Optimization and Applications Seminar, Fall 2008

ETH Zurich and University of Zurich

Flexible Control Lyapunov Functions

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Where innovation starts

Outline

- **Flexible Control Lyapunov Functions (CLFs)**

- **Trajectory-dependent CLFs: hybrid systems**

  joint work with Andrej Jokic

- **Optimized input-to-state stabilization**

  joint work with Maurice Heemels

- **Structured CLFs: decentralized control of dynamically coupled nonlinear systems**

  joint work with Andrej Jokic

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Control Lyapunov functions

$$x(k+1) = \phi(x(k), u(k)), \; x(k) \in \mathbb{X}, \; u(k) \in \mathbb{U}, \; k \in \mathbb{Z}_+$$

$V : \mathbb{R}^n \to \mathbb{R}_+, \quad \exists \alpha_1, \alpha_2, \alpha_3 \in \mathcal{K}_\infty, \; u(x) : \mathbb{X} \to \mathbb{U}$ such that

$$\alpha_1(\|x\|) \leq V(x) \leq \alpha_2(\|x\|), \quad \forall x \in \mathbb{X}$$

$$V(\phi(x, u(x))) - V(x) \leq -\alpha_3(\|x\|), \quad \forall x \in \mathbb{X}$$

$$V(\phi(x, u(x))) - \rho V(x) \leq 0, \quad \forall x \in \mathbb{X}, \; \rho \in \mathbb{R}_{[0,1)}$$

At time $k \in \mathbb{Z}_+$ measure $x(k)$ and find $u(k)$ such that

$$u(k) \in \mathbb{U}, \; \phi(x(k), u(k)) \in \mathbb{X}$$

$$V(\phi(x(k), u(k))) - V(x(k)) + \alpha_3(\|x(k)\|) \leq 0$$

Let $\pi(x(k))$ be the set of feasible control actions

Then $x(k+1) \in \phi_{\text{cl}}(x(k), \pi(x(k)))$ is AS in $\mathbb{X}$

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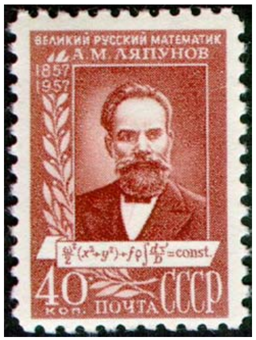
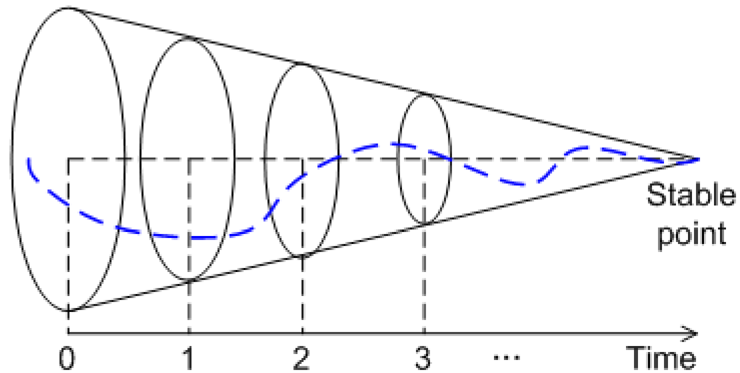
Control Lyapunov functions





Lyapunov Function = Energy

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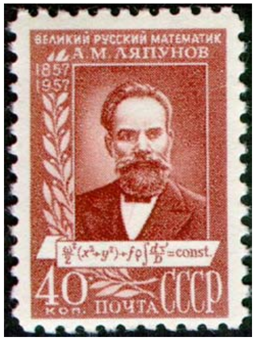
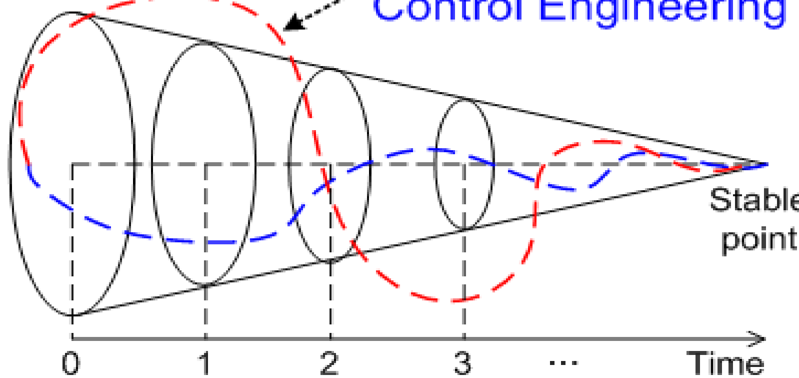
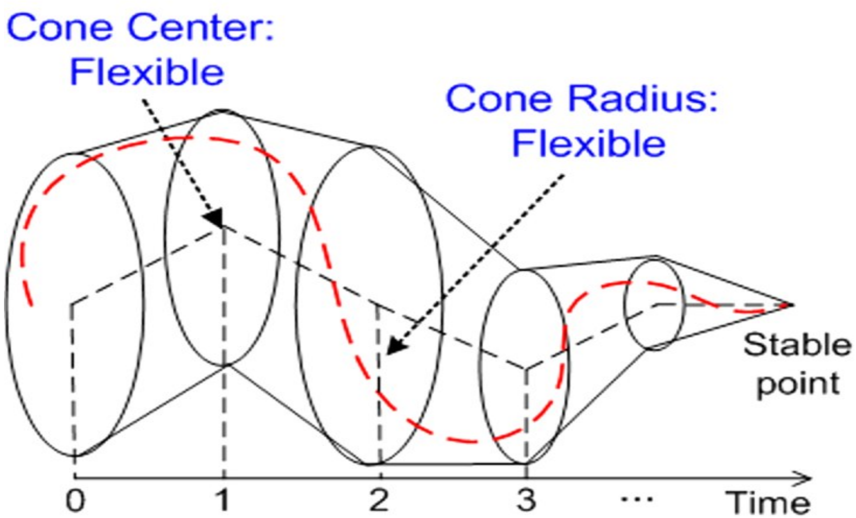
Control Lyapunov functions



90% (Qin and Badgwell, 2003)
Control Engineering Practice



Rigid Lyapunov Function



Flexible Lyapunov Function

$$V(\phi(x,u(x))) \leq \rho V(x)$$

$$V(\phi(x,u(x))) \leq \rho V(x) + \lambda(x)$$

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Flexible control Lyapunov functions

Choose a candidate CLF $V(\cdot)$ and a cost $J(\lambda(k))$

At time $k \in \mathbb{Z}_+$ measure $x(k)$, minimize
$J(\lambda(k))$ over $\lambda(k)$ and find $u(k)$ such that
$u(k) \in \mathbb{U},\ \phi(x(k), u(k)) \in \mathbb{X}$
$V(\phi(x(k), u(k))) \leq \rho V(x(k)) + \lambda(k)$

Let $\pi(x(k))$ be the set of feasible control actions
If $\lim_{k\rightarrow\infty} \lambda(k) = 0$ then $x(k+1) \in \phi_{\text{cl}}(x(k), \pi(x(k)))$ is AS in $\mathbb{X}$

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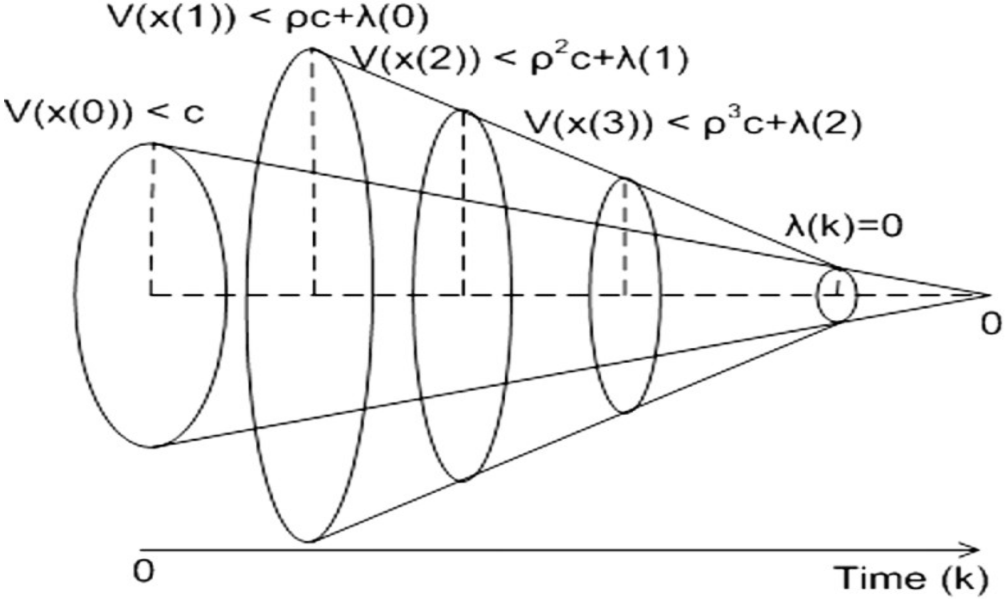
Flexible control Lyapunov functions

