Robust H_∞ vibration control of fluid/plate system

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GT MOSAR

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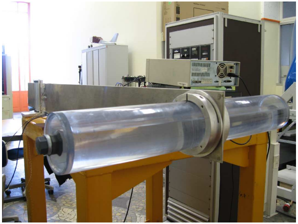
- Introduction
- Mathematical model
- Controller sinthesis
- Concluding remarks

State space representation of the system:

$$\begin{cases} \dot{X} = AX + Bu \\ y = CX \end{cases}$$



$A(\cdot)$ - state matrix, $B(\cdot)$ - input matrix, $C(\cdot)$ - output matrix,
$u(\cdot)$ - input vector (piezoelectric actuator voltage),
$y(\cdot)$ - output vector (piezoelectric sensor voltage),
$X(\cdot)$ - state vector (plate X_p + fluid X_z).

plate: $\begin{cases} \dot{X}_p = A_p X_p + B_p u \\ y = C_p X_p \end{cases}$ fluid: $\begin{cases} \dot{X}_z = A_z X_z + B_z u_z \\ y_z = C_z X_z \end{cases}$

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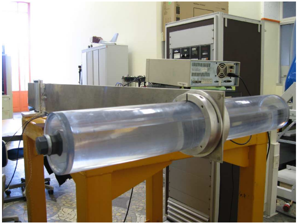

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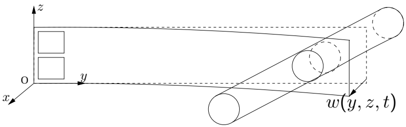

$$m_s \frac{\partial^2 w}{\partial t^2} + \zeta(w)\frac{\partial w}{\partial t} + YI_s\Delta^2 w = \frac{\partial^2 m_y}{\partial y^2} + \frac{\partial^2 m_z}{\partial z^2}$$

+ boundary conditions:

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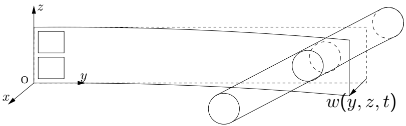
$$w = \frac{\partial w}{\partial y} = \frac{\partial w}{\partial z} = 0 \ \forall (y,z) \in \{0\} \times [0,l].$$

$$w(y,z,t) = \sum_{k=1}^{\infty} \eta_k(y,z) q_k(t) = \eta(y,z)^T \cdot q(t)$$

$\eta^T = (\eta_1(y,z), \dots, \eta_k(y,z), \dots, \eta_N(y,z))$ - Ritz functions (finite dimension),

with: $\eta_k(y,z) = Y_{i_k}(y) Z_{j_k}(z)$



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State matrix (A_p)

Plate state-space vector: $X_p = \begin{pmatrix} \dot{q_1} & \omega_1 q_1 & ... & \dot{q_N} & \omega_N q_N \end{pmatrix}$

$$A_p = \begin{pmatrix} A_{p_1} & 0 & \cdots & 0 \\ 0 & A_{p_2} & \cdots & 0 \\ & \cdots & & \\ 0 & 0 & \cdots & A_{p_N} \end{pmatrix}$$

with:

$$A_{p_k} = \begin{pmatrix} -2\zeta_k\omega_k & -\omega_k \\ \omega_k & 0 \end{pmatrix}$$

ω_k *- frequency of the* k^{th} *mode*

ζ_k *- damping of the* k^{th} *mode*

Input matrix (B_p) and Output matrix (C_p)

with piezoelectric actuators and sensors

$$B_p = (b_{p_1}, 0, ..., b_{p_k}, 0, ..., b_{p_N}, 0)^T$$

with:

$$b_{p_k} = K_b(Y'_{i_k}(y_{a2}) - Y'_{i_k}(y_{a1})) \int_{z_{a1}}^{z_{a2}} Z_{j_k}(z)dz + K_b(Z'_{j_k}(z_{a2}) - Z'_{j_k}(z_{a1})) \int_{y_{a1}}^{y_{a2}} Y_{i_k}(y)dy$$

$K_b = K_b$(plate,piezo)

$(y_{a1}, z_{a1}), (y_{a2}, z_{a2})$ actuator corners coordinates

$$C_p = (0, c_{p_1}, ..., 0, c_{p_k}, ..., 0, c_{p_N})$$

with:

$$c_{p_k} = \frac{1}{\omega_k}\frac{K_c}{C_a}\Bigg((Y'_{i_k}(y_{c2}) - Y'_{i_k}(y_{c1})) \int_{z_{c1}}^{z_{c2}} Z_{j_k}(z)dz + (Z'_{j_k}(z_{c2}) - Z'_{j_k}(z_{c1})) \int_{y_{c1}}^{y_{c2}} Y_{i_k}(y)dy\Bigg)$$

