



# Off-line Robustification of Multivariable Model Predictive Control

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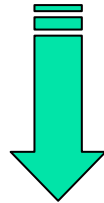
GT CPNL, 31-01-2008

- Introduction
- Multivariable MPC
- Robustness using the Youla parameter
- Robustified MIMO MPC
  - Robust stability (RS)
  - Nominal performance
- Application to a stirred tank reactor
- Conclusions

- Off-line state-space methodology for enhancing the robustness of multivariable MPC

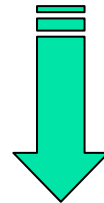
Starting point

- Initial stabilizing MIMO Model Predictive Controller



Goal

- Robustification of this initial controller by convex optimization of a multivariable Youla parameter



Means

- Off-line methodology to improve robust stability under unstructured uncertainties, while respecting nominal performance specifications  LMI tools

## ✓ Introduction

### ■ **Multivariable MPC**

- *Model description*
- *Cost function*
- *Prediction equation*
- *Control law synthesis*

- Robustness using the Youla parameter
- Robustified MIMO MPC
- Application to a stirred tank reactor
- Conclusions

- Model description (MIMO system  $m \times p$ )

$$\begin{cases} \mathbf{x}(k+1) = \mathbf{A} \mathbf{x}(k) + \mathbf{B} \mathbf{u}(k) \\ \mathbf{y}(k) = \mathbf{C} \mathbf{x}(k) \end{cases} \text{ with } \mathbf{A} \in \mathbf{R}^{n \times n}, \mathbf{B} \in \mathbf{R}^{n \times m}, \mathbf{C} \in \mathbf{R}^{p \times n}$$

- Steady-state errors cancellation

$$\mathbf{u}(k) = \mathbf{u}(k-1) + \Delta \mathbf{u}(k) \quad \Rightarrow$$

$$\begin{cases} \mathbf{x}_e(k+1) = \mathbf{A}_e \mathbf{x}_e(k) + \mathbf{B}_e \Delta \mathbf{u}(k) \\ \mathbf{y}(k) = \mathbf{C}_e \mathbf{x}_e(k) \end{cases}$$

Extended state-space  $\mathbf{x}_e(k) = \begin{bmatrix} \mathbf{x}(k) \\ \mathbf{u}(k-1) \end{bmatrix}$

- Predicted output vector

$$\hat{\mathbf{y}}(k+i) = \mathbf{C} \mathbf{A}^i \hat{\mathbf{x}}(k) + \sum_{j=0}^{i-1} \mathbf{C} \mathbf{A}^{i-j-1} \mathbf{B} \underbrace{\left[ \mathbf{u}(k-1) + \sum_{l=0}^j \Delta \mathbf{u}(k+l) \right]}_{\mathbf{u}(k+j)}$$

- Observer

$$\hat{\mathbf{x}}_e(k+1) = \mathbf{A}_e \hat{\mathbf{x}}_e(k) + \mathbf{B}_e \Delta \mathbf{u}(k) + \mathbf{K}[\mathbf{y}(k) - \mathbf{C}_e \hat{\mathbf{x}}_e(k)]$$

- Quadratic objective function minimization

$$J = \sum_{i=N_1}^{N_2} \|\hat{\mathbf{y}}(k+i) - \mathbf{y}_r(k+i)\|_{\tilde{\mathbf{Q}}_J(i)}^2 + \sum_{i=0}^{N_u-1} \|\Delta \mathbf{u}(k+i)\|_{\tilde{\mathbf{R}}_J(i)}^2$$

Output prediction horizons

Weightings

Control horizon

with  $\Delta \mathbf{u}(k+i) = 0, \forall i \geq N_u$

Future control actions

Extended state-space and future setpoints

- Objective function in the matrix formalism

$$J = \|\mathbf{Y}(k) - \mathbf{Y}_r(k)\|_{\mathbf{Q}_J}^2 + \|\Delta \mathbf{U}(k)\|_{\mathbf{R}_J}^2 = \|\Phi_{\Delta} \Delta \mathbf{U}(k) - \Theta(k)\|_{\mathbf{Q}_J}^2 + \|\Delta \mathbf{U}(k)\|_{\mathbf{R}_J}^2$$

with

$$\begin{aligned} \mathbf{Y}(k) &= \Psi \hat{\mathbf{x}}(k) + \Phi \mathbf{u}(k-1) + \Phi_{\Delta} \Delta \mathbf{U}(k) \\ \Theta(k) &= \mathbf{Y}_r(k) - \Psi \hat{\mathbf{x}}(k) - \Phi \mathbf{u}(k-1) \end{aligned}$$

$$\Downarrow \frac{\partial J}{\partial \Delta \mathbf{U}(k)} = 0$$

$\Delta \mathbf{U}(k)$

- Future control laws sequence

$$\Delta \mathbf{U}(k) = (\mathbf{R}_J + \Phi_{\Delta}^T \mathbf{Q}_J \Phi_{\Delta})^{-1} \Phi_{\Delta}^T \mathbf{Q}_J \Theta(k)$$

