Tube Model Predictive Control Using Homothety & Invariance

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Outlook

- Set Invariance – Basic Facts §1
- Invariant Tubes – General Idea §2
- Linear Time Invariant Systems §3
  - Rigid Invariant Tubes §3.1
  - Homothetic Invariant Tubes §3.2
- Extensions §4
  - Linear Difference Inclusions §4.1
  - Matched Nonlinearities §4.2
  - Piecewise Affine Systems §4.3
- Conclusions §5

Problem Formulation

Discrete Time, Time-Invariant, System

$$x^+ = f(x, u, w)$$

The state variable is $x \in R^n$
The control is $u \in R^m$
The disturbance is $w \in R^p$

System variables are constrained by $$(x, u, w) \in X \times U \times W$$

The control $u \in U$ is chosen by ***Controller***
The disturbance $w \in W$ is chosen by ***Adversary***

Controller **is concerned with:**

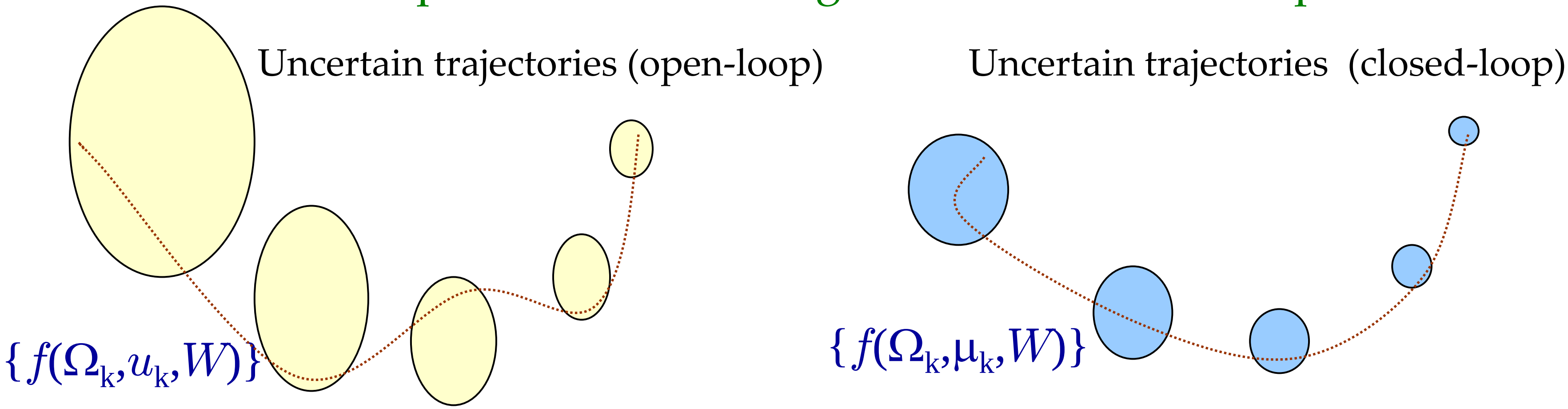
Adversary **is ``opposite'' to** ***Controller***

- **Robust constraint satisfaction from now to infinity,**
- **Robust stability (and attractivity) of an adequate set,**
- **``Optimized'' performance with respect to a cost function.**

Preliminary Facts

``Sufficiently sophisticated'' *Controller* should be aware of:

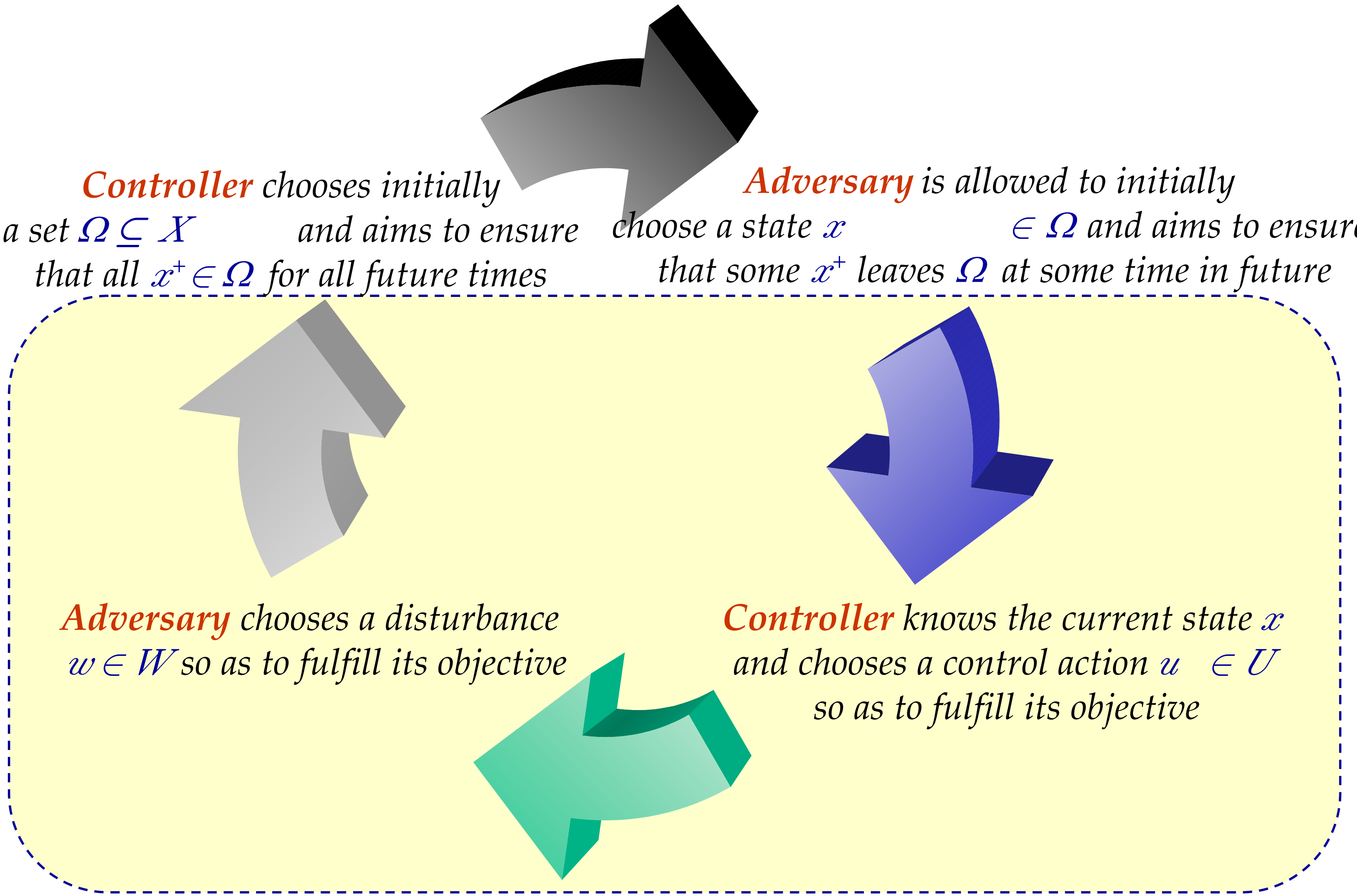
- Adversaries presence (interaction of uncertainty with cost and constraints)
- The fact that the problem he is facing is not an ``one-shot'' problem



Controller also knows that he can devise his strategy within:

- Class of open-loop control rules (sequences of control actions)
- Class of closed-loop control rules (sequences of control laws)

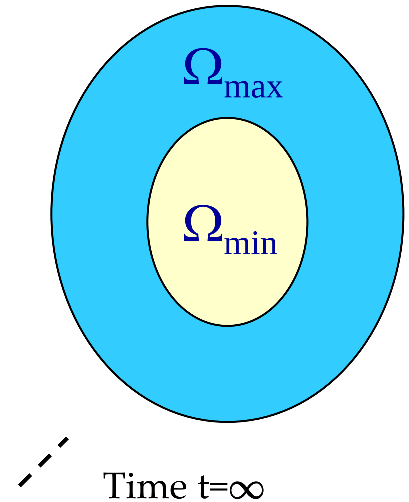
Control Synthesis: A Simple Desirable Scenario

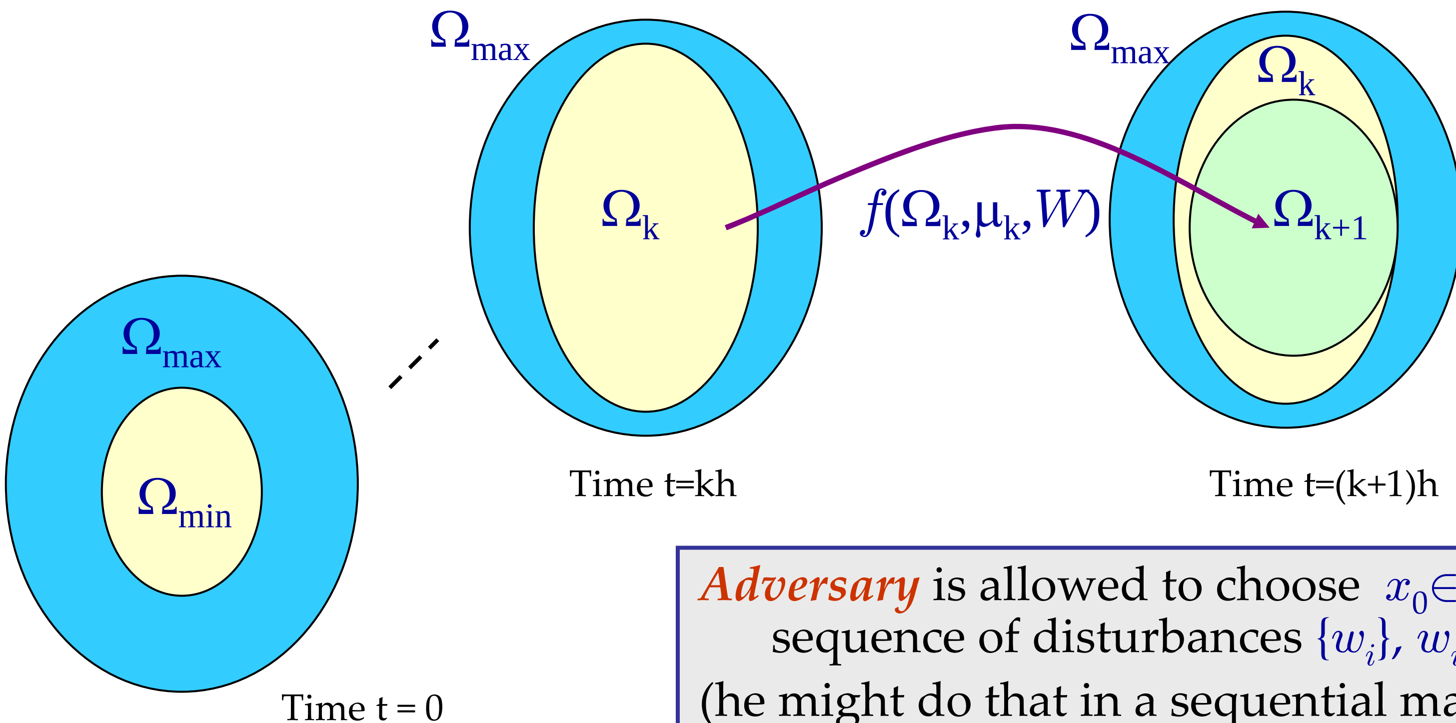


Control Synthesis: A More Preferable Scenario

Controller identifies two sets Ω_{min} and Ω_{max} such that $\Omega_{min} \subseteq \Omega_{max} \subseteq X$ and chooses a sequence of control laws $\{\mu_i(\cdot)\}$ (he might do that sequentially) so that