At time $k \in \mathbb{Z}_+$ measure $x(k)$, minimize $J(\lambda(k))$ over $\lambda(k)$ and find $u(k)$ such that

$$u(k) \in \mathbb{U},\ \phi(x(k), u(k)) \in \mathbb{X}$$

$$V(\phi(x(k), u(k))) \leq \rho V(x(k)) + \lambda(k)$$

$$\lambda(k) \leq \mu \lambda(k-1),\ k \in \mathbb{R}_{\geq 1}$$



Flexible Radius Lyapunov Function: V(x(k))

$$\lim_{k \to \infty} \lambda(k) = 0$$

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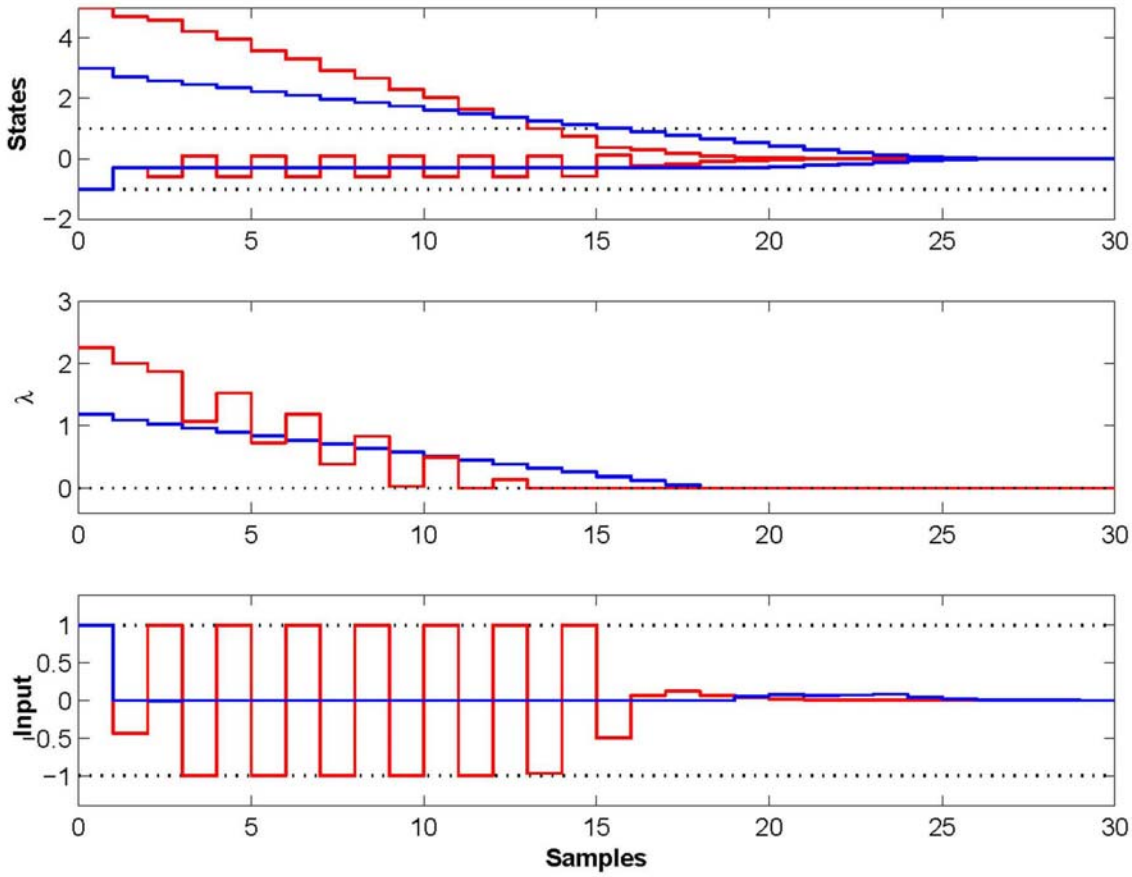
Flexible control Lyapunov functions



$$x(k+1) = f(x(k)) + g(x(k))u(k)$$
$$= \begin{pmatrix} [x]_1(k)+0.7[x]_2(k)+([x]_2(k))^2 \\ [x]_2(k) \end{pmatrix} + \begin{pmatrix} 0.245+\sin([x]_2(k)) \\ 0.7 \end{pmatrix} u(k)$$

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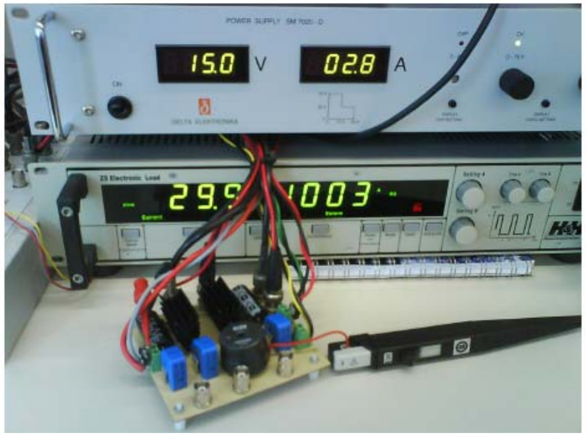
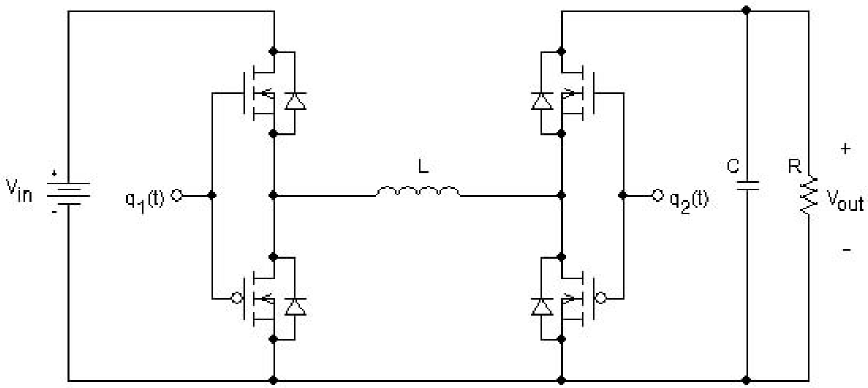
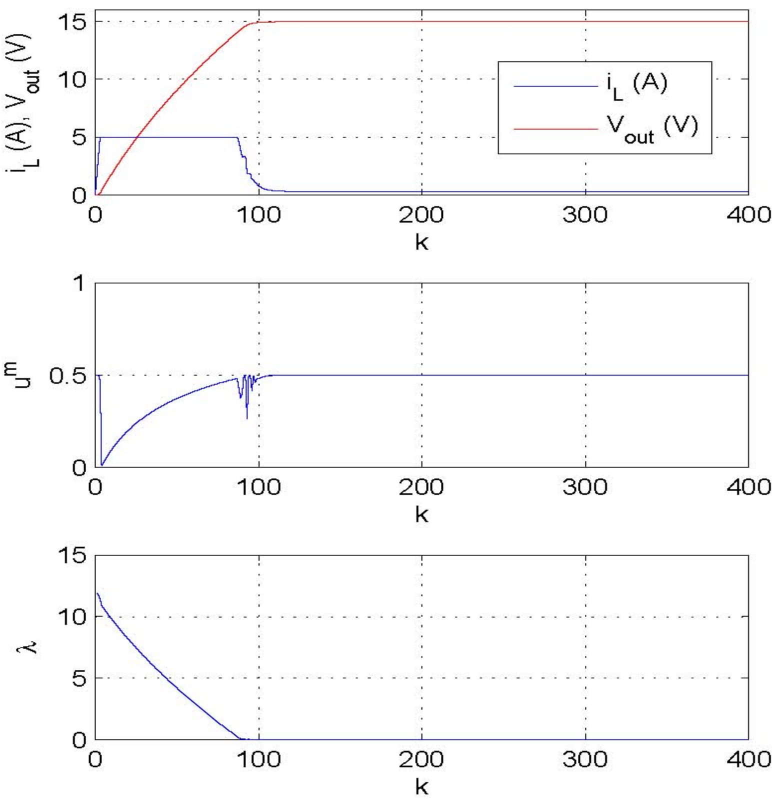
Power converters: 0.1ms







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Trajectory-dependent CLFs

$$x(k+1) = \phi(x(k), u(k)),\ x(k) \in \mathbb{X},\ u(k) \in \mathbb{U},\ k \in \mathbb{Z}_+$$

Time-variant Control Lyapunov Function:

$V : \mathbb{Z}_+ \times \mathbb{R}^n \rightarrow \mathbb{R}_+, \quad \exists \alpha_1, \alpha_2 \in \mathcal{K}_\infty,\ u(x) : \mathbb{X} \rightarrow \mathbb{U}$ s.t.