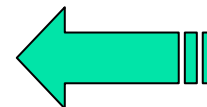
with

$$\Psi = \begin{bmatrix} \mathbf{C} \mathbf{A}^{N_1} \\ \vdots \\ \mathbf{C} \mathbf{A}^{N_2} \end{bmatrix}, \Phi = \begin{bmatrix} \Sigma_{N_1-1} \\ \vdots \\ \Sigma_{N_2-1} \end{bmatrix}, \Phi_{\Delta} = \begin{bmatrix} \Sigma_{N_1-1} & \cdots & \Sigma_0 & 0 & \cdots & 0 \\ \vdots & \cdots & \vdots & \vdots & \ddots & \vdots \\ \Sigma_{N_2-1} & \cdots & \Sigma_{N_2-N_1} & \Sigma_{N_2-N_1-1} & \cdots & \Sigma_{N_2-N_u} \end{bmatrix}$$

where  $\Sigma_i = \mathbf{C} \sum_{j=0}^i \mathbf{A}^{i-j} \mathbf{B}$

- The first component of each future control sequence

$$\Delta \mathbf{u}(k) = \boldsymbol{\mu} \Theta(k)$$

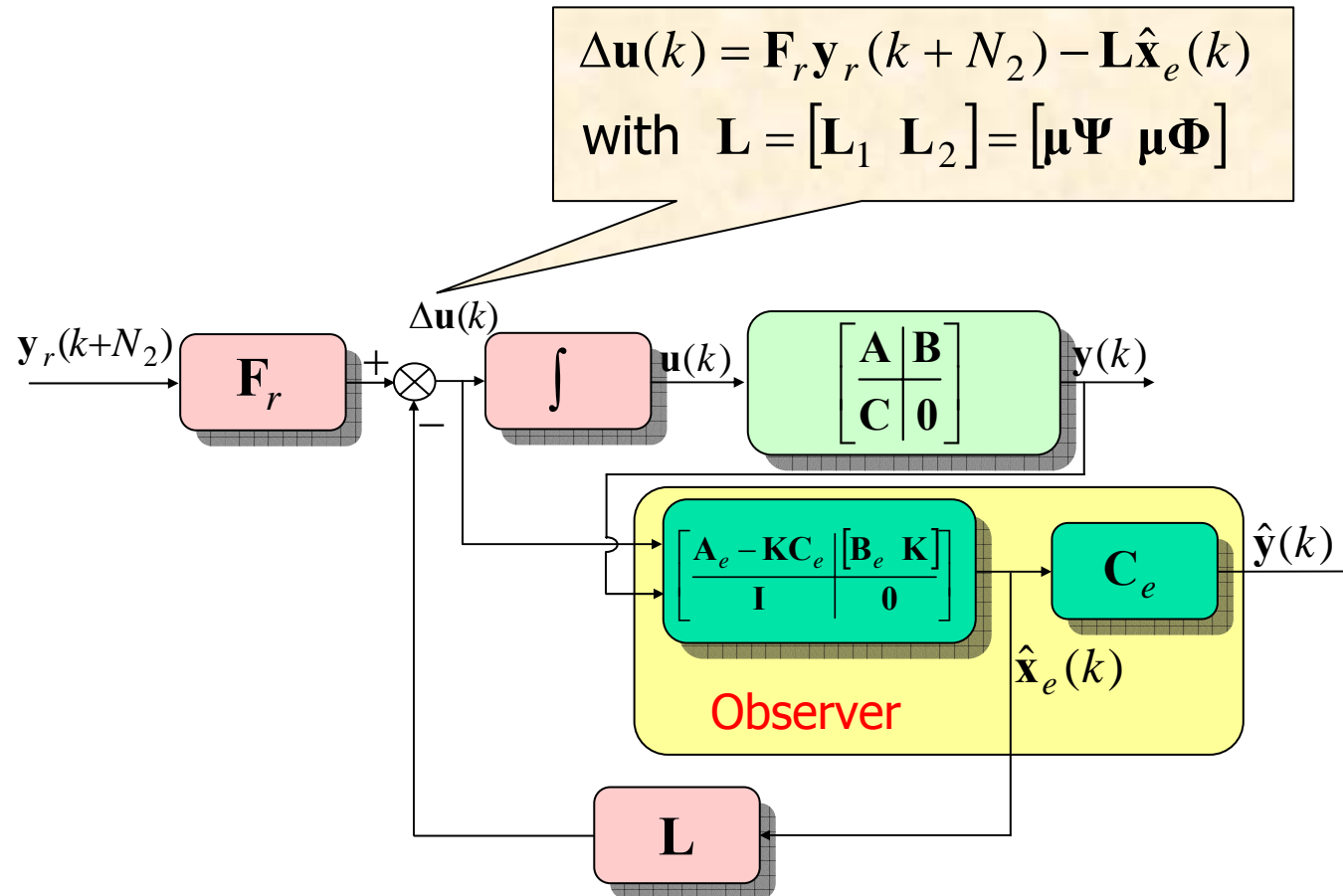


Receding horizon principle

with  $\boldsymbol{\mu} = \begin{bmatrix} \mathbf{I}_m & \mathbf{0}_{m, m(N_u-1)} \end{bmatrix} (\mathbf{R}_J + \Phi_{\Delta}^T \mathbf{Q}_J \Phi_{\Delta})^{-1} \Phi_{\Delta}^T \mathbf{Q}_J$

# Multivariable MPC

- Block diagram of MIMO MPC

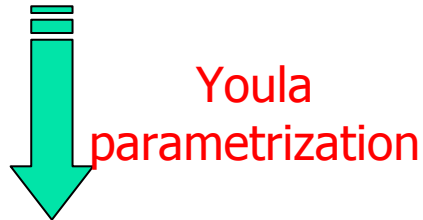




- ✓ Introduction
- ✓ Multivariable MPC
- **Robustness using the Youla parameter**
  - *Initial stabilizing control law*
  - *Robust stability under frequency constraints*
  - *Nominal performance specifications*
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# Robustness using the Youla parameter