$f(\Omega_k, \mu_k, W) \subseteq \Omega_{k+1} \subseteq \Omega_k$, $\mu_k(\Omega_k) \subseteq U$ for all k and $\Omega_\infty \rightarrow \Omega_{min}$ and performance is optimized with respect to a cost function



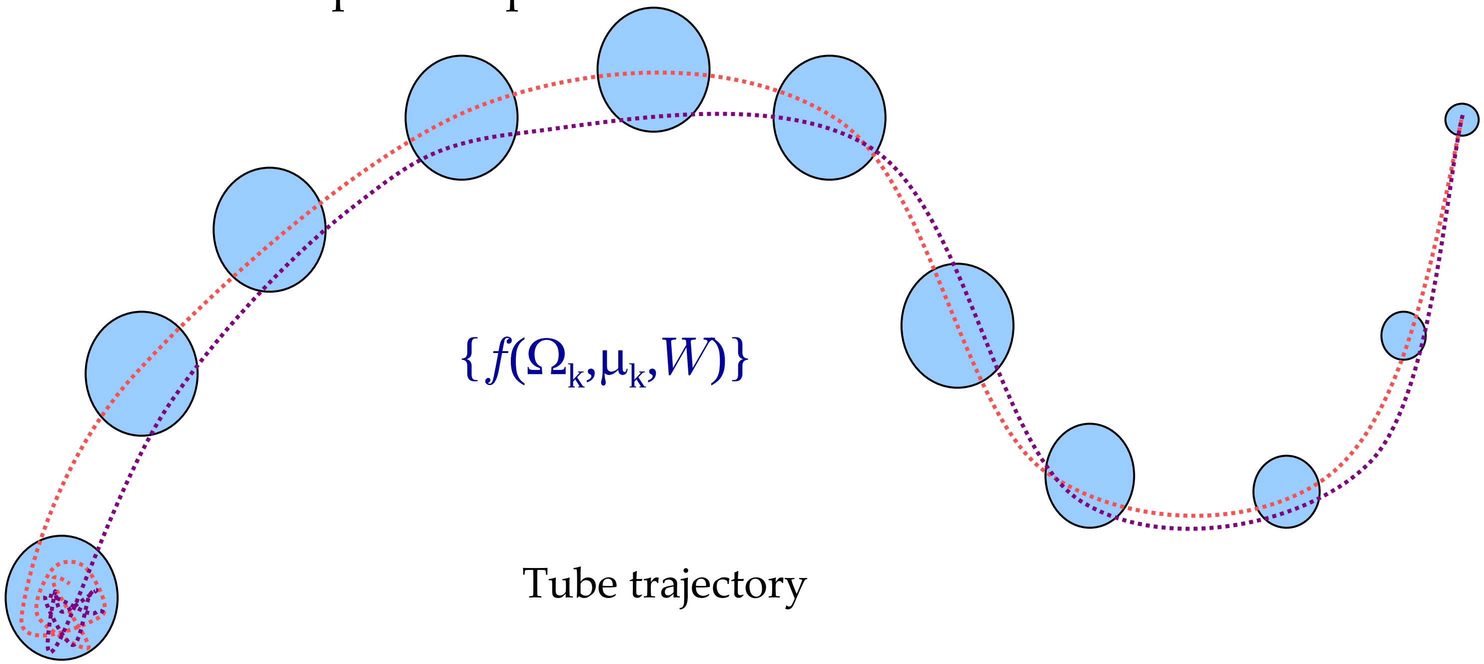


Adversary is allowed to choose $x_0 \in \Omega_{max}$ and a sequence of disturbances $\{w_i\}$, $w_i \in W$ (he might do that in a sequential manner)

Control Synthesis: Controllers Tools

Controller decides to use:

- Set Invariance to ensure:
  - Robust Constraint Satisfaction and Recursive Feasibility,
  - Robust Stability (and Attractivity) of an adequate set.
- Model Predictive Control to optimize performance in some sense.



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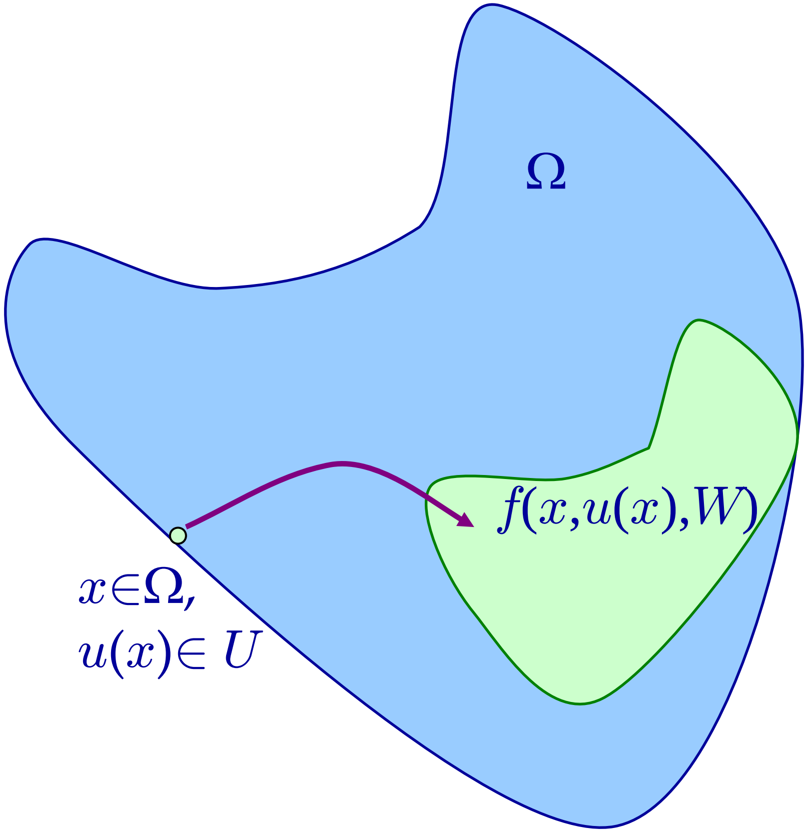
Set Invariance : Questions and Facts

When a set is invariant?



A set $\Omega \subseteq R^n$ is an invariant set for system $x^+=f(x,u,w)$ and constraint set (X,U,W) if and only if

1) $\Omega \subseteq X$ and

2) for all $x\in\Omega$ there exists a $u\in U$ such that $f(x,u,w)\in\Omega$ for all $w\in W$

Set Invariance : Questions and Facts

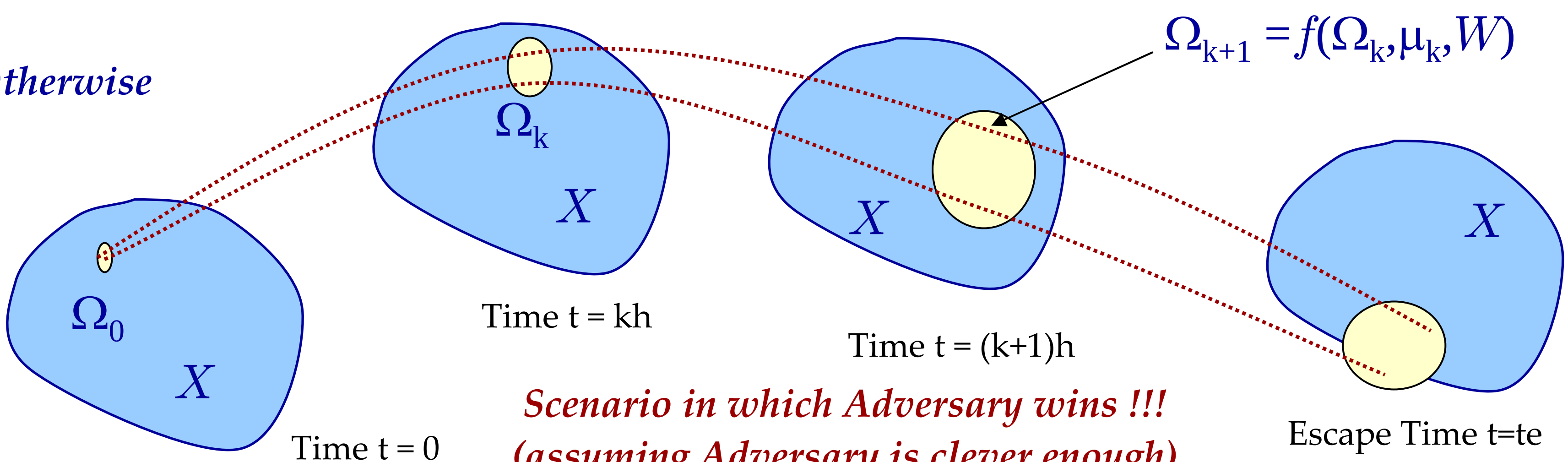
Why be bothered with invariant sets?

Fundamental Fact:

Control of uncertain (and deterministic), constrained discrete time systems makes sense if and only if

- there exists an invariant set and
- initial condition (which could be a set) belongs to such set !