$K_c = K_c$(plate,piezo)
C_a capacity of the charge amplifier

$(y_{c1}, z_{c1}), (y_{c2}, z_{c2})$ sensor corners coordinates

ϕ - velocity potential

- Laplace equation for $(x, y, z, t) \in (0, a) \times (0, b) \times (0, h) \times (0, \infty)$:

$$\Delta \phi = 0$$

- Linearised equation of fluid motion:

$$\frac{\partial \phi}{\partial t} + \frac{p}{\rho} + g(z - h) - C_0 x = 0$$

where $p =$ pression, $\rho =$ density and $C_0 =$ external horizontal acceleration

- Tank walls boundary conditions:

$$\frac{\partial \phi}{\partial x}(x = 0) = \frac{\partial \phi}{\partial x}(x = a) = \frac{\partial \phi}{\partial y}(y = 0) = \frac{\partial \phi}{\partial y}(y = b) = \frac{\partial \phi}{\partial z}(z = 0) = 0$$

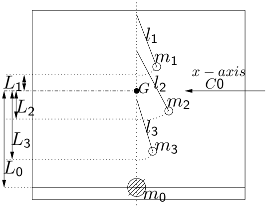
- Free surface boundary conditions: (free surface: $z = h + \xi(x, y, t)$)

$$\frac{\partial \xi}{\partial t} = \frac{\partial \phi}{\partial z}$$

$$\frac{\partial \phi}{\partial t} + g\xi - C_0 x = 0 \quad \text{with } p = 0$$

Equivalent mechanical model

- Mass - pendulum systems to model liquid sloshing



- equivalence of forces and moments
- $G = (y_G, z_G)$ - liquid gravity center in steady motion

1 mass - pendulum system to represent liquid sloshing.

$$\dot{X}_z = \begin{pmatrix} -2\zeta\sqrt{\frac{g}{l_1}} & -\sqrt{\frac{g}{l_1}} \\ \sqrt{\frac{g}{l_1}} & 0 \end{pmatrix} X_z + \begin{pmatrix} -\frac{1}{l_1} \\ 0 \end{pmatrix} C_0$$

- Plate movement influence on the liquid sloshing

Complete representation of the tank with liquid:

$$\dot{X}_z = \begin{pmatrix} -2\xi\sqrt{\frac{g}{l_1}} & -\sqrt{\frac{g}{l_1}} \\ \sqrt{\frac{g}{l_1}} & 0 \end{pmatrix} X_z + \begin{pmatrix} -\frac{1}{l_1} \\ 0 \end{pmatrix} K_G(A_pX_p + B_pu)$$

where: $K_G = \begin{pmatrix} Y_1(y_G)Z_1(z_G) & 0 \end{pmatrix}$

- Liquid sloshing influence on the plate movement

Complete representation of the plate:

$$\begin{cases} \dot{X}_p = A_pX_p + B_pu + B_{pend}X_z \\ y = C_pX_p \end{cases}$$

where: $B_{pend} = \begin{pmatrix} b_{pend} \\ 0 \end{pmatrix}$

State - space representation of the total system

State - space vector:

$$X = \begin{pmatrix} X_p & X_z \end{pmatrix}^T = \begin{pmatrix} \begin{pmatrix} \dot{q}_1 & \omega_{1q} q_1 \end{pmatrix} & \begin{pmatrix} \dot{z}_1 & \omega_{1z} z_1 \end{pmatrix} \end{pmatrix}^T$$

$$\begin{cases} \begin{pmatrix} \dot{X}_p \\ \dot{X}_z \end{pmatrix} = \begin{pmatrix} A_p & B_{pend} \\ A^\sharp & A_z \end{pmatrix} \begin{pmatrix} X_p \\ X_z \end{pmatrix} + \begin{pmatrix} B_p \\ B^\sharp \end{pmatrix} u \\ y = \begin{pmatrix} C_p & 0 \end{pmatrix} \begin{pmatrix} X_p \\ X_z \end{pmatrix} \end{cases}$$

where: $A^\sharp = \begin{pmatrix} -\frac{1}{l_1} K_G A_p \\ 0 \end{pmatrix}$, $B^\sharp = \begin{pmatrix} -\frac{1}{l_1} K_G B_p \\ 0 \end{pmatrix}$, $A_z = \begin{pmatrix} -2\zeta\sqrt{\frac{g}{l_1}} & -\sqrt{\frac{g}{l_1}} \\ \sqrt{\frac{g}{l_1}} & 0 \end{pmatrix}$