- Initial stabilizing controller  $K_0$



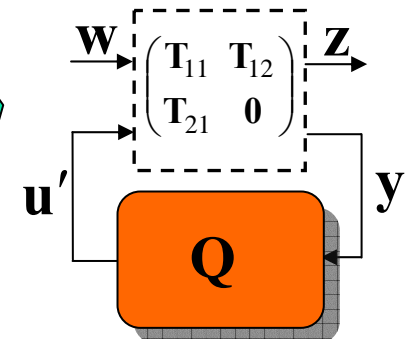
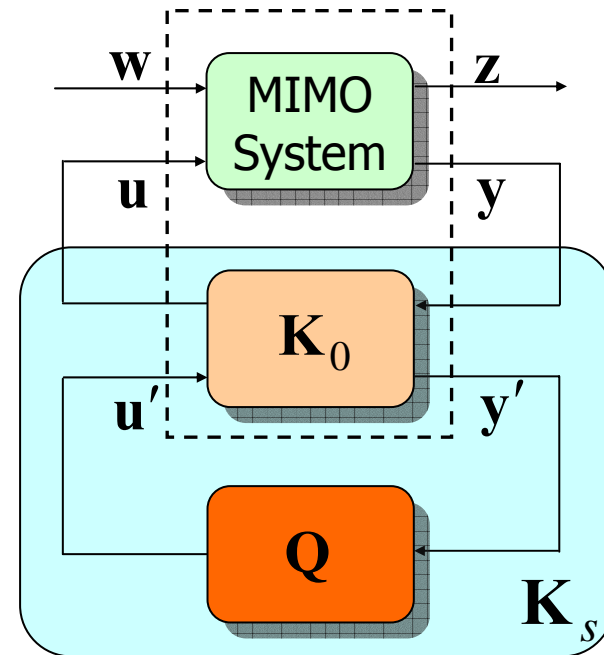
- Set of stabilizing controllers  $K_s$

- $Q \in \mathcal{RH}_\infty$   
Affine dependence in  $Q$

$$T_{zw} = T_{11} + T_{12}QT_{21}$$

$Q = ?$

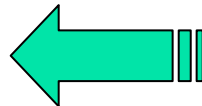
Convex specifications in closed-loop



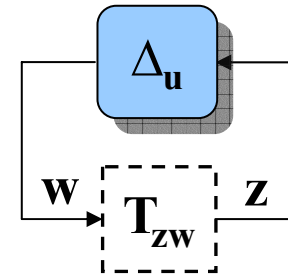
# Robustness using the Youla parameter

- Robustification under unstructured uncertainties  $\Delta_u$

$$\min_{Q \in \mathcal{RH}_\infty} \|T_{zw}\|_\infty$$



Small gain theorem



- Theorem

*A discrete-time system  $(A_{cl}, B_{cl}, C_{cl}, D_{cl})$  is stable and admits a  $H_\infty$  norm lower than  $\gamma$  if and only if*

$$\exists X_1 = X_1^T \succ 0 / \begin{bmatrix} -X_1^{-1} & A_{cl} & B_{cl} & 0 \\ A_{cl}^T & -X_1 & 0 & C_{cl}^T \\ B_{cl}^T & 0 & -\gamma I & D_{cl}^T \\ 0 & C_{cl} & D_{cl} & -\gamma I \end{bmatrix} \prec 0$$



Transformation  
into LMI1

- New optimization problem

$$\min_{LMI_1} \gamma$$

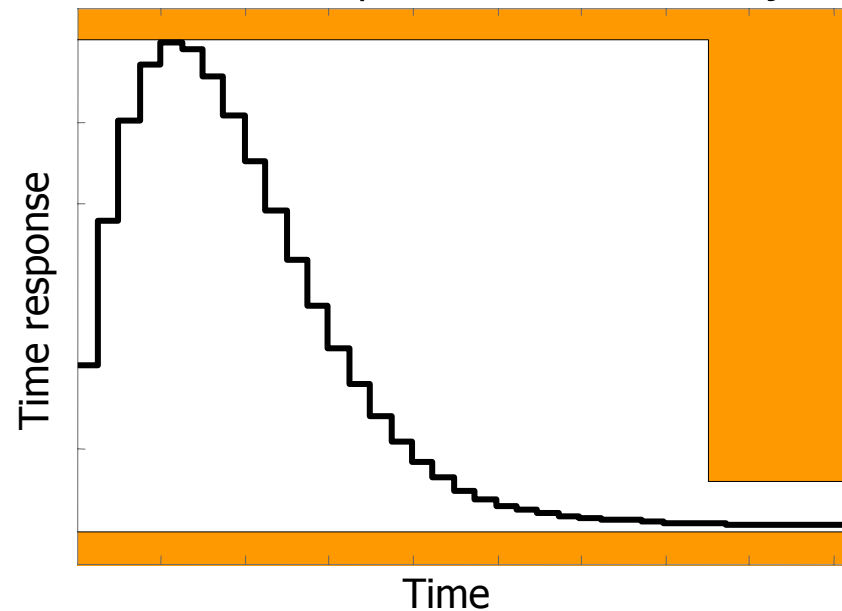
- Nominal performance specifications as output time-domain templates for disturbances rejection

$$\mathbf{y}_{\min}(k) \leq \mathbf{y}(k) \leq \mathbf{y}_{\max}(k), \quad \forall k / 0 \leq k \leq N_t$$