Otherwise



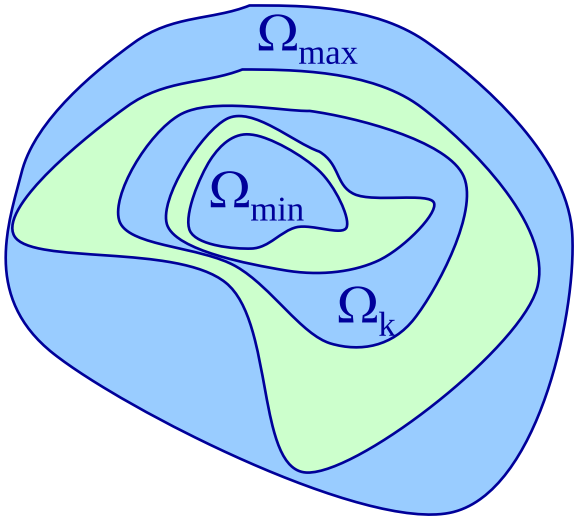
Scenario in which Adversary wins !!!
(assuming Adversary is clever enough)

Set Invariance : Questions and Facts

What are important invariant sets?

Any invariant set, but there are two special invariant sets (under suitable assumptions).

- The maximal invariant set (well understood, solid theory)
- The minimal invariant set (theory emerged only recently!)



$\Omega_0 := \Omega_{max}$

$\{\Omega_k\},\ \Omega_{k+1} = f(\Omega_k, \mu_k, W)$

$\Omega_k \subseteq X,\ \mu_k(\Omega_k) \subseteq U$

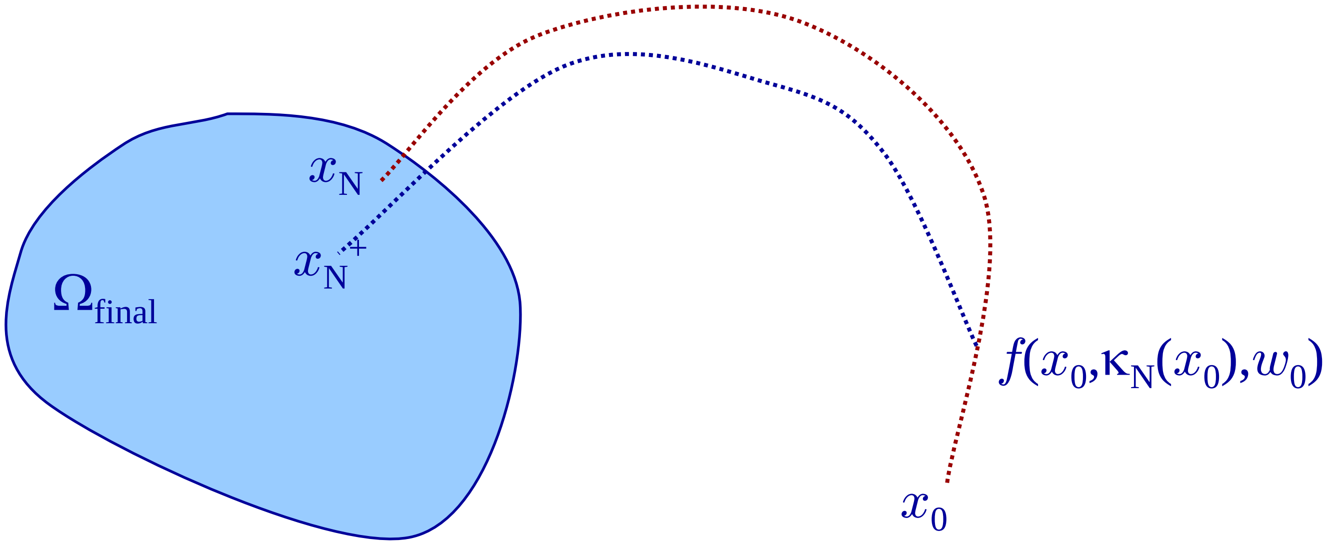
$\Omega_k \rightarrow \Omega_{min}$ as $k \rightarrow \infty$

Set Invariance & Model Predictive Control

What invariant sets do for model predictive control?

Invariant sets are used in model predictive control in order to:

- Ensure recursive feasibility,
- Guarantee *a-priori* stability and attractivity,
- Obtain quantitative information of the domain of attraction,
- Design Robust MPC via Invariant Tubes



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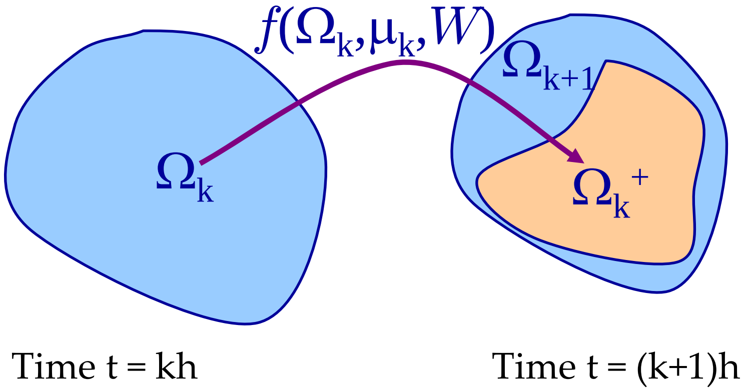
Invariant Tubes

What are invariant tubes?



$$\Omega_k^{+} \subseteq \Omega_{k+1},\ \mu_k(\Omega_k) \subseteq U$$

$$\Omega_k^{+} := \{ f(x,\mu_k(x),w) :\ (x,w) \in \Omega_k \times W \},$$
$$\mu_k(\Omega_k) := \{ \mu_k(x) :\ x \in \Omega_k \}.$$

Observe that tube dynamics are different

A set sequence $\{\Omega_k\}$, $\Omega_k \subseteq R^n$ is an invariant tube for system $x^{+}=f(x,u,w)$ and constraint set (X,U,W) if and only if

1) $\Omega_k \subseteq X$ and

2) for all $x \in \Omega_k$ there exists a $u \in U$ such that $f(x,u,w) \in \Omega_{k+1}$ for all $w \in W$

Invariant Tubes

Why be bothered with invariant tubes?

Fact I Invariant tubes are related to dynamic programming

Dynamic programming recursion for sets and corresponding set-valued feedbacks (equation for the value function is omitted) is:

$$X_k := \{x \in X : \exists\, u \in U \text{ such that } f(x,u,w) \in X_{k-1} \text{ for all } w \in W\},$$
$$\Theta_k(x) := \{u \in U : f(x,u,w) \in X_{k-1} \text{ for all } w \in W\},\ x \in X_k$$

with boundary conditions $X_0 := \Phi,\ \Theta_0(X_0) \subseteq U$

If Φ is invariant for some feedback $\theta_0(x) \in \Theta_0(x),\ x \in X_0$ we have:

1) $X_k \subseteq X_{k+1}$ and more importantly
2) There exists a pair invariant tube –control policy (sequence of control laws) $\{\Omega_k\}$ and $\{\mu_k\}$ such that additionally $\Omega_k \rightarrow X_0$ as $k \rightarrow \infty$

Hint : Invariant tube is constructed from sets $\{X_k\}$ and control policy from set-valued controllers $\{\Theta_k\}$.

Invariant Tubes

What do invariant tubes offer that DP does not?

Fact II Invariant tubes ``simplify'' dynamic programming

Invariant tubes help us:

- Not to consider exact reachable sets, exact dynamics are replaced by inclusions,
- To exploit the structure of the underplaying uncertain constrained system,
- To consider suitable parameterizations of invariant tube-control policy pairs,
- To reduce the corresponding on-line computational burden.

Potential drawbacks of invariant tubes are:

- They might be somehow conservative,
- We might have to sacrifice optimality (compared to DP).

Invariant Tubes & MPC

Idea for tube based model predictive control:

Apply truncated invariant tubes

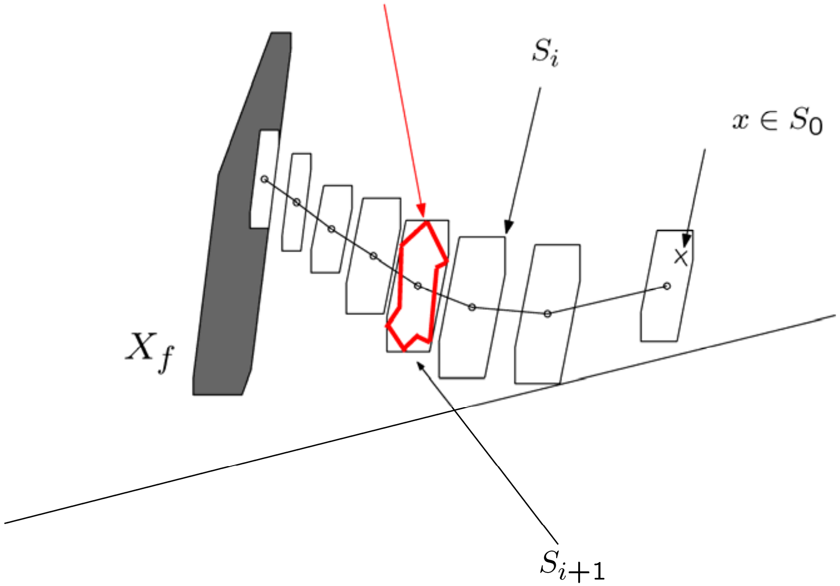
by using model predictive control

$$
\begin{aligned}
x &\in S_0, \\
S_i &\subseteq X, \quad \forall i \in \mathbf{N}_{N-1}, \\
S_N &\subseteq X_f \subseteq X, \\
U(S_i, \mu_i) &\subseteq U, \quad \forall i \in \mathbf{N}_{N-1} \\
\mathcal{F}(S_i, \mu_i, W) &\subseteq S_{i+1}, \quad \forall i \in \mathbf{N}_{N-1}
\end{aligned}
$$

where $\mathbf{N}_{N-1} := \{0, 1, \ldots, N-1\}$ and

$$
\begin{aligned}
U(S_i, \mu_i) &:= \{\mu_i(x) \ : \ x \in S_i\} \\
\mathcal{F}(S_i, \mu_i, W) &:= \{f(x, \mu_i(x), w) \ : \ (x, w) \in S_i \times W\}
\end{aligned}
$$