$\alpha_1(\|x\|) \leq V(k,x) \leq \alpha_2(\|x\|), \quad \forall x \in \mathbb{X}, \forall k \in \mathbb{Z}_+$

$\forall x(0) \in \mathbb{X},\ V(k+1, \phi(x(k), u(x(k)))) \leq \rho V(k, x(k))\ \forall k \in \mathbb{Z}_+$

At time $k \in \mathbb{Z}_+$ measure $x(k)$ and find $u(k)$ such that

$u(k) \in \mathbb{U},\ \phi(x(k), u(k)) \in \mathbb{X}$

$V(k, \phi(x(k), u(k))) \leq \rho V(k, x(k))$

$V(k, x(k)) \leq V(k-1, x(k)),\ \forall k \in \mathbb{Z}_{\geq 1}$

Let $\pi(x(k), V(k-1, \cdot))$ be the set of feasible control actions

Then $x(k+1) \in \phi_{\text{cl}}(x(k), \pi(x(k)))$ is AS in $\mathbb{X}$

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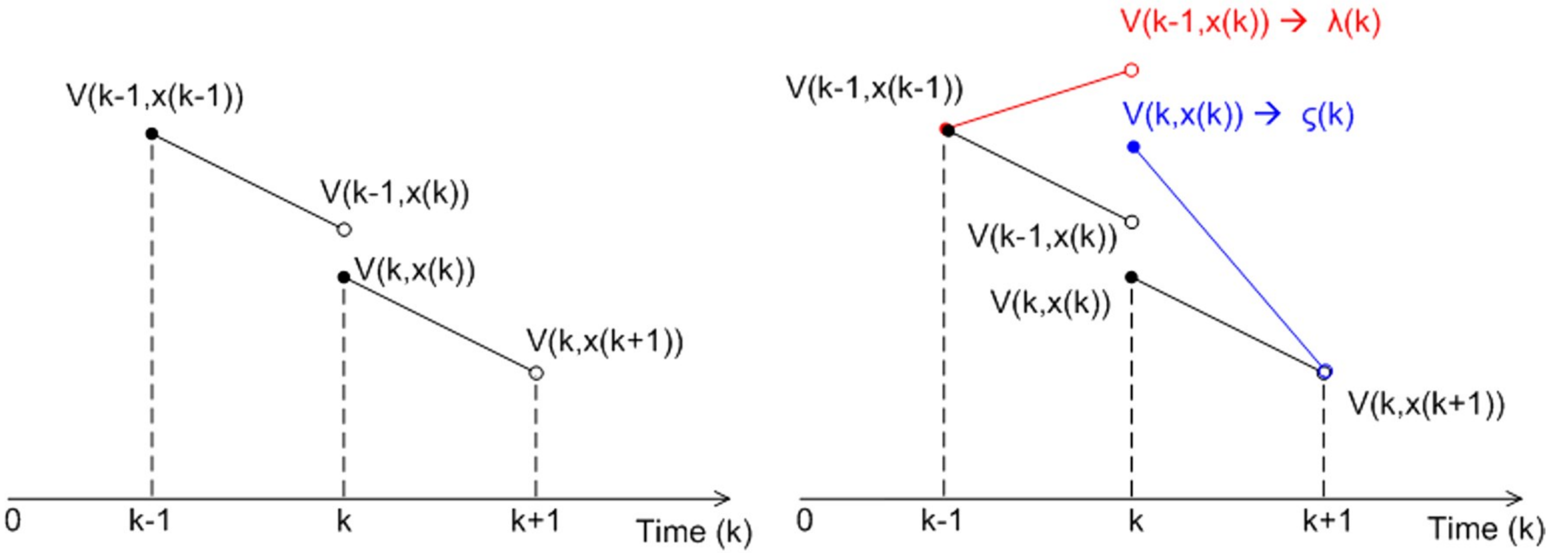
Trajectory-dependent CLFs



Monotone time-variant CLF

Non-monotone time-variant CLF

Particular cases:

$\mathcal{S}$-procedure relaxation (Johansson and Rantzer, 1998)

Multiple Lyapunov functions (Branicky et al., 1998)

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A flower system for synthesis

$$x_{k+1} = \begin{cases} A_1 x_k + B u_k & \text{if } E_1 x_k > 0 \\ A_2 x_k + B u_k & \text{if } E_2 x_k \geq 0 \\ A_3 x_k + B u_k & \text{if } E_3 x_k > 0 \\ A_4 x_k + B u_k & \text{if } E_4 x_k \geq 0 \end{cases}$$

$$x_k \in \mathbb{X} = [-10, 10]^2, \quad u_k \in \mathbb{U} = [-1, 1]$$

$$A_1 = A_3 = \begin{bmatrix} 0.5 & 0.61 \\ 0.9 & 1.345 \end{bmatrix}, A_2 = A_4 = \begin{bmatrix} -0.92 & 0.644 \\ 0.758 & -0.71 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$E_1 = -E_3 = \begin{bmatrix} -1 & 1 \\ -1 & -1 \end{bmatrix}, E_2 = -E_4 = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}, \; Q = 10^{-4} I_2, \; R = 10^{-3}$$

(Lazar et al., ACC 2005)

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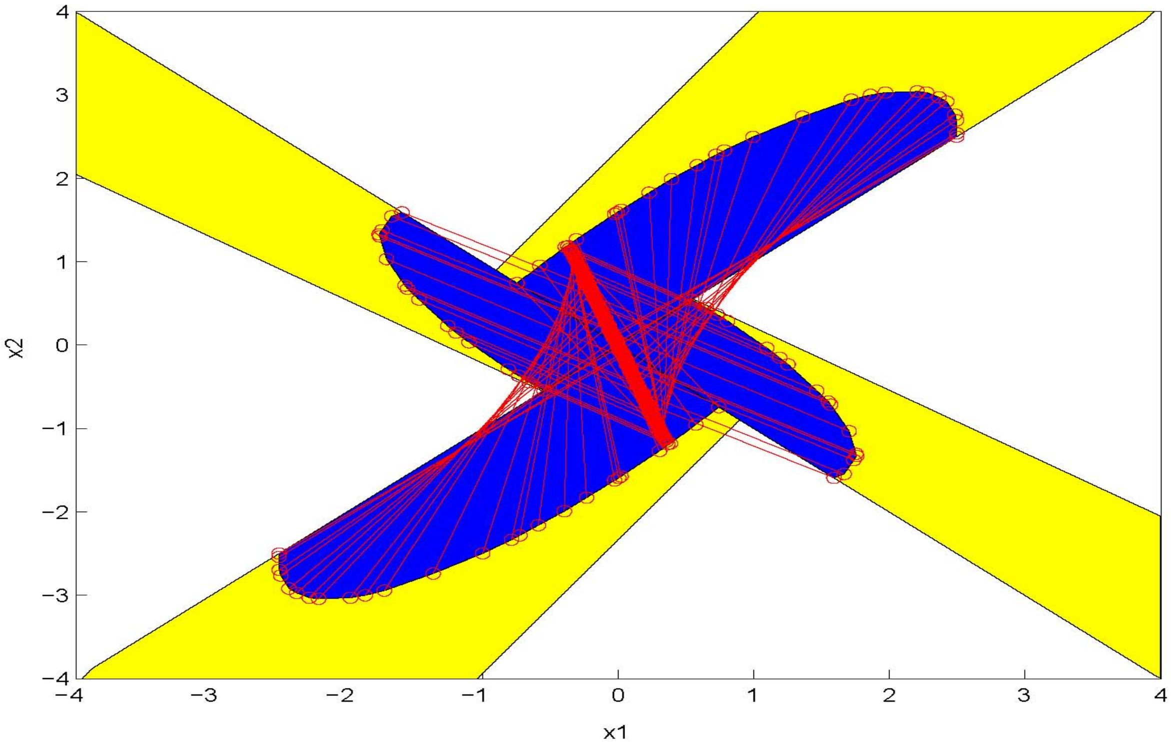
A flower system for synthesis