Transformation  
into LMI2

$$\min_{LMI_1, LMI_2} \gamma$$

Time-domain template for disturbance rejection



- ✓ Introduction
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- **Robustified MIMO MPC**
  - *Initial stabilizing control law*
  - *Robust stability under frequency constraints*
  - *Nominal performance specifications*
- Application to a stirred tank reactor
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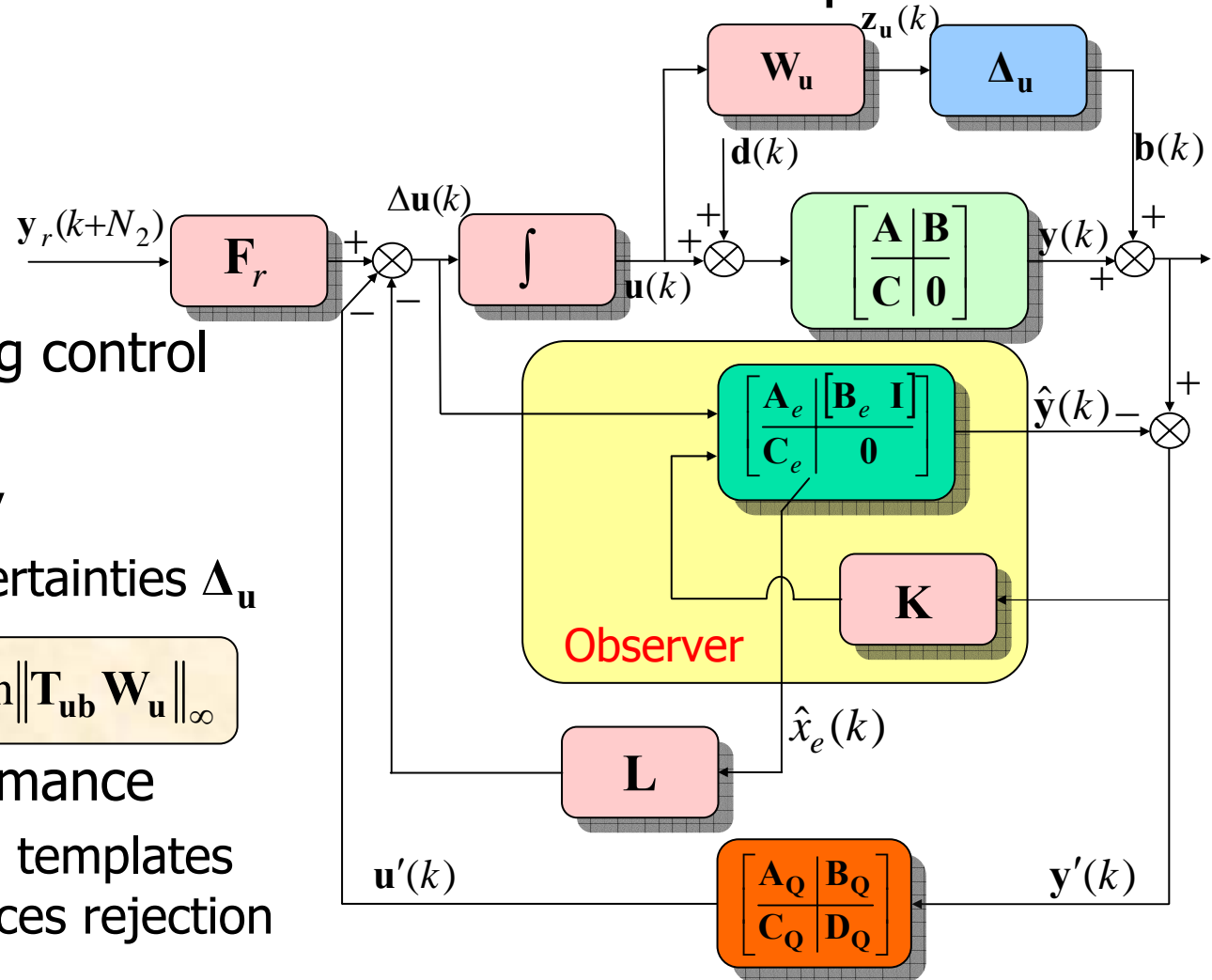
# Robustified MIMO MPC

- Block diagram of MIMO MPC with the  $Q$  parameter

- Initial stabilizing control
  - MIMO MPC
- Robust stability
  - Additive uncertainties  $\Delta_u$
- Nominal performance
  - Time-domain templates for disturbances rejection

$$\min \|T_{z_u b}\|_{\infty} = \min \|T_{u b} W_u\|_{\infty}$$

$$d(k) \rightarrow y(k)$$



# Robustified MIMO MPC

- Goal  $\mathbf{b}(k) \rightarrow \mathbf{z}_u(k)$

- Multivariable Model Predictive Control law

$$\Delta \mathbf{u}(k) = \mathbf{F}_r \mathbf{y}_r(k + N_2) - \mathbf{L} \hat{\mathbf{x}}_e(k) - \mathbf{u}'(k)$$

with the observer

$$\hat{\mathbf{x}}_e(k + 1) = \mathbf{A}_e \hat{\mathbf{x}}_e(k) + \mathbf{B}_e \Delta \mathbf{u}(k) + \mathbf{K}[\mathbf{y}(k) - \mathbf{C}_e \hat{\mathbf{x}}_e(k) + \mathbf{b}(k)]$$

- Error  $\boldsymbol{\varepsilon}(k) = \mathbf{x}_e(k) - \hat{\mathbf{x}}_e(k)$

- Weighting

$$\mathbf{W}_u : \begin{bmatrix} \mathbf{A}_w & \mathbf{B}_w \\ \mathbf{C}_w & \mathbf{D}_w \end{bmatrix}$$

- Youla parameter  $\mathbf{u}'(k) = \mathbf{Q}\mathbf{y}'(k)$

Extended state-space  
of MIMO system

# Robustified MIMO MPC

- Aim: find the Youla parameter  $Q \in \mathcal{RH}_\infty$  which minimizes the norm  $\|\mathbf{T}_{z,u,b}\|_\infty$   $\Rightarrow$  *convex optimization problem*
- Sub-optimal solution
  - for each pair  $(i, j)$   $\Rightarrow$  a polynomial or a FIR filter  $Q^{ij}$

unknown parameters

$$Q^{ij} = \sum_{l=0}^{n_Q} q_l^{ij} q^{-l}, i = \overline{1, m}, j = \overline{1, p}$$