$$S_i^+ := \{f(y, \mu_i(y), w),\ y \in S_i,\ w \in W\} \subseteq S_{i+1}$$



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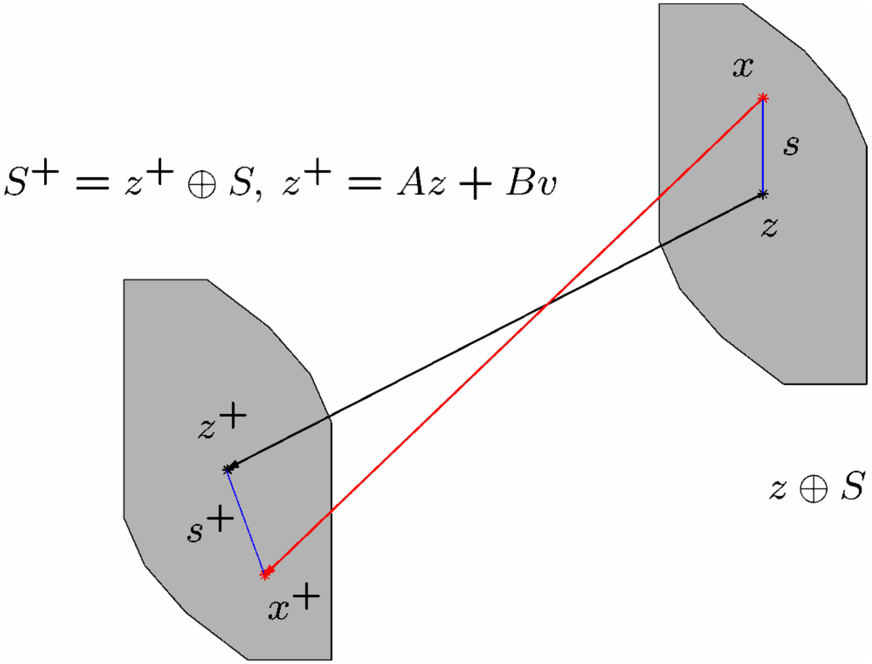
Invariant Tubes & MPC
Linear Time Invariant Systems

Linearity permits a very simple ``two-phase'' design under fairly modest assumptions

Set Dynamics Observation



If the set-control law pair $(S, \nu(\cdot))$ satisfy:

$$Ax + B\nu(x) + w \in S,\ \forall (x, w) \in S \times W$$

we have

$$Ax + B\mu(x, z, v) + w \in S^+$$

for all $(x, w) \in (z \oplus S) \times W$, where

$$\begin{aligned} S^+ &= z^+ \oplus S, \\ z^+ &= Az + Bv, \\ \mu(x, z, v) &= v + \nu(x - z) \end{aligned}$$

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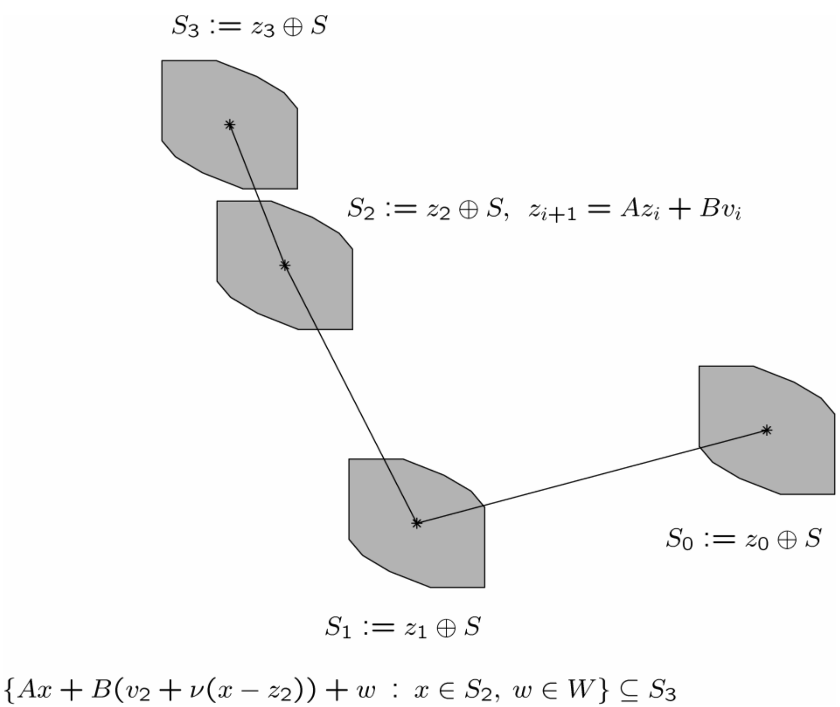
Tube Based MPC
Linear Time Invariant Systems

Simple Tube Based MPC for linear case uses set dynamics observation and results in a simple optimal control problem for on-line implementation

Extended Set Dynamics Observation



Robust MPC design requires:

Off-line (fairly straight-forward in linear case):

Construction of pair $(S, \nu(\cdot))$ such that

$$Ax + B\nu(x) + w \in S, \ \forall (x, w) \in S \times W$$

and sets Z and V such that

$$z \oplus S \subseteq X, \ \forall z \in Z$$
$$v + \nu(x) \in U, \ \forall (v, x) \in V \times S$$

On-line (fairly straight-forward in linear case):

``Standard'' MPC for deterministic linear systems with minor modifications

Tube Based MPC
Modified Problem

The resulting optimal control problem is

$\mathbb{P}_N(x)$

$$V_N^0(x) := \inf_{(z,\mathbf{v})} \{V_N(z,\mathbf{v}) \; : \; (z,\mathbf{v}) \in \mathcal{V}_N(x)\}$$
$$(z^0(x), \mathbf{v}^0(x)) \in \arg \inf_{(z,\mathbf{v})} \{V_N(z,\mathbf{v}) \; : \; (z,\mathbf{v}) \in \mathcal{V}_N(x)\}$$

where $\mathbf{v} := \{v_0, v_1, \ldots, v_{N-1}\}$, $V_N(z,\mathbf{v})$ is a cost function (selection criterion) and

$$\mathcal{V}_N(x) := \{(z,\mathbf{v}) \; : (z_i, v_i) \in Z \times V, \; z_{i+1} = Az_i + Bv_i, \; \forall i \in \mathbf{N}_{N-1}, \; z_N \in Z_f, \; z_0 := z, \; x \in z \oplus S\}$$

Under fairly modest assumptions $\mathbb{P}_N(x)$ is a strictly convex quadratic programming problem!

Tube Based MPC Controller

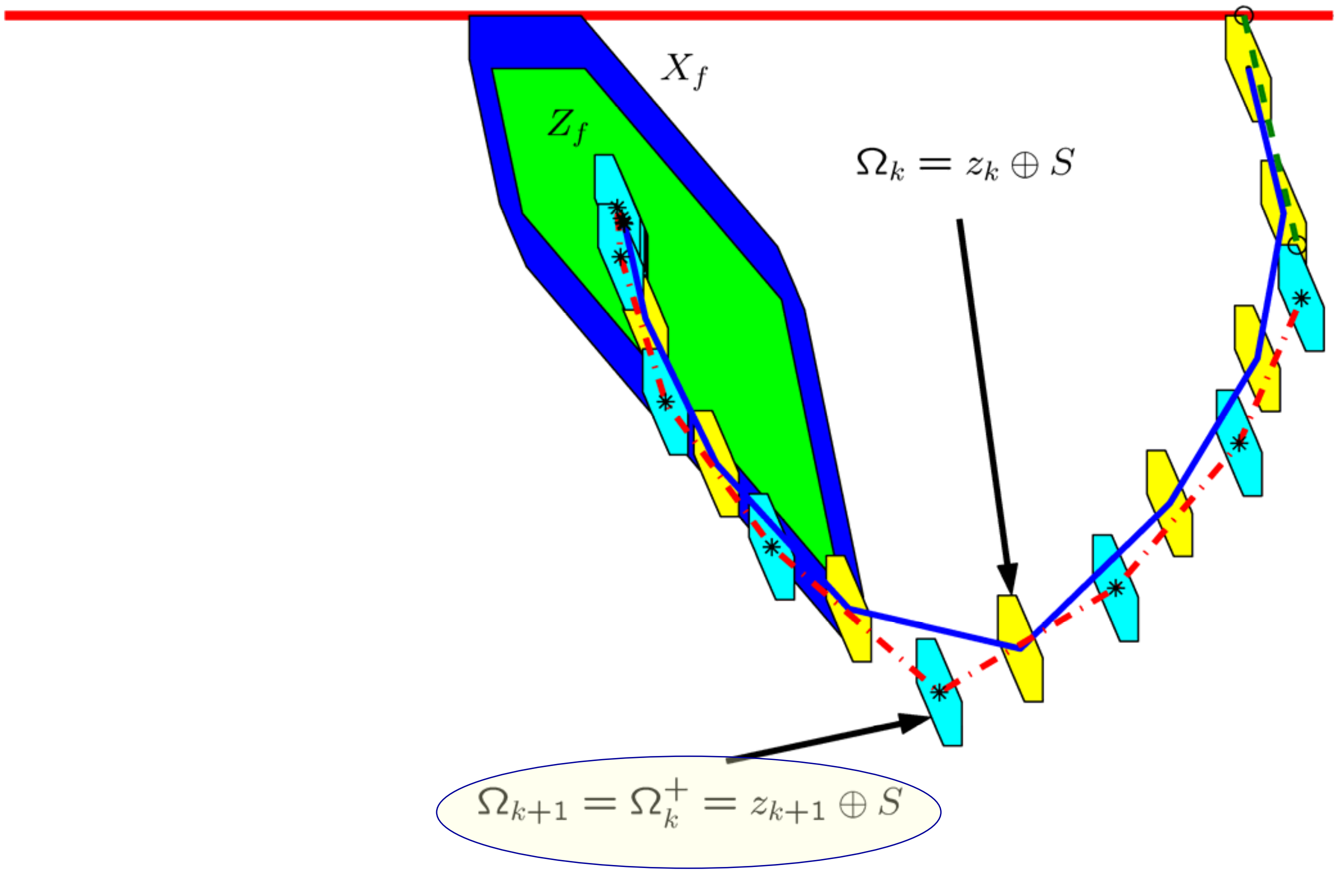
The resulting controller is $\kappa_N^0(x) := v_0^0(x) + \nu(x - z^0(x))$

The domain of attraction is $\mathcal{X}_N := \{x \, : \, \mathcal{V}_N(x) \neq \emptyset\}$

The proposed controller is computationally simple and guarantees a-priori (under standard MPC assumptions):

- Robust Constraint Satisfaction and Robust Recursive Feasibility
- Robust Stability (and Attractivity) of an adequate set S

Tube Based MPC
How does it work?