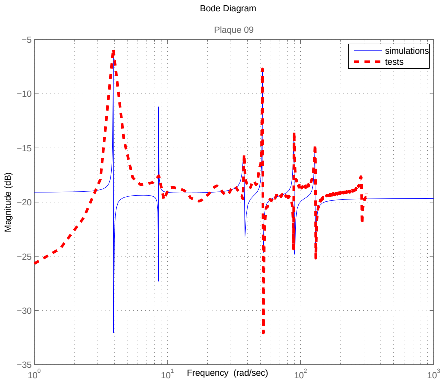
Imposibility to build a perfect model

$\Longleftrightarrow$ Amplitude and frequency discrepancy

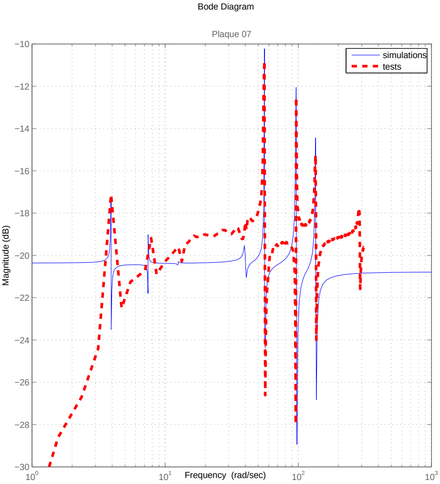
$\Longrightarrow$ Model matching.

- Tuning of A_p, B_p, C_p matrices;
- Feedforeward matrix $D_p \neq 0$.

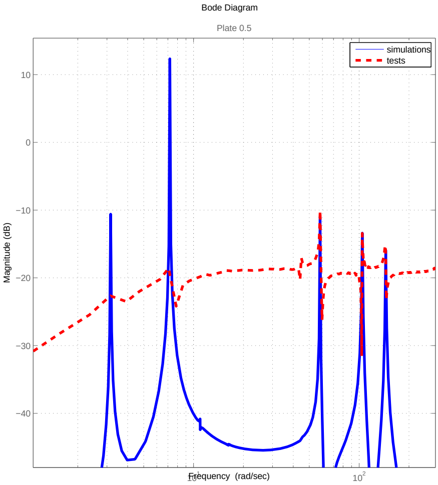
Model matching for 90% filled tank



Model matching for 70% filled tank

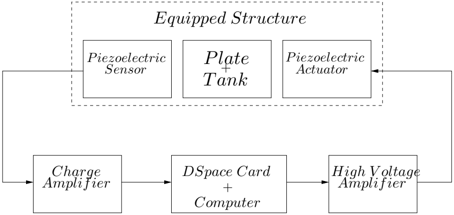


Model matching for half filled tank



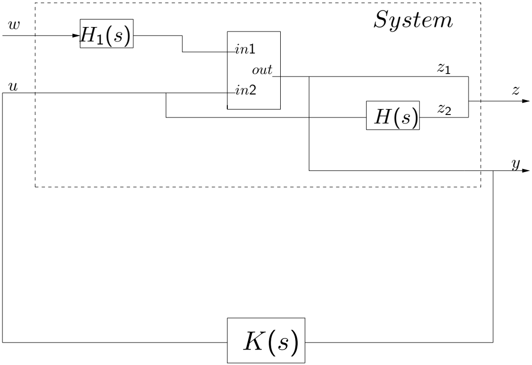
Experimental setup

- High voltage amplifier $\pm 100V$;
- Charge amplifier 2635 from Brüer & Kjaer;
- DSpace card NI BNC-2110.



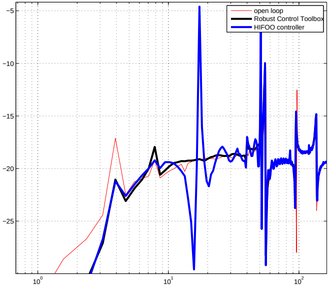
Synthesis of the controller

- $X_p \in \mathbb{R}^{10}$ and $X_z \in \mathbb{R}^4$;
- $H(s)$ roll-off filter.



Robust controller with Matlab

- Robust Control Toolbox and HIFOO algorithm:
  - $X_p \in \mathbb{R}^2$ and $X_z \in \mathbb{R}^2$;
  - NO filters;
  - Suitable actuator limits.



$\Longrightarrow$ HIFOO better than Robust Control Toolbox

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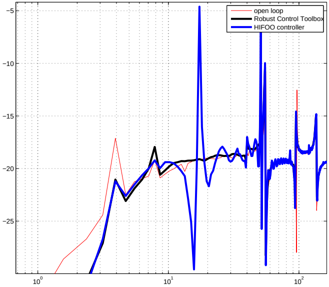

$\Longrightarrow$ HIFOO better than Robust Control Toolbox

Choice of the HIFOO order

- Comparison between a 4^{th} order and a 1^{st} order controller.
- Same H - infinity performance for controller order $\geq$ 4.

- 1^{st} order HIFOO controller.