- State-space representation

order of Youla parameter

fixed	=	$\begin{bmatrix} \mathbf{A}_Q & \mathbf{B}_Q \\ \mathbf{C}_Q & \mathbf{D}_Q \end{bmatrix}$	=	$\left[ \begin{array}{cc cc} \mathbf{a}_Q & \mathbf{0} & \mathbf{b}_Q & \mathbf{0} \\ & \ddots & & \\ \mathbf{0} & \mathbf{a}_Q & \mathbf{0} & \mathbf{b}_Q \\ \hline \mathbf{c}_Q^{11} & \dots & \mathbf{c}_Q^{1p} & d_Q^{11} & \dots & d_Q^{1p} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{c}_Q^{m1} & \dots & \mathbf{c}_Q^{mp} & d_Q^{m1} & \dots & d_Q^{mp} \end{array} \right]$
variable				

$$\mathbf{a}_Q = \begin{bmatrix} \mathbf{0}_{1, n_Q-1} & 0 \\ \mathbf{I}_{n_Q-1} & \mathbf{0}_{n_Q-1, 1} \end{bmatrix}, \mathbf{b}_Q = \begin{bmatrix} 1 \\ \mathbf{0}_{n_Q-1, 1} \end{bmatrix}, \mathbf{c}_Q^{ij} = [q_1^{ij} \ \dots \ q_{n_Q}^{ij}], d_Q^{ij} = q_0^{ij}$$



# Robustified MIMO MPC

- Closed-loop system in state-space formulation

$$\mathbf{b}(k) \rightarrow \mathbf{z}_u(k) \quad \Rightarrow \quad \left[ \begin{array}{c|c} \mathbf{A}_{cl} & \mathbf{B}_{cl} \\ \hline \mathbf{C}_{cl} & \mathbf{D}_{cl} \end{array} \right] = \left[ \begin{array}{ccc|c} \bar{\mathbf{A}}_1 & \bar{\mathbf{A}}_3 - \bar{\mathbf{B}}_{u_1} \mathbf{D}_Q \mathbf{C}_e & -\bar{\mathbf{B}}_{u_1} \mathbf{C}_Q & -\bar{\mathbf{B}}_{u_1} \mathbf{D}_Q \\ \mathbf{0} & \mathbf{A}_2 & \mathbf{0} & -\mathbf{K} \\ \mathbf{0} & \mathbf{B}_Q \mathbf{C}_e & \mathbf{A}_Q & \mathbf{B}_Q \\ \hline \bar{\mathbf{C}}_1 & \bar{\mathbf{C}}_2 - \mathbf{D}_w \mathbf{D}_Q \mathbf{C}_e & -\mathbf{D}_w \mathbf{C}_Q & -\mathbf{D}_w \mathbf{D}_Q \end{array} \right]$$

- Maximization of robust stability under  $\Delta_u$  :

$$\min \gamma \quad \text{LMI}_1 \quad \leftarrow \quad \min \|\mathbf{T}_{z_u b}\|_\infty$$

$$\left[ \begin{array}{ccc|ccc|ccc} \mathbf{R}_1 & \mathbf{0} & \mathbf{0} & \bar{\mathbf{A}}_1 \mathbf{R}_1 & \bar{\mathbf{A}}_1 \mathbf{S}_{11} - \mathbf{S}_{11} \mathbf{A}_2 - \mathbf{S}_{12} \mathbf{B}_Q \mathbf{C}_e + \bar{\mathbf{A}}_3 - \bar{\mathbf{B}}_{u_1} \mathbf{D}_Q \mathbf{C}_e & \bar{\mathbf{A}}_1 \mathbf{S}_{12} - \mathbf{S}_{12} \mathbf{A}_Q - \bar{\mathbf{B}}_{u_1} \mathbf{C}_Q & -\bar{\mathbf{B}}_{u_1} \mathbf{D}_Q + \mathbf{S}_{11} \mathbf{K} - \mathbf{S}_{12} \mathbf{B}_Q & \mathbf{0} \\ * & -\mathbf{T}_{11} & -\mathbf{T}_{12} & \mathbf{0} & \mathbf{T}_{11} \mathbf{A}_2 + \mathbf{T}_{12} \mathbf{B}_Q \mathbf{C}_e & \mathbf{T}_{12} \mathbf{A}_Q & -\mathbf{T}_{11} \mathbf{K} + \mathbf{T}_{12} \mathbf{B}_Q & \mathbf{0} \\ * & * & -\mathbf{T}_{22} & \mathbf{0} & \mathbf{T}_{12}^T \mathbf{A}_2 + \mathbf{T}_{22} \mathbf{B}_Q \mathbf{C}_e & \mathbf{T}_{22} \mathbf{A}_Q & -\mathbf{T}_{12}^T \mathbf{K} + \mathbf{T}_{22} \mathbf{B}_Q & \mathbf{0} \\ \hline * & * & * & -\mathbf{R}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{R}_1 \mathbf{C}_1^T \\ * & * & * & * & -\mathbf{T}_{11} & -\mathbf{T}_{12} & \mathbf{0} & \mathbf{S}_{11}^T \bar{\mathbf{C}}_1^T + \bar{\mathbf{C}}_2^T - \mathbf{C}_e^T \mathbf{D}_Q^T \mathbf{D}_w^T \\ * & * & * & * & * & -\mathbf{T}_{22} & \mathbf{0} & \mathbf{S}_{12}^T \bar{\mathbf{C}}_1^T - \mathbf{C}_Q^T \mathbf{D}_w^T \\ \hline * & * & * & * & * & * & -\gamma \mathbf{I} & -\mathbf{D}_Q^T \mathbf{D}_w^T \\ * & * & * & * & * & * & * & -\gamma \mathbf{I} \end{array} \right] \preceq \mathbf{0}$$