Rigid invariant tubes do not employ the full power of invariant tubes since tube centers satisfy an ``exact'' difference equation!

Rigid Invariant Tubes
Linear Time Invariant Systems

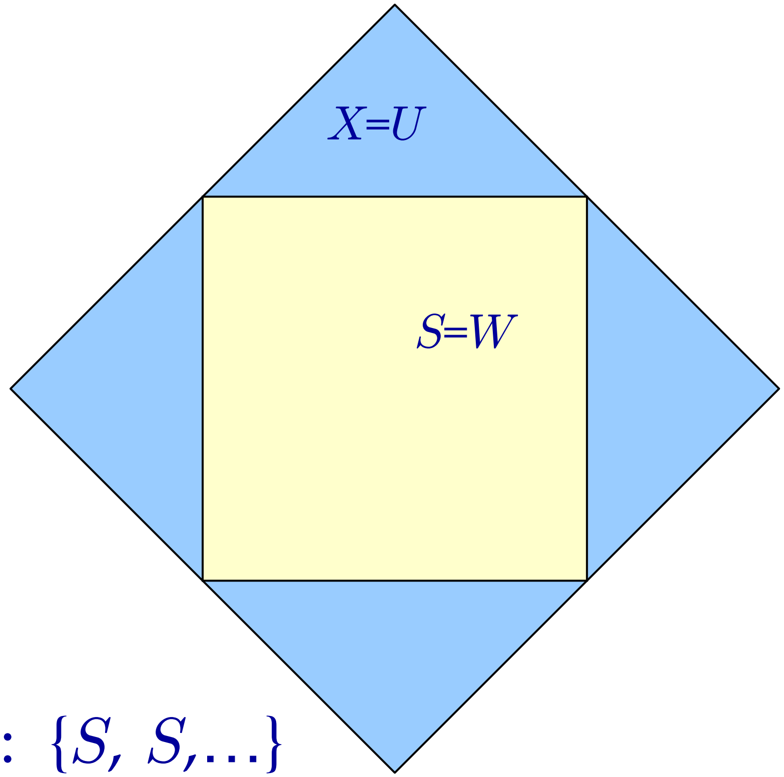
Rigid invariant tubes : sensitivity to geometry

System :

$x^+ = x + u + w$

Constraints :

$X = U = 2B_1$
$W = B_\infty$



Invariant Tube for $S=W$: $\{S, S, \ldots\}$

Time Varying Invariant Tube for $S_0=X$, $S_{i>0}=W$: $\{X, S, S, \ldots\}$

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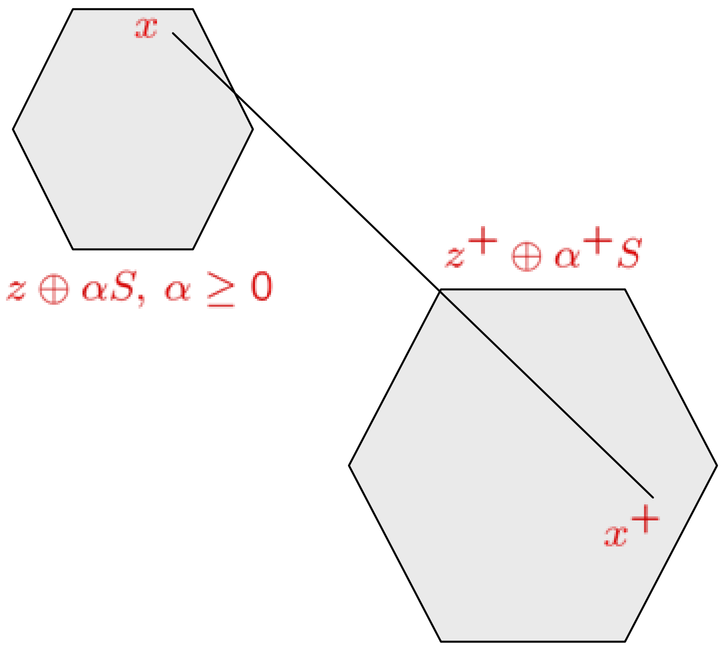
Homothetic Invariant Tubes
Linear Time Invariant Systems

Homothetic Invariant Tubes

Set Dynamics Observation - Revisited

It is not necessary to require $z^+ = Az + Bv$

nor $\alpha = \alpha^+ = 1$



If the set-control law pair $(S, \nu(\cdot))$ satisfy:

$$Ax + B\nu(x) + w \in S, \ \forall (x, w) \in S \times W$$

we have (with extra assumptions on $(S, \nu(\cdot))$ or with the use of convexification)

$$Ax + B\mu(x, z, v) + w \in S^+$$

for all $(x, w) \in (z \oplus \alpha S) \times W$, where

$$S^+ = z^+ \oplus \alpha^+ S,$$
$$\mu(x, z, v) = v + \alpha \nu(x - z)$$

for some (z^+, α^+)

Homothetic Invariant Tubes Linear Time Invariant Systems

Homothetic Invariant Tubes

Set Dynamics Observation – Revisited and Simplified

Suppose that S and W are C-set and $\nu(x) = Kx$ (for simplicity) and

$$(A + BK)S \oplus W \subseteq S$$

Then

$$(A + BK)\alpha S \oplus W \subseteq \alpha^+ S$$

with

$$\alpha^+ \leq \alpha \text{ when } \alpha \geq 1$$
$$\alpha^+ \geq \alpha \text{ when } 0 \leq \alpha \leq 1$$

``Practical'' Invariant Tubes
Linear Time Invariant Systems

How to get homothetic invariant tubes?

Use one step tube dynamics constraints (assumed X and U are convex)

Let $\Omega_k = z_k \oplus \alpha_k S$ and $\Omega_{k+1} = z_{k+1} \oplus \alpha_{k+1} S$

We wish to ensure that $\Omega_k^+ \subseteq \Omega_{k+1}$

We have

$$\Omega_k^+ = Az_k + Bv_k \oplus \alpha_k (A + BK)S \oplus W$$

Consequently

$$\Omega_k^+ \subseteq \Omega_{k+1} \Leftrightarrow Az_k + Bv_k \oplus \alpha_k (A + BK)S \oplus W \subseteq z_{k+1} \oplus \alpha_{k+1} S$$

which is convex constraint in $z_k, v_k, \alpha_k, z_{k+1}, \alpha_{k+1}$

Similarly $\Omega_k = z_k \oplus \alpha_k S \subseteq X$ and $v_k \oplus \alpha_k KS \subseteq U$ are convex in z_k, v_k, α_k

Homothetic Invariant Tubes
Linear Time Invariant Systems

Homothetic Invariant Tubes

The problem reduces to choosing sequences $\{z_k\}$, $\{v_k\}$ and $\{\alpha_k \geq 0\}$ satisfying the following constraints for $k \in \mathbf{N}_{N-1} := \{0, 1, \ldots, N-1\}$

$$x \in z_0 \oplus \alpha_0 S$$
$$Az_k + Bv_k \oplus \alpha_k (A + BK)S \oplus W \subseteq z_{k+1} \oplus \alpha_{k+1} S$$
$$z_k \oplus \alpha_k S \subseteq X$$
$$v_k \oplus \alpha_k KS \subseteq U$$
$$z_N \oplus \alpha_N S \subseteq X_f$$

Set of $\{z_k\}$, $\{v_k\}$ and $\{\alpha_k \geq 0\}$ is convex (polytopic) if constraints X, X_f, U and W are convex (polytopic) . Hence, homothetic invariant tube can be obtained from a solution of standard convex (linear/quadratic) programming problem.

In fact, under mild assumptions, all the properties (recursive feasibility, robust constraint satisfaction, robust stability and attractivity) of rigid invariant tubes are guaranteed.

``Practical'' Invariant Tubes
Linear Time Invariant Systems

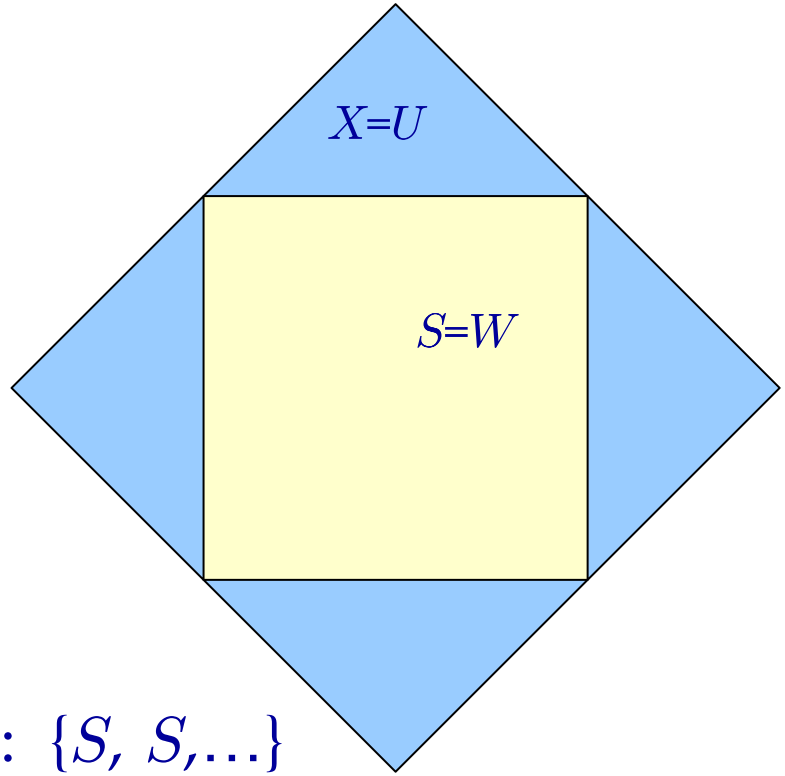
Improved simple invariant tubes versus simple invariant tubes

System :

$x^+ = x + u + w$

Constraints :

$X = U = 2B_1$
$W = B_\infty$



Simple invariant tube for $S=W$: $\{S, S, \ldots\}$

are not applicable if $x \in X \setminus S$

Improved simple invariant tube $S=W$: $\{\{x\}, S, S, \ldots\}$ for any $x \in X$

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``Practical'' Invariant Tubes

Question: Are practical invariant tubes restricted to Linear Case?

Answer: No

There are some results and ideas towards more complex systems, such as:

- Linear Difference Inclusions
- Matched Nonlinearity Case
- Piecewise affine systems

Issues and ideas are outlined next but details and technicalities are omitted.