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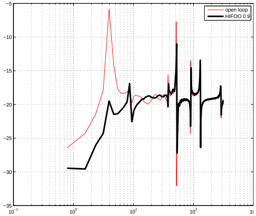
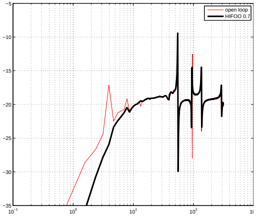
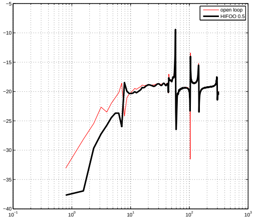
HIFOO controller

Three different filling levels for the tank: 90%, 70% and 50% filled.





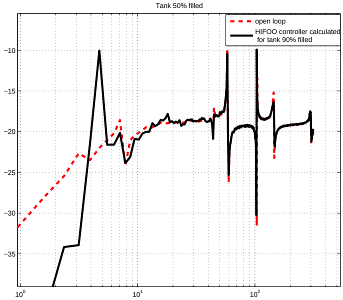


First order HIFOO controller

- First order HIFOO controller;
- $X_p \in \mathbb{R}^{10}$ and $X_z \in \mathbb{R}^4$;
- Flexion and torsion mode attenuation.

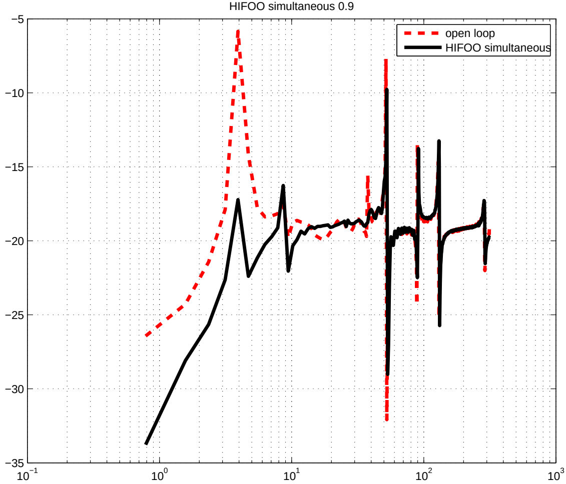
Simultaneous HIFOO controller

- HIFOO calculated for tank 90% filled tested on tank 50% filled.

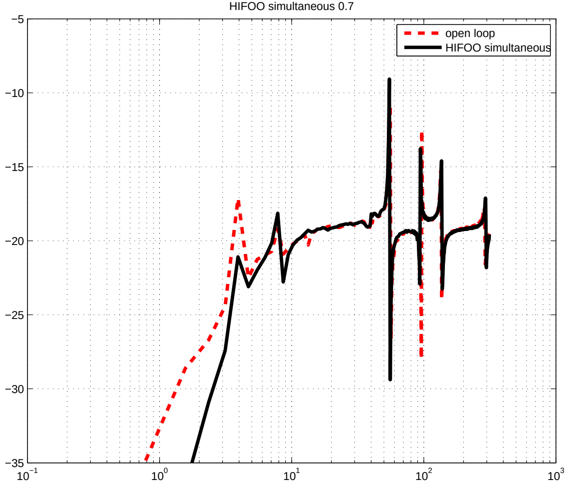


- Conclusion: NOT robust enough.

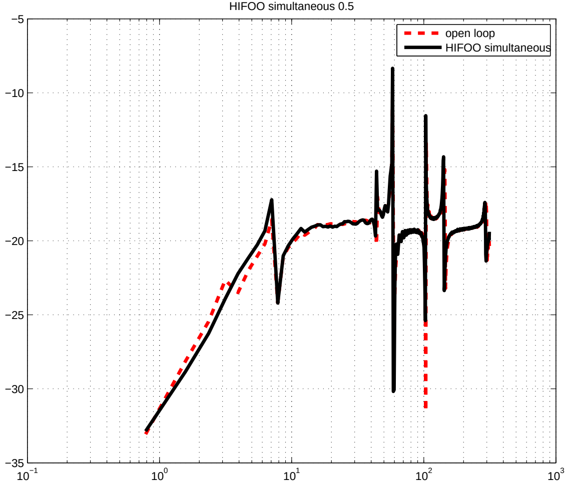
Simultaneous HIFOO controller - Tank 90% filled



Simultaneous HIFOO controller - Tank 70% filled



Simultaneous HIFOO controller - Tank 50% filled



Conclusions

- Analytical model of the total system (plate + liquid + coupling);
- Model matching;
- Robust controller using Robust control Toolbox and HIFOO library;
- HIFOO first order controller;
- Simultaneous HIFOO controller.

Work in progress:

- Weight for the simultaneous HIFOO controller.

Perspectives:

- Parametric uncertainty.

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