- Nominal performance for disturbances rejection

- Transfer  $\mathbf{d}(k) \rightarrow \mathbf{y}(k)$  for  $k = \overline{0, N_t}$

$$\begin{cases} \begin{bmatrix} \mathbf{y} \\ \mathbf{y}' \end{bmatrix} = \begin{bmatrix} \mathbf{T}_{11_{yd}} & \mathbf{T}_{12_{yd}} \\ \mathbf{T}_{21_{yd}} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{d} \\ \mathbf{u}' \end{bmatrix} \\ \mathbf{u}' = \mathbf{Q}\mathbf{y}' \end{cases} \Rightarrow \mathbf{y}(k) = \mathbf{T}_{11_{yd}} \mathbf{d}(k) + \mathbf{T}_{12_{yd}} \mathbf{Q} \mathbf{T}_{21_{yd}} \mathbf{d}(k)$$

Affine in  $\mathbf{Q}$

- Time domain templates

$$\mathbf{y}_{\min}(k) \leq \mathbf{y}(k) \leq \mathbf{y}_{\max}(k)$$

$$\begin{cases} \mathbf{T}_{11_{yd}} \mathbf{d}(k) + \mathbf{T}_{12_{yd}} \mathbf{Q} \mathbf{T}_{21_{yd}} \mathbf{d}(k) - \mathbf{y}_{\max}(k) \leq \mathbf{0} \\ \mathbf{T}_{11_{yd}} \mathbf{d}(k) + \mathbf{T}_{12_{yd}} \mathbf{Q} \mathbf{T}_{21_{yd}} \mathbf{d}(k) + \mathbf{y}_{\min}(k) \leq \mathbf{0} \end{cases} \xrightarrow{\text{Manipulations}} LMI_2 \Rightarrow \min_{LMI_1, LMI_2} \gamma$$

- ✓ Introduction
- ✓ Multivariable MPC
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- **Application to a stirred tank reactor**

- *System description & MPC parameters*
- *Robust stability under frequency constraints*
- *Nominal performance specifications*
  - *Decoupled system(2 templates)*
  - *Coupled system(4 templates)*

- **Conclusions**

# Application to a stirred tank reactor

- Simplified MIMO model of reactor

$$\begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix} = \begin{bmatrix} \frac{1}{1+0.7s} & \frac{5}{1+0.3s} \\ \frac{1}{1+0.5s} & \frac{2}{1+0.4s} \end{bmatrix} \begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix}$$

State-space representation

$$n = 4, m = 2, p = 2$$

$$\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{0} \end{bmatrix}$$

- Discretized for  $T_e = 0.03$  min

- MPC parameters

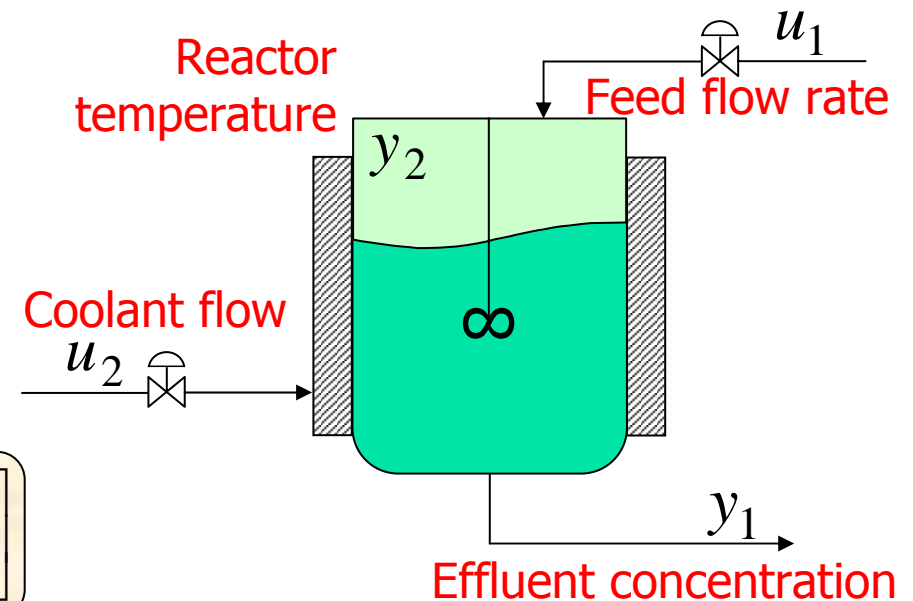
$$N_1 = 1, N_2 = 3, N_u = 2$$

$$\tilde{\mathbf{R}}_J = 0.05 \mathbf{I}_{N_u}, \tilde{\mathbf{Q}}_J = \mathbf{I}_{N_2 - N_1 + 1}$$