$$P_1 = \begin{bmatrix} 12.9707 & 10.9974 \\ 10.9974 & 14.9026 \end{bmatrix}, P_2 = \begin{bmatrix} 7.9915 & -5.5898 \\ -5.5898 & 5.3833 \end{bmatrix}, P_3 = P_1, \quad P_4 = P_2,$$

$$K_1 = \begin{bmatrix} -0.7757 & -1.0299 \end{bmatrix}, K_2 = \begin{bmatrix} 0.6788 & -0.4302 \end{bmatrix}, K_3 = K_1, \quad K_4 = K_2$$

$$U_{11} = \begin{bmatrix} 0.4596 & 1.9626 \\ 1.9626 & 0.0198 \end{bmatrix}, U_{12} = \begin{bmatrix} 0.4545 & 2.0034 \\ 2.0034 & 0.0250 \end{bmatrix},$$

$$U_{21} = \begin{bmatrix} 0.0542 & 0.0841 \\ 0.0841 & 0.0506 \end{bmatrix}, U_{22} = \begin{bmatrix} 0.0599 & 0.0914 \\ 0.0914 & 0.0565 \end{bmatrix}$$

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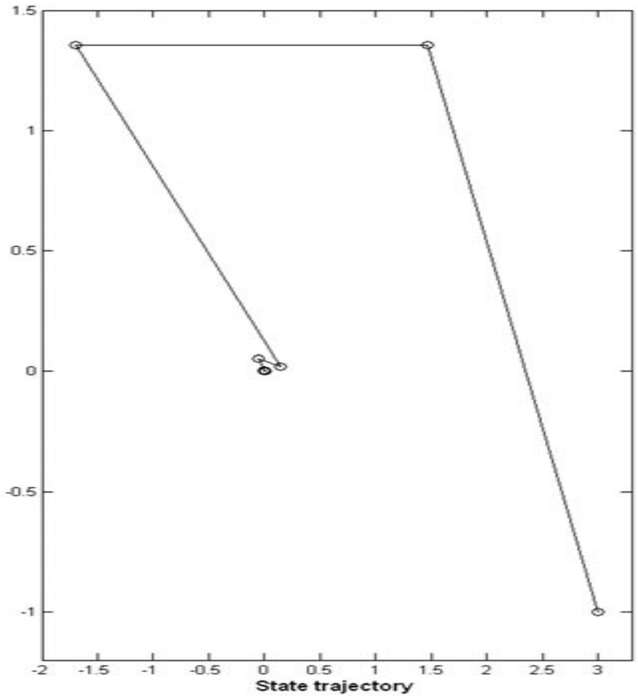
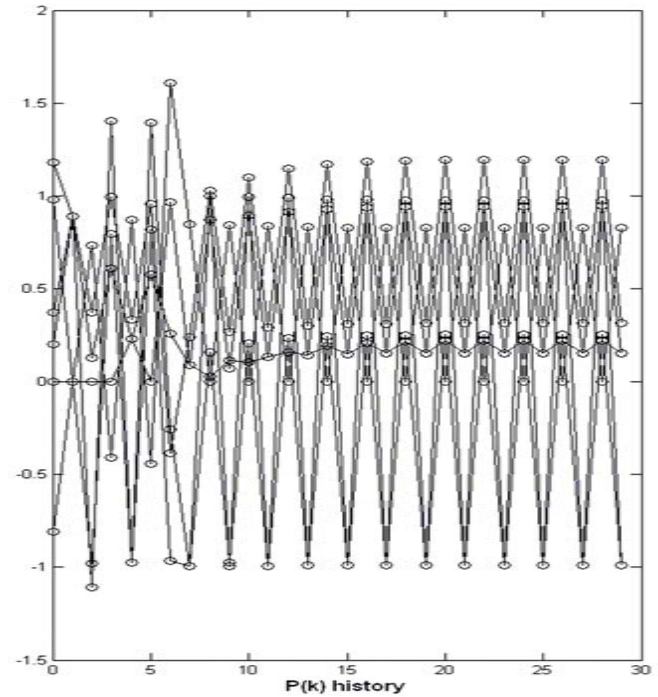
Solution based on tdCLFs





Trajectory-dependent CLFs can cope with non-trivial stabilization problems in hybrid systems

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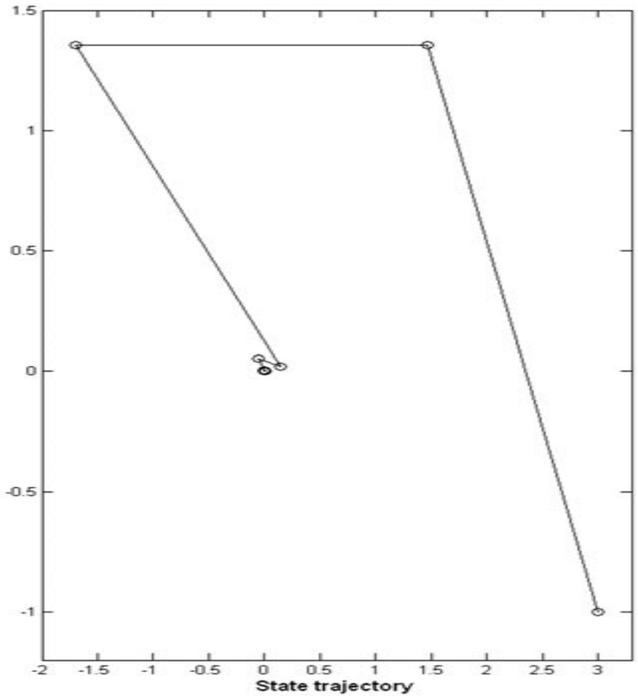
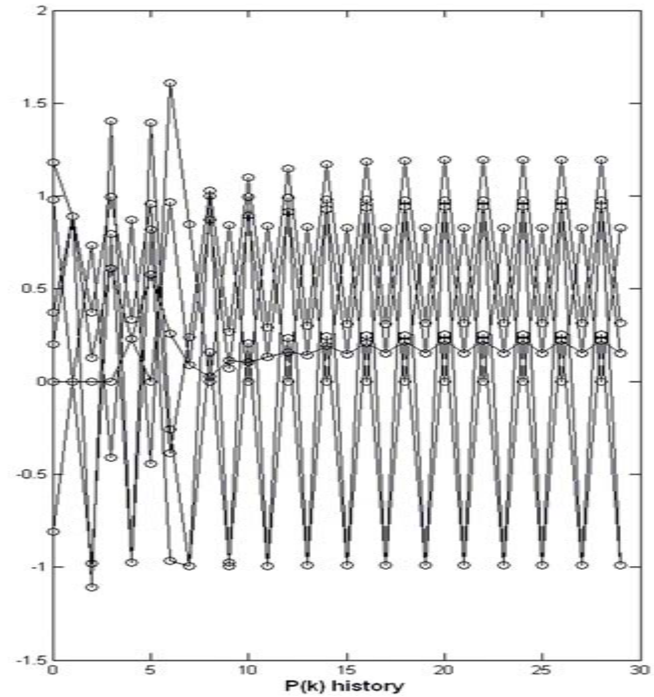
Solution based on tdCLFs





Concluding remark:

Flexible/trajectory-dependent CLFs do not necessarily imply existence of a global CLF in the classical sense

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Input-to-state stability

System: $x(k+1) \in \Phi(x(k), w(k)); \quad k \in \mathbb{Z}_+$

Input-to-State Stability (ISS)

(Sontag, 1989, 1990), (Jiang and Wang, 2001)

$$\|x(k)\| \leq \beta(\|x(0)\|, k) + \gamma(\|\mathbf{w}_{[k-1]}\|), \quad \beta \in \mathcal{KL}, \ \gamma \in \mathcal{K}$$