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Homothetic Invariant Tubes
Linear Difference Inclusions

How to handle Linear Difference Inclusions?

In this case

$$x^+ = Ax + Bu + w$$
$$A = \bar{A} + \Delta A \text{ and } B = \bar{B} + \Delta B$$
$$(\Delta A, \Delta B) \in M := \overline{\text{convh}}\{(\Delta A_i, \Delta B_i) \ : \ i \in \mathbf{N}_q\}$$

Additional problem due to $(\Delta A, \Delta B) \in M$

so that disturbance depends on the current state and control

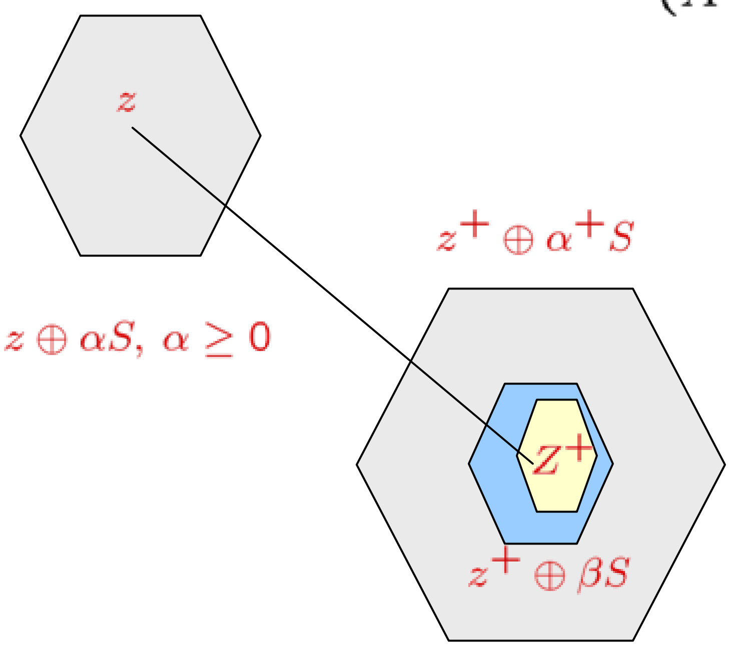
Adversary has more tricks at his disposal

Homothetic Invariant Tubes
Linear Difference Inclusions

Set Dynamics Observation and ``magic'' constraint

Standard assumptions (as in the linear case)

$$(A + BK)S \oplus W \subseteq S$$



$$Z^+ := Az + Bv \subseteq z^+ \oplus \beta S$$

Let

$$\mu(x, z, v) = v + \alpha K(x - z)$$

$$z^+ = \bar{A}z + \bar{B}v$$

$$x \in z \oplus \alpha S$$

Then

$$x^+ = \bar{A}z + \bar{B}v + \Delta Az + \Delta Bv + \alpha(A + BK)s + w$$

Can get ``nice'' constraints as follows

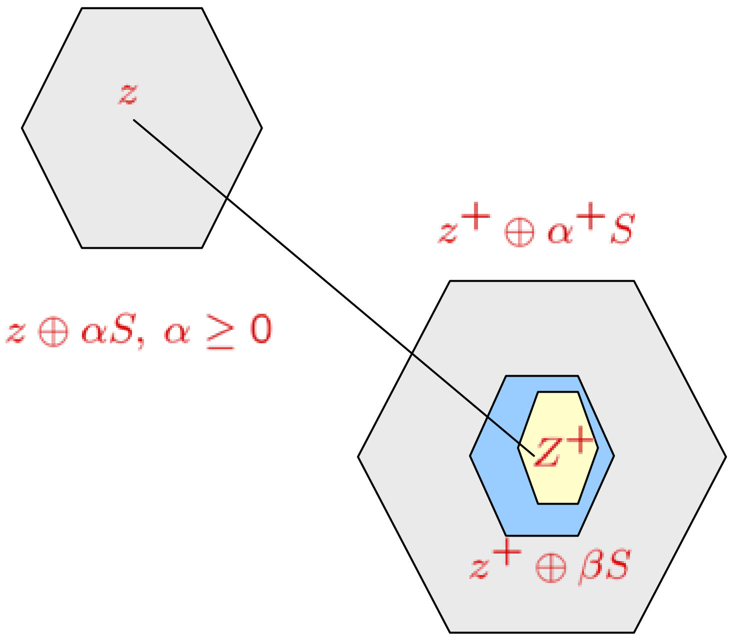
$$\bar{A}z + \bar{B}v + \Delta Az + \Delta Bv \subseteq z^+ \oplus \beta S,\ \forall(\Delta A, \Delta B) \in M$$

Since we know that

$$\alpha(A + BK)S \oplus W \subseteq \gamma S$$

Homothetic Invariant Tubes
Linear Difference Inclusions

Homothetic invariant tubes for linear difference inclusions are similar to homothetic invariant tubes for linear case



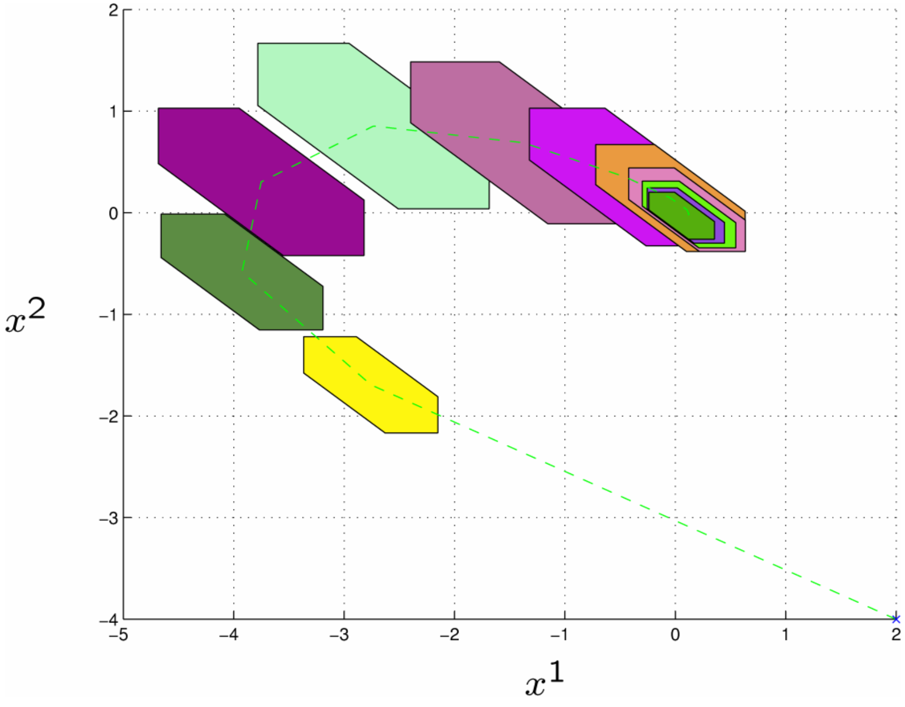
$Z^+ := Az + Bv \subseteq z^+ \oplus \beta S$

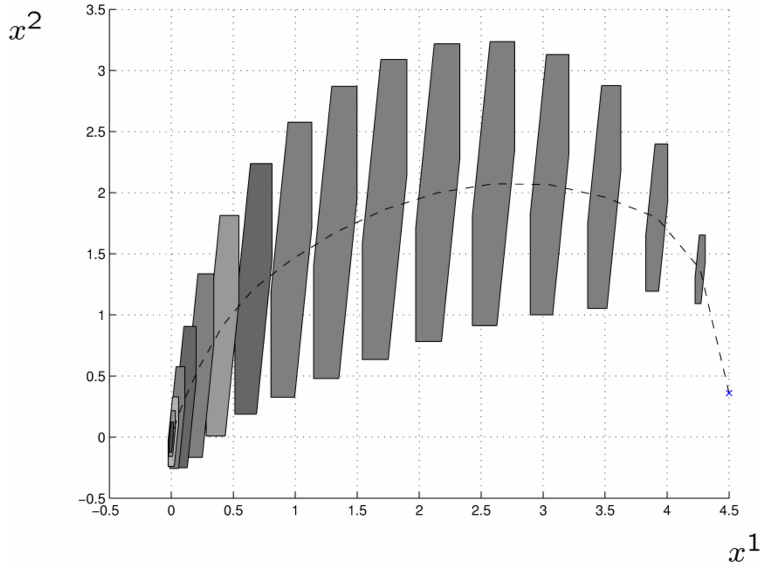
A single tractable CP/QP/LP can be formulated for selection of an invariant tube and the design of robust MPC.

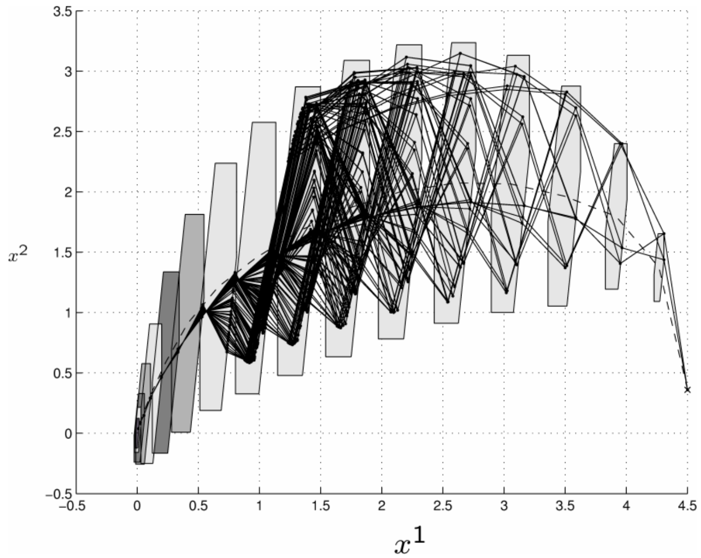
The formulation offers relevant system-theoretic properties and avoid exponential complexity usually present in the methods reported in the literature.