- Weightings

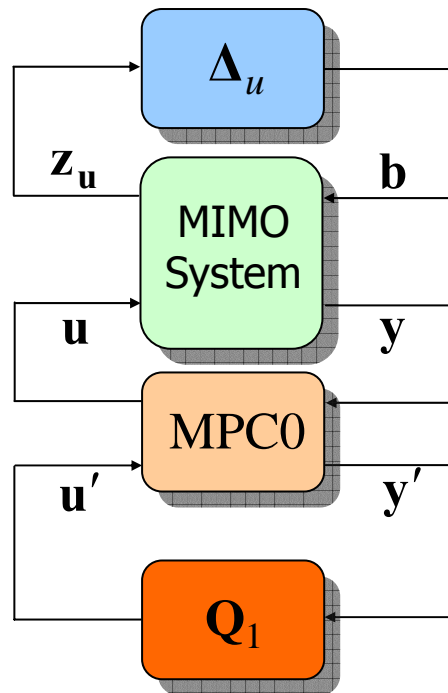
$$\mathbf{W}_u = \mathbf{I}_2 (1 - 0.7q^{-1}) / 0.3$$

$$\begin{bmatrix} \mathbf{A}_w & \mathbf{B}_w \\ \mathbf{C}_w & \mathbf{D}_w \end{bmatrix}$$

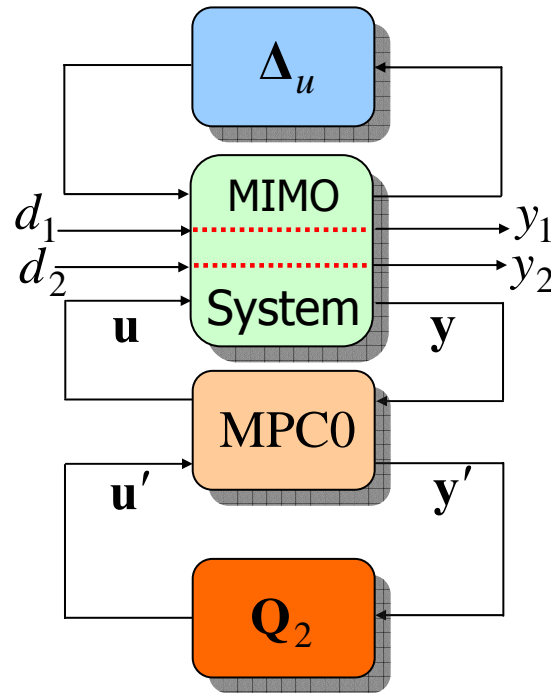


# Application to a stirred tank reactor

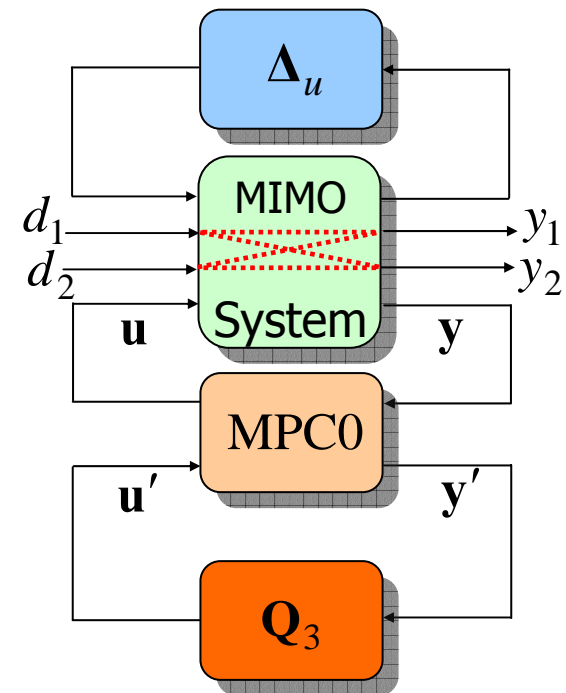
- **MPC0** : initial MPC controller (before robustification)



- **MPC1** : robustness under frequency constraints only



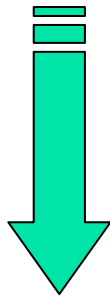
- **MPC2** : robustness under frequency and time-domain constraints – 2 templates



- **MPC3** : robustness under frequency and time-domain constraints – 4 templates

# Application to a stirred tank reactor

- Robustification using a Youla parameter of order  $n_Q = 40$
- Decreasing  $H_\infty$  norm