ISS gain

Sufficient conditions for Lyapunov stability and ISS:

$$\alpha_1(\|x\|) \leq V(x) \leq \alpha_2(\|x\|)$$

$$V(x^+) - V(x) \leq -\alpha_3(\|x\|) + \sigma(\|w\|)$$

for all $x^+ \in \Phi(x, w)$, $\forall x, w$

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Inherent input-to-state stability

$$x(k+1) = \phi(x(k), u(k), w(k)), \; x(k) \in \mathbb{X}, \; u(k) \in \mathbb{U}, \; w(k) \in \mathbb{W}$$
$$= f(x(k), u(k)) + g(x(k))w(k), \quad k \in \mathbb{Z}_+$$

$\mathbb{X}$ compact, $V(\cdot)$ continuous on $\mathbb{X}$ for all $u \in \mathbb{U}$, $\|g(x)\| \leq M$

$$|V(\phi(x,u,w)) - V(\phi(x,u,0))| = |V(f(x,u) + g(x)w) - V(f(x,u))|$$
$$\leq \sigma_V(M\|w\|) =: \sigma(\|w\|)$$

$$V(\phi(x,u,0)) - V(x) \leq -\alpha_3(\|x\|)$$

$$\Downarrow$$

$$V(\phi(x,u,w)) - V(x) \leq -\alpha_3(\|x\|) + \sigma(\|w\|), \quad \forall x \in \mathbb{X}, \; \forall w \in \mathbb{W}$$

Then $x(k+1) \in \phi_{\text{cl}}(x(k), \pi_0(x(k)), w(k))$ is ISS$(\mathbb{X}, \mathbb{W})$

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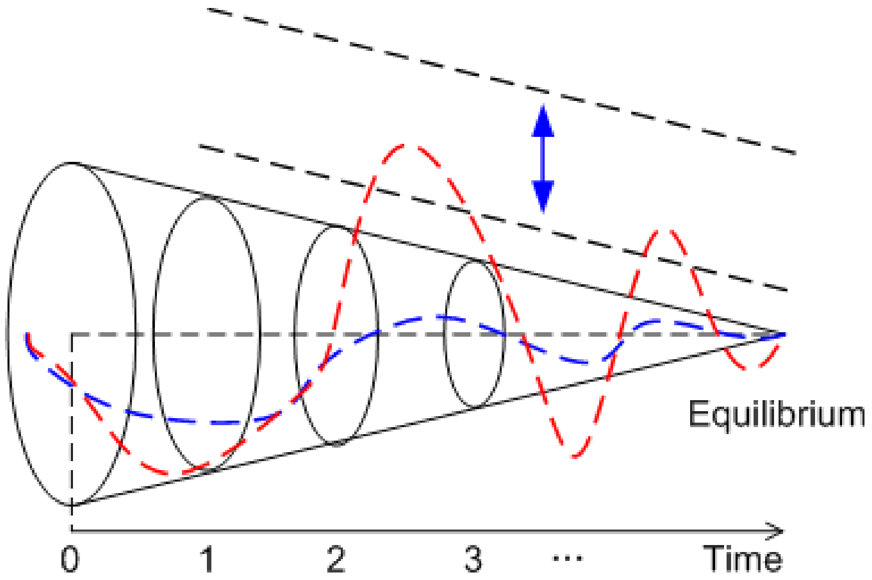
Feasibility versus performance



Constant closed-loop ISS gain

Rigid design!

$$V(\phi(x,u,0)) - V(x) \leq -\alpha_3(\|x\|)$$

$$V(\phi(x,u,w)) - V(x) \leq -\alpha_3(\|x\|) + \sigma(\|w\|), \quad \forall x \in \mathbb{X}, \; \forall w \in \mathbb{W}$$

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Optimized input-to-state stabilization

$$x(k+1) = \phi(x(k), u(k), w(k)),\ x(k) \in \mathbb{X},\ u(k) \in \mathbb{U},\ w(k) \in \mathbb{W}$$
$$= f(x(k), u(k)) + g(x(k))w(k), \quad k \in \mathbb{Z}_+$$

At time $k \in \mathbb{Z}_+$ measure $x(k)$, find $u(k)$ and minimize the gain of σ ($\sigma(s) = \eta(k)s^\lambda$) such that

$u(k) \in \mathbb{U},\ \phi(x(k), u(k), w(k)) \in \mathbb{X}$

Flexible design!

$$V(\phi(x(k), u(k), w(k))) - V(x(k)) + \alpha_3(\|x(k)\|) - \sigma(\|w(k)\|) \leq 0$$

$\forall w(k) \in \mathbb{W}$ (Lazar and Heemels, ACC 2008)

ISS inequality - not convex in w

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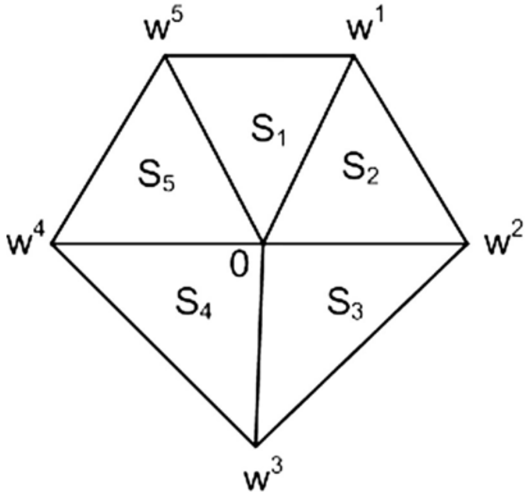
Exploiting the input bounds

$\mathbb{W}$ is a polyhedron with a non-empty interior containing the origin

$w^e,\ e = 1, \ldots, E$ are its vertices



Simplicial partition: $S_1, \ldots, S_M,\ \cup_{i=1}^{M} S_i = \mathbb{W}$

$S_i = \text{co}\{0, w^{e_{i,1}}, \ldots, w^{e_{i,l}}\}$

$\{w^{e_{i,1}}, \ldots, w^{e_{i,l}}\} \subseteq \{w^1, \ldots, w^E\}$

Example in the figure: $S_1, S_2, \ldots, S_5,$ where the simplex S_3 is generated by $0, w^{e_{3,1}}, w^{e_{3,2}}$ $(e_{3,1} = 3,\ e_{3,2} = 2)$

For each simplex S_i, $\quad W_i := [w^{e_{i,1}}\ \ldots\ w^{e_{i,l}}]$

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Finite number of inequalities

Associate with each w^e an optimization variable $\lambda_e(k)$, $k \in \mathbb{Z}_+$

$V(\cdot)$ is convex and continuous

For known $x(k)$ suppose that $u(k), \lambda_1(k), \ldots, \lambda_E(k)$ satisfy:

$$V(\phi(x(k), u(k), 0)) - V(x(k)) + \alpha_3(\|x(k)\|) \leq 0$$

$$V(\phi(x(k), u(k), w^e)) - V(x(k)) + \alpha_3(\|x(k)\|) - \lambda_e(k) \leq 0$$

for all $e = 1, \ldots, E$

$$\bar{\lambda}_i(k)[\mu_1 \ \ldots \ \mu_l]^\top = \bar{\lambda}_i(k) W_i^{-1} w(k)$$

Then:

$$V(\phi(x(k), u(k), w(k))) - V(x(k)) + \alpha_3(\|x(k)\|) - \sigma(\|w(k)\|) \leq 0$$

with $\sigma(s) = \eta(k)s$ and $\eta(k) := \max_{i=1,\ldots,M} \|\bar{\lambda}_i(k) W_i^{-1}\|$

where $\bar{\lambda}_i(k) := [\lambda_{e_{i,1}}(k) \ \ldots \ \lambda_{e_{i,l}}(k)] \in \mathbb{R}^{1 \times l}$

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Finite dimensional problem

$J : \mathbb{R}^E \rightarrow \mathbb{R}_+, \quad \alpha_4(\|\bar{\lambda}\|) \leq J(\lambda_1, \ldots, \lambda_E) \leq \alpha_5(\|\bar{\lambda}\|)$

At time $k \in \mathbb{Z}_+$ measure $x(k)$,

and minimize $J(\lambda_1(k), \ldots, \lambda_E(k))$ subject to

$u(k) \in \mathbb{U}, \ \lambda_e(k) \geq 0, \ f(x(k), u(k)) \in \mathbb{X} \sim \mathbb{W}_{x(k)}$