Homothetic Invariant Tubes
Linear Difference Inclusions

Few example just to illustrate method







Outlook - §4.2

- Set Invariance – Basic Facts §1
- Invariant Tubes – General Idea §2
- Linear Time Invariant Systems §3
  - Simple Invariant Tubes §3.1
  - Improved Simple Invariant Tubes §3.2
- Extensions §4
  - Linear Difference Inclusions §4.1
  - **Matched Nonlinearities §4.2**
  - Piecewise Affine Systems §4.3
- Conclusions §5

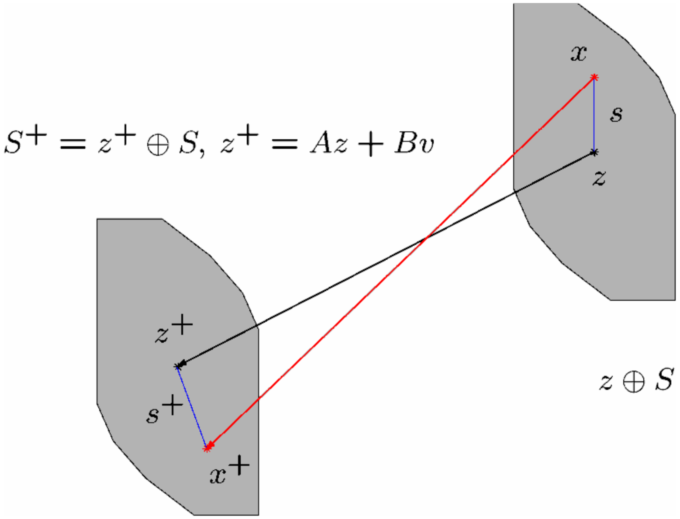
Homothetic Invariant Tubes
Matched Nonlinearities

How to handle some classes of nonlinear systems?

Particular Case $$x^+ = Ax + B[g(x)u + \varphi(x)] + w$$

Includes special case of piecewise affine systems

Set Dynamics Observation as in the linear case



If the set-control law pair $(S, \nu(\cdot))$ satisfy:

$$Ax + B\nu(x) + w \in S,\ \forall (x, w) \in S \times W$$

we have

$$Ax + B[g(x)\mu(x, z, v) + \varphi(x)] + w \in S^+$$

for all $(x, w) \in (z \oplus S) \times W$, where

$$S^+ = z^+ \oplus S,$$
$$z^+ = Az + Bv,$$
$$\mu(x, z, v) = g(x)^{-1}[-\varphi(x) + v + \nu(x - z)]$$

Homothetic Invariant Tubes
Matched Nonlinearities

Simple invariant tubes for this case almost identical as the ones for the linear case, only challenge is the construction of sets Z, Z_f and V

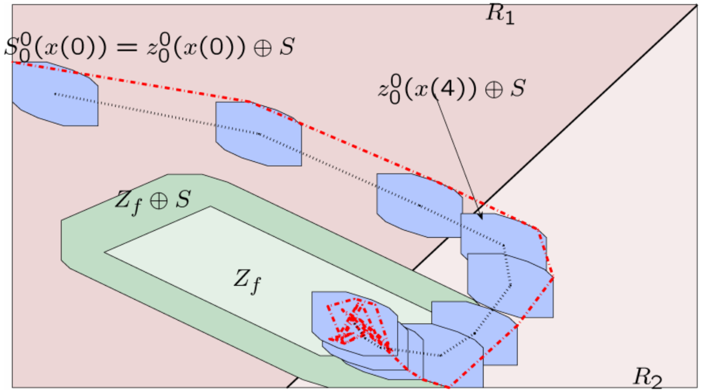
An example just to indicate what is all about

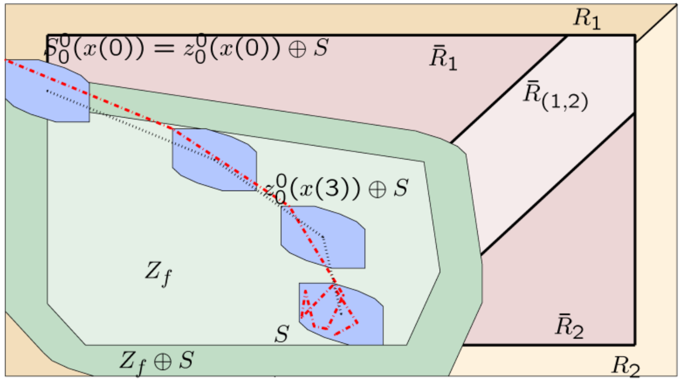
System:

$$x^+ = \begin{cases} \begin{bmatrix} \frac{5}{4} & \frac{3}{4} \\ \frac{1}{4} & \frac{3}{4} \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u + w \text{ when } x^1 - x^2 \leq 0 \\ \begin{bmatrix} \frac{3}{4} & \frac{5}{4} \\ -\frac{1}{4} & \frac{5}{4} \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u + w \text{ when } -x^1 + x^2 \leq 0 \end{cases}$$

Constraints:

$$X := \{ x \in R^2 \ : \ |x|_\infty \leq 10, \ x^2 \geq -2 \}$$
$$U := \{ u \in R \ : \ -7 \leq u \leq 7 \}, W := \{ w \in R^2 \ : \ |w|_\infty \leq \frac{1}{2} \}$$





Two Versions of
Simple Invariant Tubes

Outlook - §4.3

- Set Invariance – Basic Facts §1
- Invariant Tubes – General Idea §2
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  - Simple Invariant Tubes §3.1

  - Improved Simple Invariant Tubes §3.2

- Extensions §4

  - Linear Difference Inclusions §4.1

  - Matched Nonlinearities §4.2

  - **Piecewise Affine Systems §4.3**

- Conclusions §5

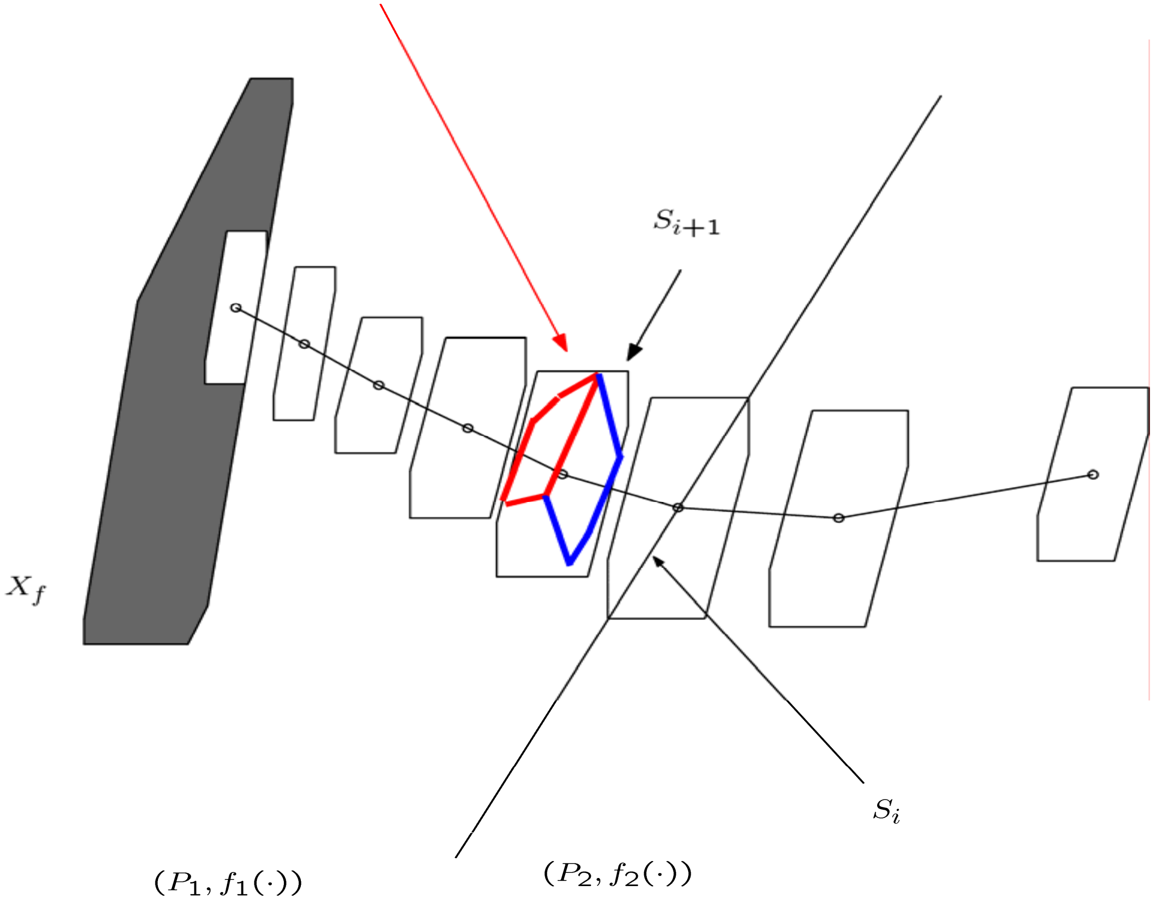
Homothetic Invariant Tubes
Piecewise Affine Systems

Main issues with piecewise affine systems

$\{f(z,\mu_i(z),w)\ :\ z \in S_i \cap P_1,\ w \in W\} \cup \{f(z,\mu_i(z),w)\ :\ z \in S_i \cap P_2,\ w \in W\} \subset S_{i+1}$

Issue 1 :

