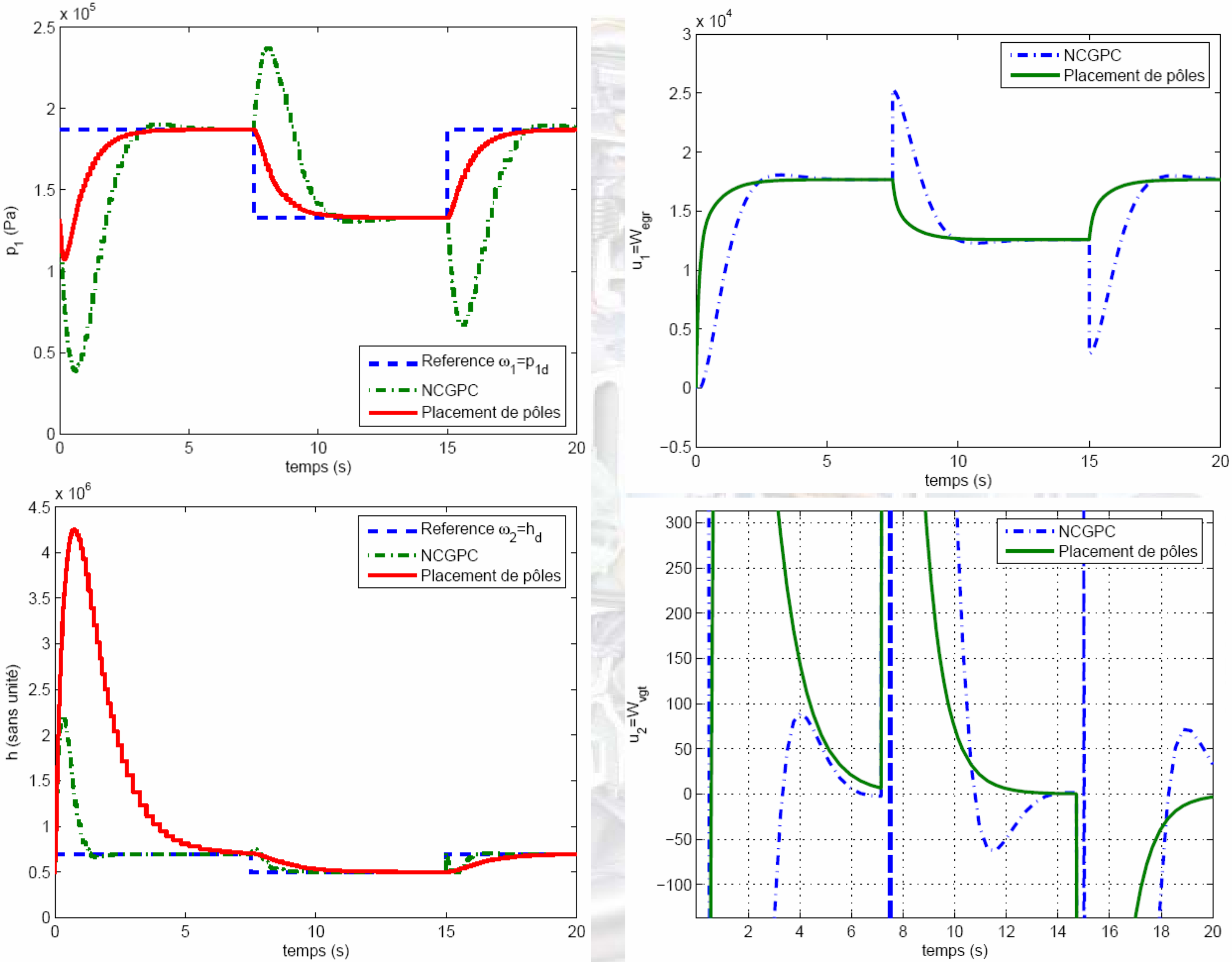
$L_{g_2} h = 0$

First outputs → **Not accessible For measurements** → New outputs → **Non minimum Phase system** → Change Of outputs → **Non-defined vector relative degree** → Dynamic Extension → **Trivial zero dynamics / Well defined vector relative degree** → Asymptotic tracking / Unconstrained NCGPC





V- Simulation results



VI- Conclusion and future works

VI-1 Conclusion

Comparative study between Asymptotic Tracking and unconstrained NCGPC applied to the 3rd order nonlinear TDE model with:

- **change of the vector output**
- **dynamic extension**

VI-2 Future works

Using a nonlinear observer

Designing an algorithm without changing vector output (avoid eventual problems of measurements)

Thank you for attention