Inherent ISS

$V(\phi(x(k), u(k), 0)) - V(x(k)) + \alpha_3(\|x(k)\|) \leq 0$

$V(\phi(x(k), u(k), w^e)) - V(x(k)) + \alpha_3(\|x(k)\|) - \lambda_e(k) \leq 0$

for all $e = 1, \ldots, E$

Always feasible

Let $\pi(x(k))$ be the set of feasible control actions

Then $x(k+1) \in \phi_{\text{cl}}(x(k), \pi(x(k)), w(k))$ is ISS$(\mathbb{X}, \mathbb{W})$

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Finite dimensional problem

$J:\mathbb{R}^E \to \mathbb{R}_+, \quad \alpha_4(\|\bar{\lambda}\|) \leq J(\lambda_1,\ldots,\lambda_E) \leq \alpha_5(\|\bar{\lambda}\|)$

At time $k \in \mathbb{Z}_+$ measure $x(k)$,

and minimize $J(\lambda_1(k),\ldots,\lambda_E(k))$ subject to

$u(k) \in \mathbb{U},\ \lambda_e(k) \geq 0,\ f(x(k),u(k)) \in \mathbb{X} \sim \mathbb{W}_{x(k)}$

$V(\phi(x(k),u(k),0)) - V(x(k)) + \alpha_3(\|x(k)\|) \leq 0$

$V(\phi(x(k),u(k),w^e)) - V(x(k)) + \alpha_3(\|x(k)\|) - \lambda_e(k) \leq 0$

for all $e = 1,\ldots,E$

$$\lambda^* := \max_{x \in \mathrm{cl}(\mathbb{X}),\, u \in \mathrm{cl}(\mathbb{U}),\, e=1,\ldots,E} \{V(\phi(x,u,w^e)) - V(x) + \alpha_3(\|x\|)\}$$

The developed optimization problem, although it inherently guarantees a constant ISS gain, it provides freedom to optimize the ISS gain of the closed-loop system

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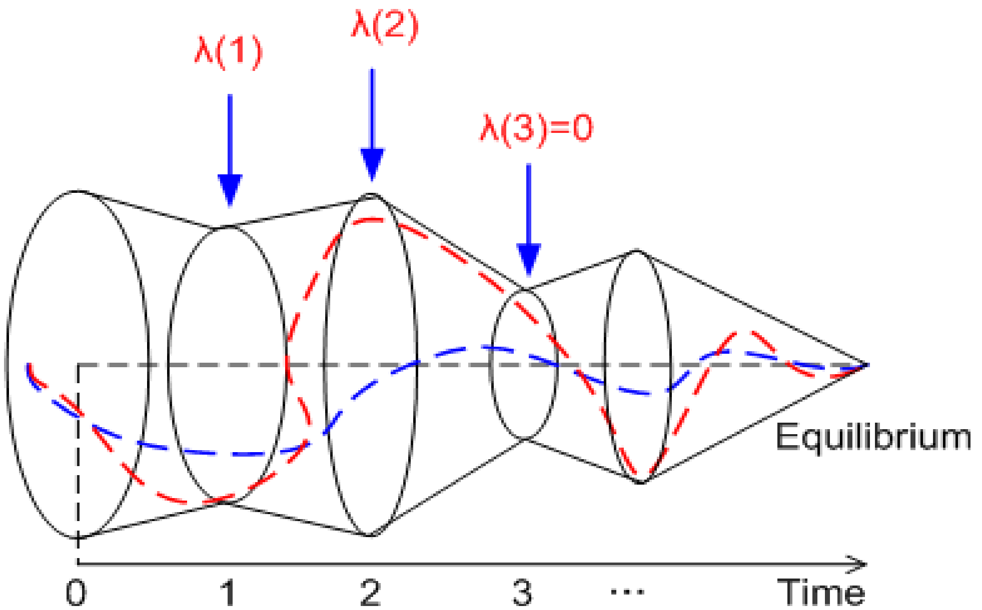
Optimized ISS - Flexible!



Optimized closed-loop ISS gain

The trade-off between feasibility and disturbance attenuation is optimized on-line

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A single linear program

$$\begin{aligned} x(k+1) &= \phi(x(k), u(k), w(k)),\ x(k) \in \mathbb{X},\ u(k) \in \mathbb{U},\ w(k) \in \mathbb{W} \\ &= f(x(k), u(k)) + g(x(k))w(k) \\ &= f_1(x(k)) + f_2(x(k))u(k) + g(x(k))w(k) \end{aligned}$$

$V(x) = \|Px\|_\infty$

Infinity norms as Lyapunov functions (Kiendl, 1992)

(Polanski, 1997; Bemporad, 2000; Lazar, 2006; Christophersen, 2007)

$\|x\|_\infty := \max_{i \in \mathbb{Z}_{[1,n]}} |[x]_i|, \ \ \|x\|_\infty \leq c \Leftrightarrow \pm[x]_i \leq c, \forall i \in \mathbb{Z}_{[1,n]}$

$$\pm [P(f_1(x(k)) + f_2(x(k))u(k))]_i - V(x(k)) + \alpha_3(\|x(k)\|) \leq 0,$$

$$\pm [P(f_1(x(k)) + f_2(x(k))u(k) + g(x(k))w^e)]_i - V(x(k)) + \alpha_3(\|x(k)\|) - \lambda_e(k) \leq 0,$$

$\forall i \in \mathbf{Z}_{[1,p]}, \ \ e = 1, \ldots, E$

$2p(E+1)$ linear inequalities

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Still a single LP or QP

MPC cost for performance:

$$J_{\mathsf{RHC}}(x(k),\bar{\mathbf{u}}(k),\bar{\lambda}(k)) := F(\bar{x}(k+N)) + \sum_{i=0}^{N-1} L(\bar{x}(k+i),\bar{u}(k+i)) + J(\bar{\lambda}(k))$$

Lower complexity for "any" model class: $(V(x) = \|Px\|_\infty)$

1a. $N = 1:$ **Quadratic costs, nonlinear model** $\Rightarrow$ **QP**

1b. $N = 1:$ **Infinity norm costs, nonlinear model** $\Rightarrow$ **LP**

2a. $N \in \mathbb{Z}_{\geq 1}:$ **Linear model** $\Rightarrow$ **QP or LP**

2b. $N \in \mathbb{Z}_{\geq 1}:$ **PWA model** $\Rightarrow$ **MIQP or MILP**

2c. $N \in \mathbb{Z}_{\geq 1}:$ **Nonlinear model** $\Rightarrow$ **Linear stability constraints**

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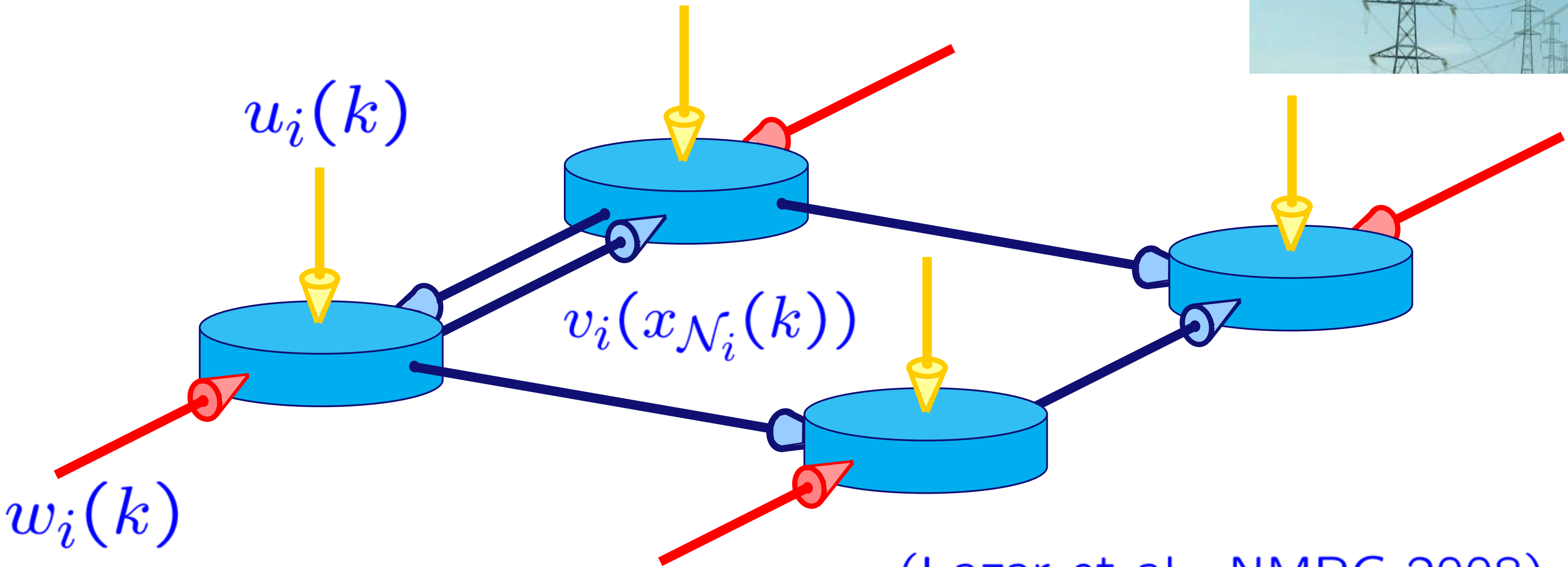
Decentralized formulation