Uncertainty can cause undesired switching of the dynamics



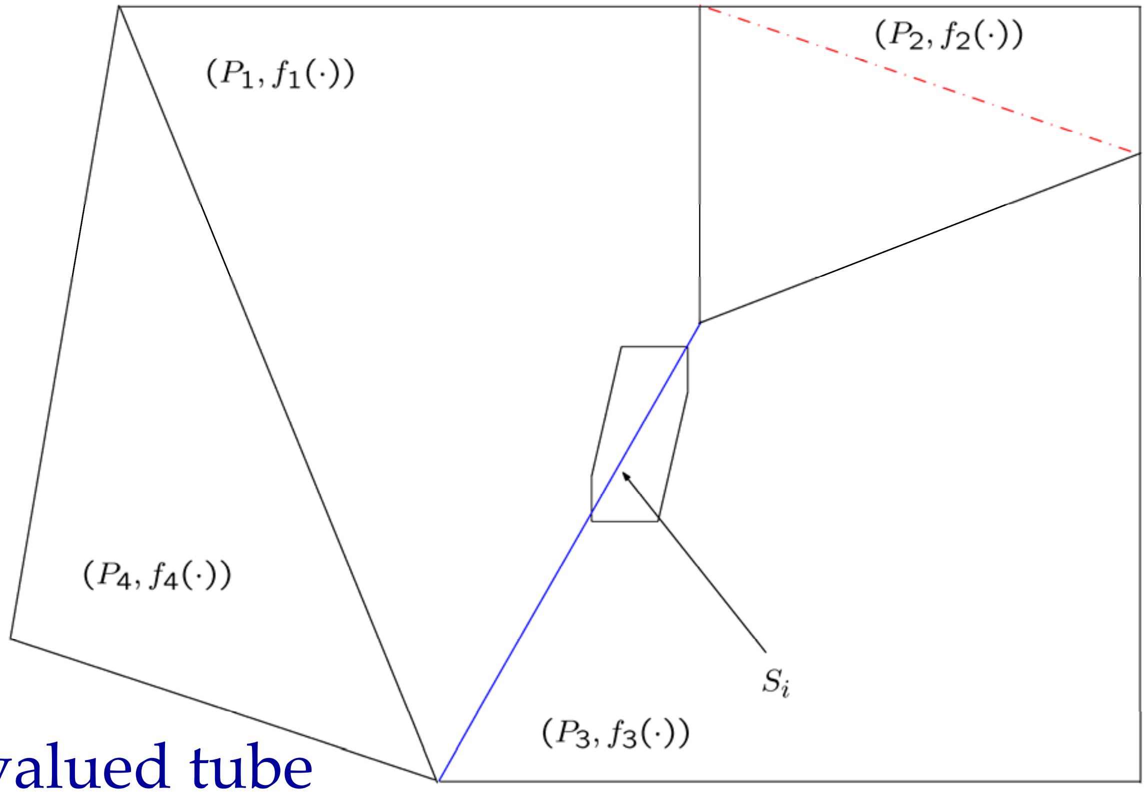
Homothetic Invariant Tubes Piecewise Affine Systems

Main issues with piecewise affine systems

$P_{s_i} = P_1 \cup P_3,\ P_{s_i} \subseteq \bar{\text{convh}}(P_{s_i})$ $s_i^c = (1, 2, 3),\ P_{s_i^c} = P_1 \cup P_2 \cup P_3,\ \bar{\text{convh}}(P_{s_i}) \subseteq P_{s_i^c}$

Issue 2 :

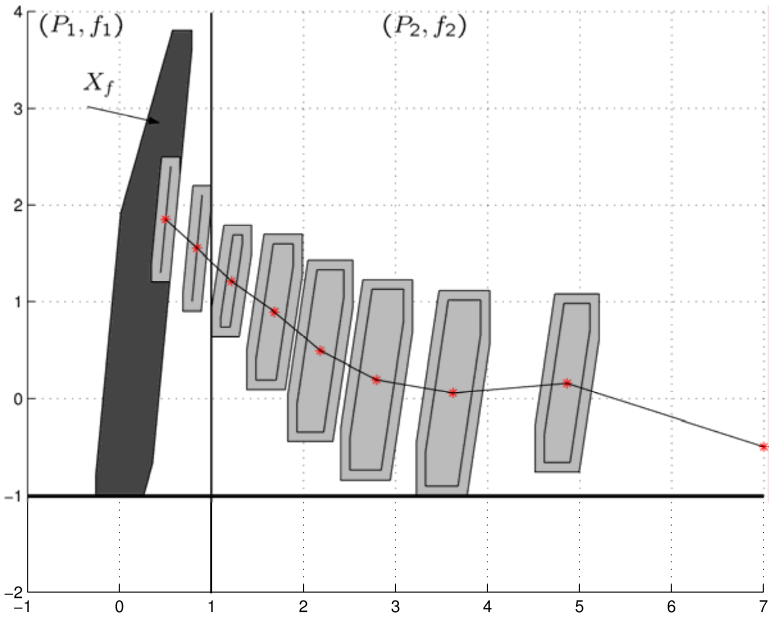
Non-convexity of PWA systems leads to more problems

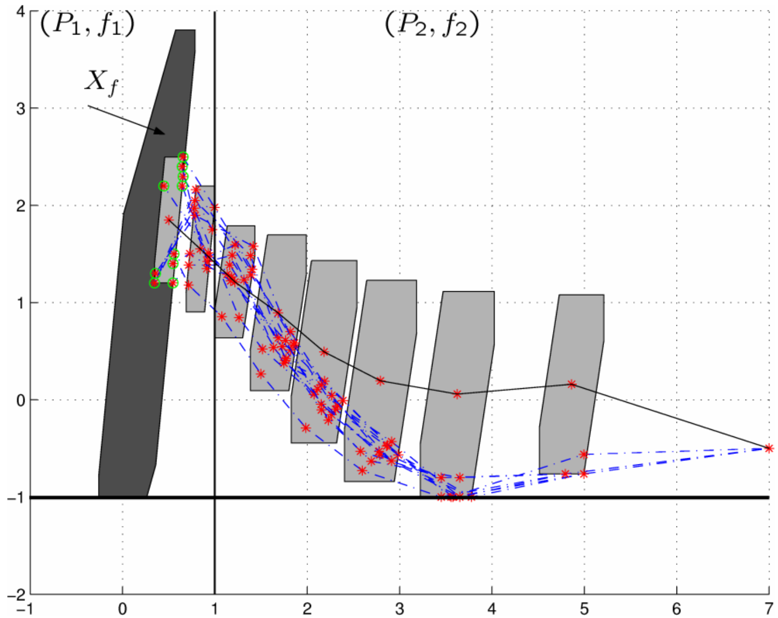


Partial solution via set-valued tube robust and convexified switching sequences

Homothetic Invariant Tubes
Linear Difference Inclusions

Few example just to indicate that it is doable but by no means complete

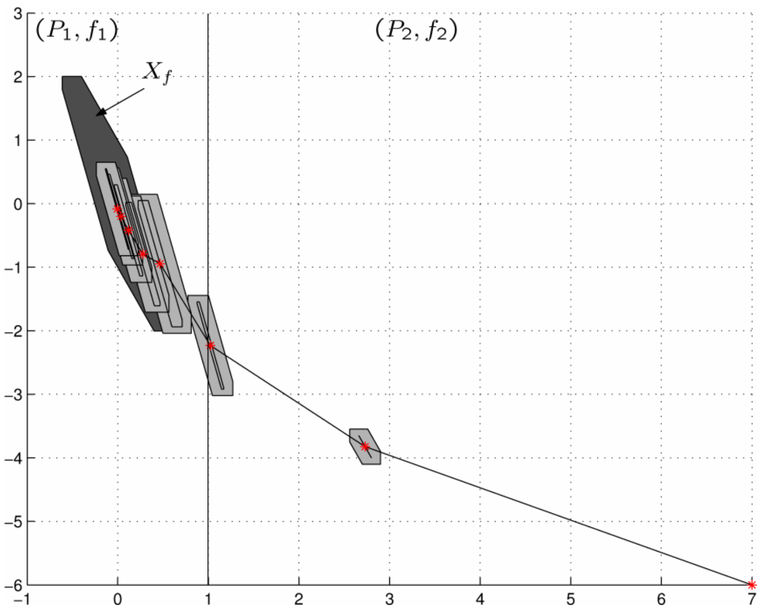


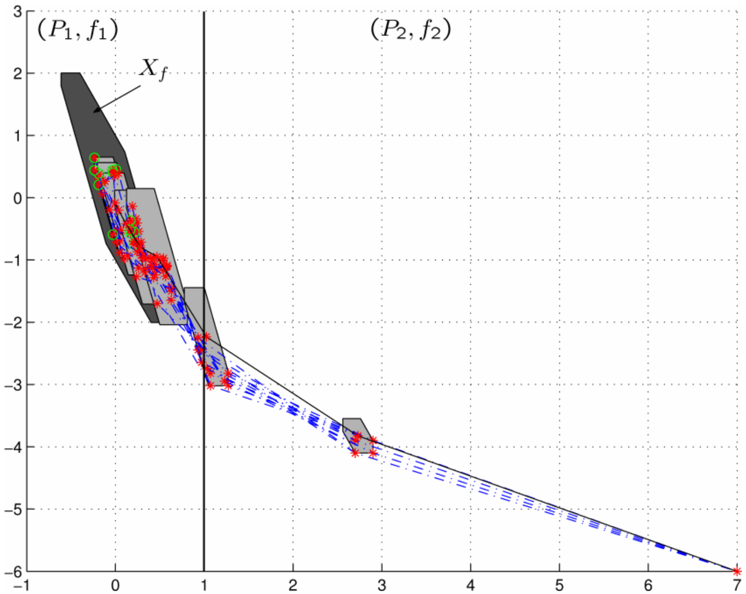


Robust Invariant Tube Switching Type I $s := \{2, 2, 2, 2, 2, 2, 2, 1, 1\}$

Homothetic Invariant Tubes
Linear Difference Inclusions

Few example just to indicate that it is doable but by no means complete





Robust Invariant Tube Switching Type II $\mathbf{s} := \{2, 2, (1, 2), 1, 1, 1, 1, 1, 1\}$

Outlook - §5

- Set Invariance – Basic Facts §1
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Conclusions and Research Directions

Invariant Tubes and Robust MPC

- Control Tube of Trajectories
- Include Tube of Trajectories into Invariant Tubes
- Design Simple Invariant Tubes
- Simplify Robust MPC synthesis by understanding of the underlying process (Set dynamics induced by feedback and disturbance set)
- Reconsider open-loop and min-max robust MPC formulations (Robustness will not be granted for free and DP/feedback MPC might be difficult to solve)

Conclusions and Research Directions

What is already done?

- Homothetic Invariant Tubes
- Invariant Tubes for Linear Difference Inclusions
- Invariant Tubes for Output Feedback Problems

What should be done?

- Invariant Tubes for Nonlinear System
  (State Feedback Linearization)
- Invariant Tubes for Piecewise Affine Systems

What is wishful thinking?

- Simple Invariant Tubes for General Nonlinear Case

Homothetic Invariant Tubes
Where do we stand?

Theoretical Aspects:

Appear to be fairly solid with clear directions for further contributions and extensions

Computational Aspects:

Appear to offer space for contributions and improvements

Practical Aspects:

Several applications of invariant tubes emerged recently.

These include robust MPC for subsystems of nuclear-power plants, power electronics, …

Acknowledgements

Some of the presented ideas are already completed and some are on-going projects.

Some of the initial concepts with respect to simple invariant tubes were developed in collaboration with:

- Mayne (linear case, several papers)
- Fiacchini (linear and LDI case)
- Teel, Mayne and Astolfi (matched nonlinearities, cdc)

The End & Discussion Time

Thank you for attention!

Questions are welcome.