Dynamically coupled nonlinear systems



(Lazar et al., NMPC 2008)

$$x_i(k+1) = \phi_i(x_i(k), u_i(k), v_i(x_{\mathcal{N}_i}(k)), w_i(k)), \quad k \in \mathbb{Z}_+$$

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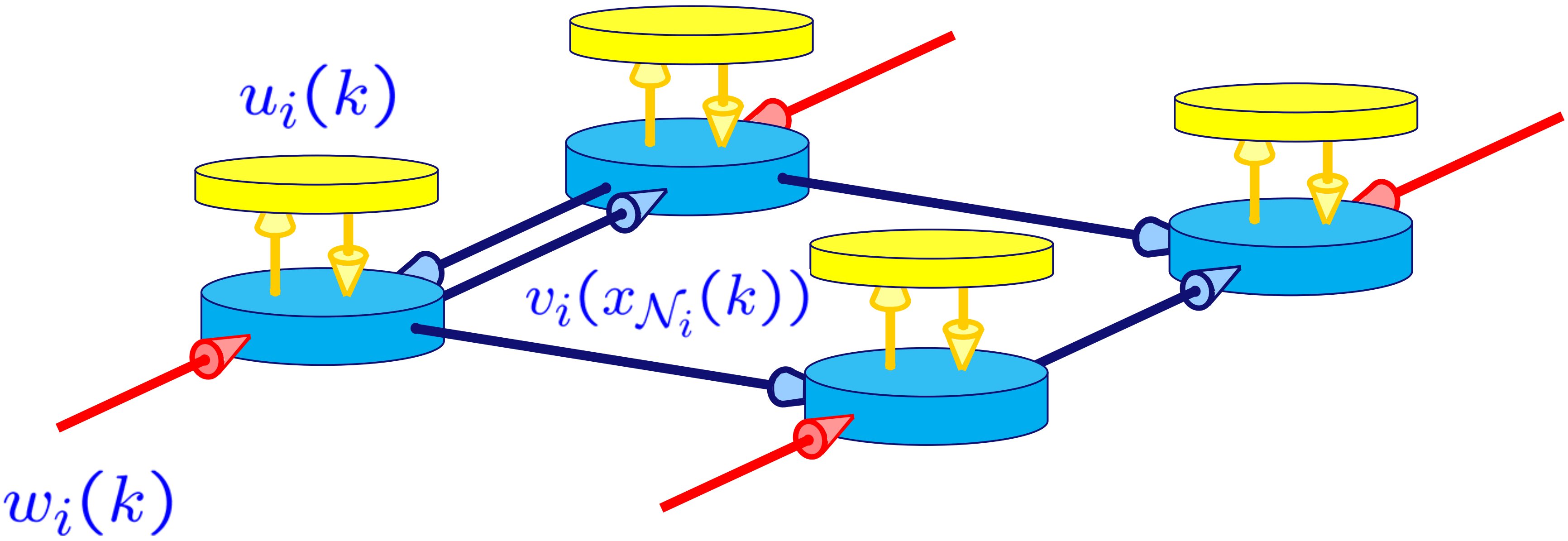
Decentralized implementation



$$u_i(k) \in \mathbb{U},\ \lambda_e^i(k) \geq 0,\ \phi_i(x_i(k), u_i(k), v_i(x_{\mathcal{N}_i}(k)), 0) \in \mathbb{X}_i \sim \mathbb{W}_{x_i(k)}$$

$$V_i(\phi_i(x_i(k), u_i(k), v_i(x_{\mathcal{N}_i}(k)), 0)) - V_i(x_i(k)) + \alpha_3^i(\|x_i(k)\|) \leq 0$$

$$V_i(\phi_i(x_i(k), u_i(k), v_i(x_{\mathcal{N}_i}(k)), w_i^e)) - V_i(x_i(k)) + \alpha_3^i(\|x_i(k)\|) - \lambda_e^i(k) \leq 0,$$

$$\forall e = \overline{1, E_i}$$

Decentralized feasibility implies global ISS

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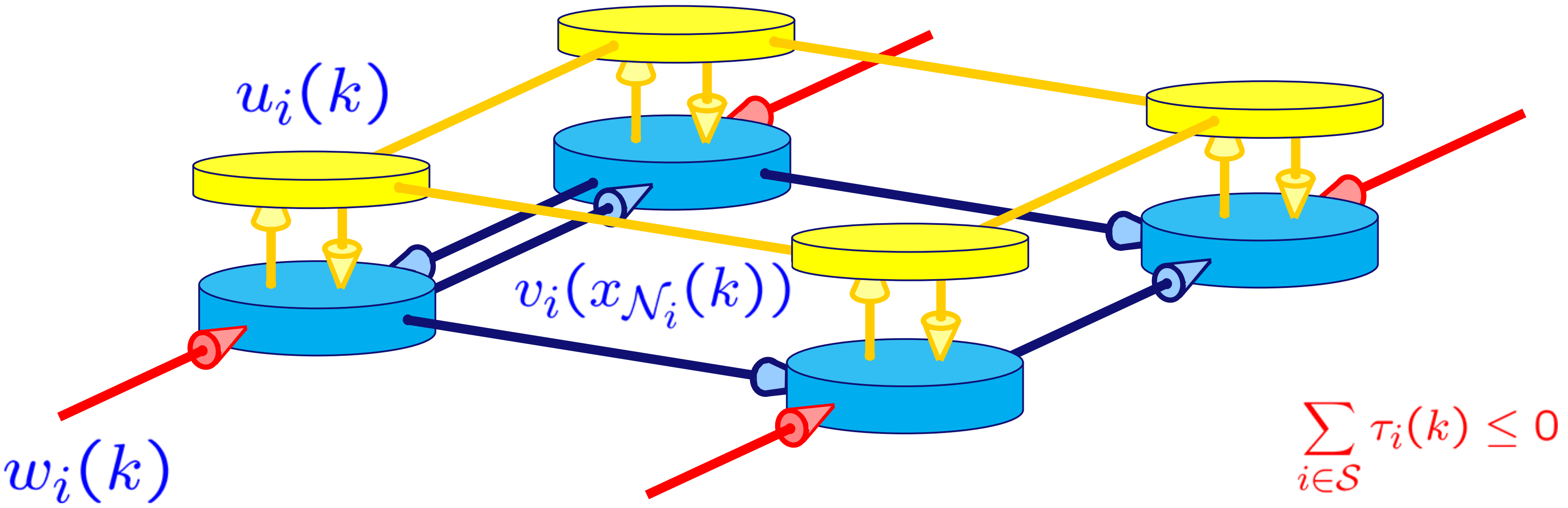
Decentralized implementation



$u_i(k) \in \mathbb{U},\ \lambda_e^i(k) \geq 0,\ \phi_i(x_i(k), u_i(k), v_i(x_{\mathcal{N}_i}(k)), 0) \in \mathbb{X}_i \sim \mathbb{W}_{x_i(k)}$

$V_i(\phi_i(x_i(k), u_i(k), v_i(x_{\mathcal{N}_i}(k)), 0)) - V_i(x_i(k)) + \alpha_3^i(\|x_i(k)\|) \leq \tau_i(k)$

$V_i(\phi_i(x_i(k), u_i(k), v_i(x_{\mathcal{N}_i}(k)), w_i^e)) - V_i(x_i(k)) + \alpha_3^i(\|x_i(k)\|) - \lambda_e^i(k) \leq 0,$

$\forall e = \overline{1, E_i}$

Further relaxations - distributed MPC

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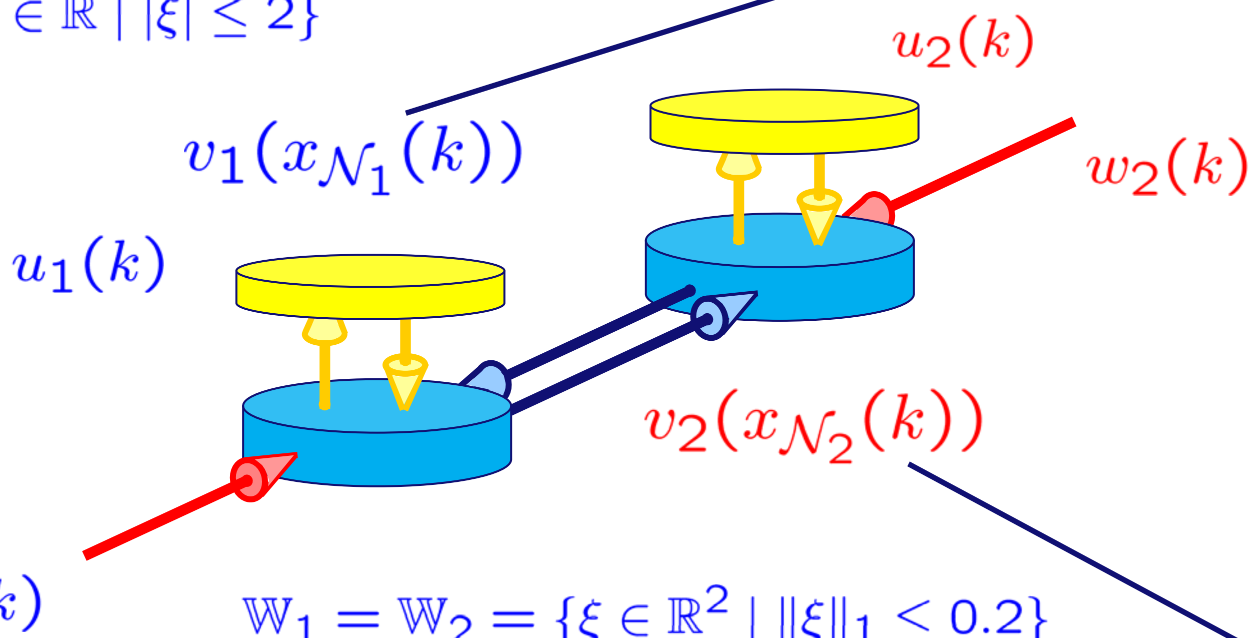
Case study: system dynamics

$$\phi_1(x_1, u_1, v_1(x_{\mathcal{N}_1}), w_1) := \begin{bmatrix} 1 & 0.7 \\ 0 & 1 \end{bmatrix} x_1 + \begin{bmatrix} \sin([x_1]_2) \\ 0 \end{bmatrix} + \begin{bmatrix} 0.245 \\ 0.7 \end{bmatrix} u_1 + \begin{bmatrix} 0 \\ ([x_2]_1)^2 \end{bmatrix} + w_1$$

$$\mathbb{X}_1 = \mathbb{X}_2 = \{\xi \in \mathbb{R}^2 \mid \|\xi\|_\infty \leq 5\}$$

$$\mathbb{U}_1 = \mathbb{U}_2 = \{\xi \in \mathbb{R} \mid |\xi| \leq 2\}$$



$$\mathbb{W}_1 = \mathbb{W}_2 = \{\xi \in \mathbb{R}^2 \mid \|\xi\|_1 \leq 0.2\}$$

$$\phi_2(x_2, u_2, v_2(x_{\mathcal{N}_2}), w_2) := \begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix} x_2 + \begin{bmatrix} \sin([x_2]_2) \\ 0 \end{bmatrix} + \begin{bmatrix} 0.125 \\ 0.5 \end{bmatrix} u_2 + \begin{bmatrix} 0 \\ [x_1]_1 \end{bmatrix} + w_2$$

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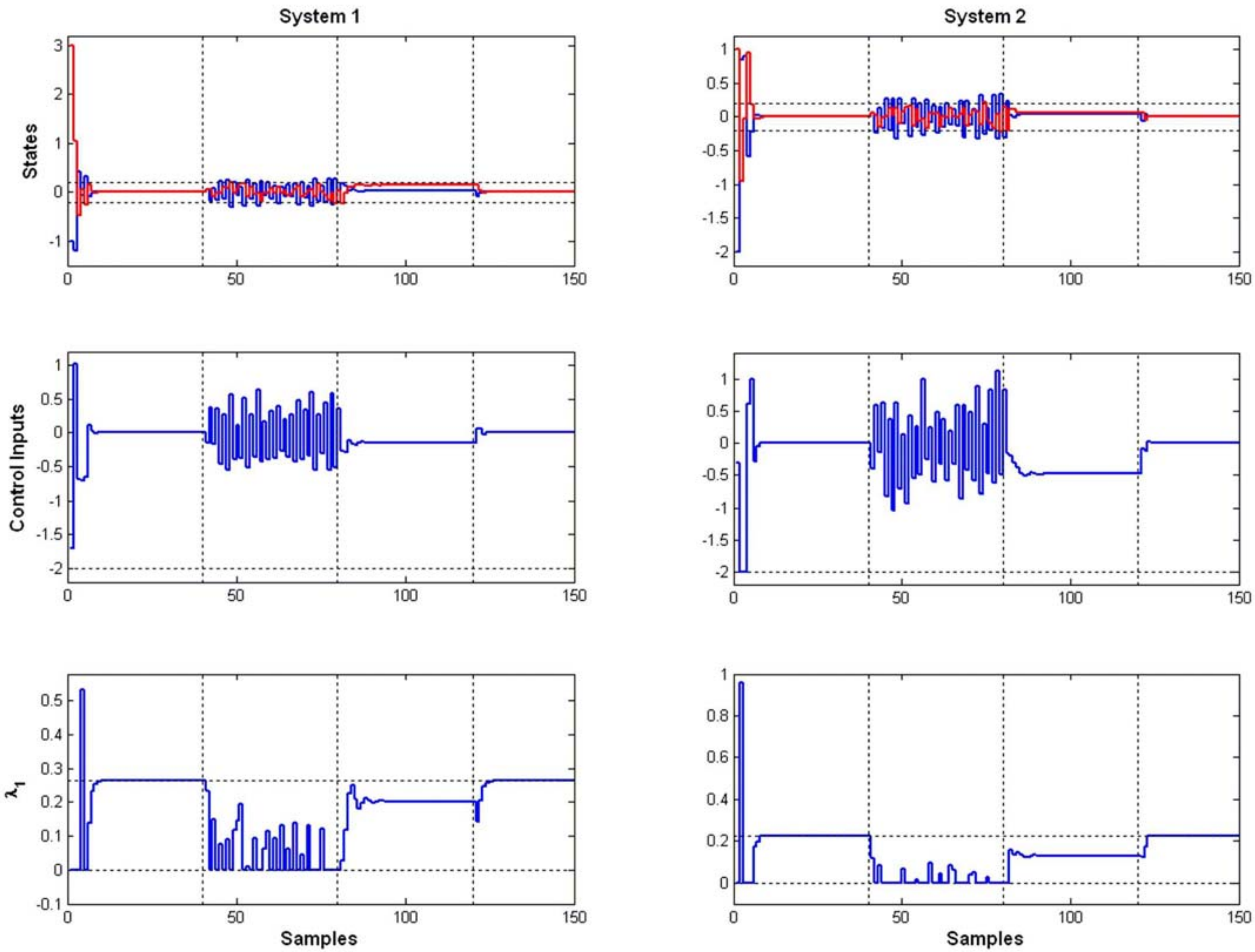
Case study: simulation results



Worst case CPU time: 5 milliseconds

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Concluding remarks: Flexible CLFs

- **Theoretically appealing:**

  - offer “least conservative” synthesis methods

  - strong guarantees of stability and “optimized” ISS

- **Suitable for real-time control:**

  - Flexible CLFs = improved feasibility

  - low complexity: a single LP

  - applications in mechatronics and power electronics

- **Great potential for decentralized control:**

  - applications in electricity networks

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