- Increasing stability robustness

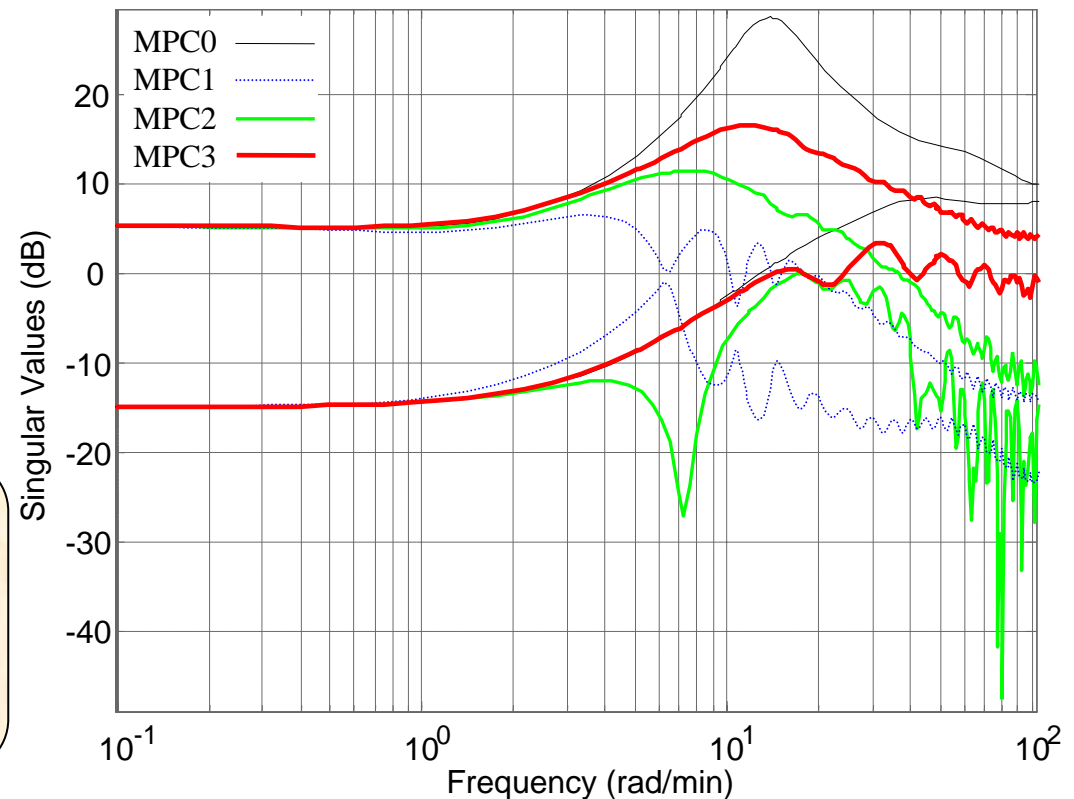
MPC0 = initial MPC

MPC1 = MPC0+LMI1

MPC2 = MPC0+LMI1+LMI2 (2 templates)

MPC3 = MPC0+LMI1+LMI2 (4 templates)

Singular values of  $\mathbf{T}_{ub}$

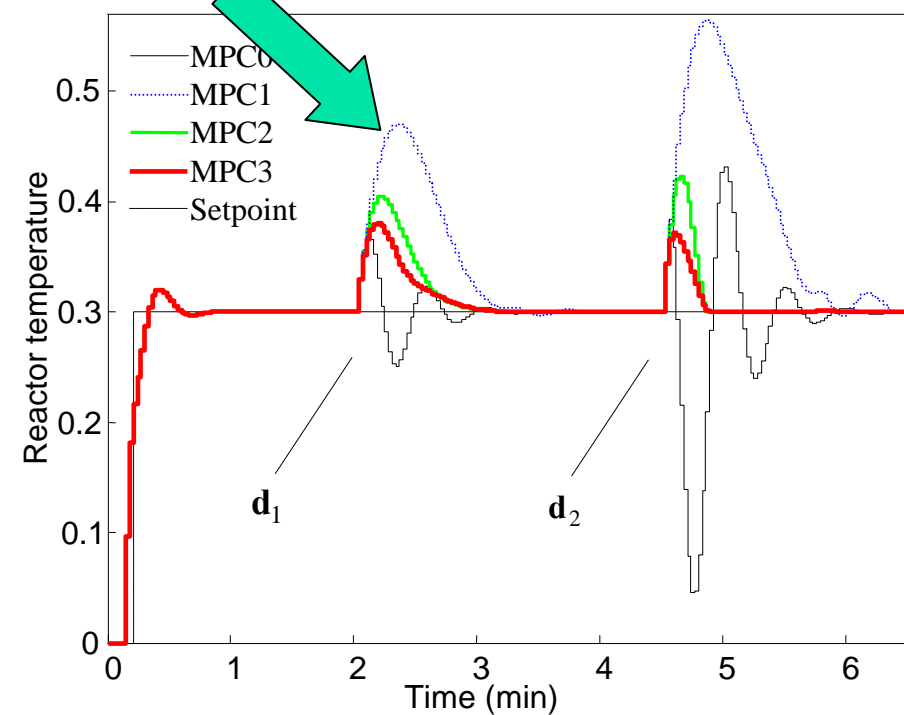
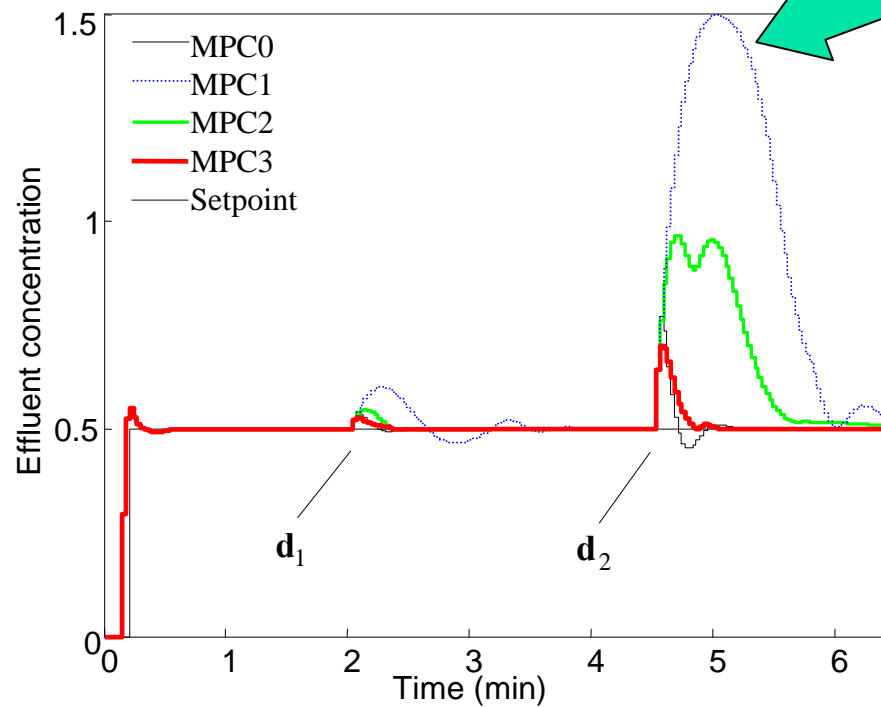


# Application to a stirred tank reactor

- Time-domain responses
- Disturbances rejection

$$y_{r_1} = 0.5, y_{r_2} = 0.3$$
$$d_1 = 0.5, d_2 = 0.3$$

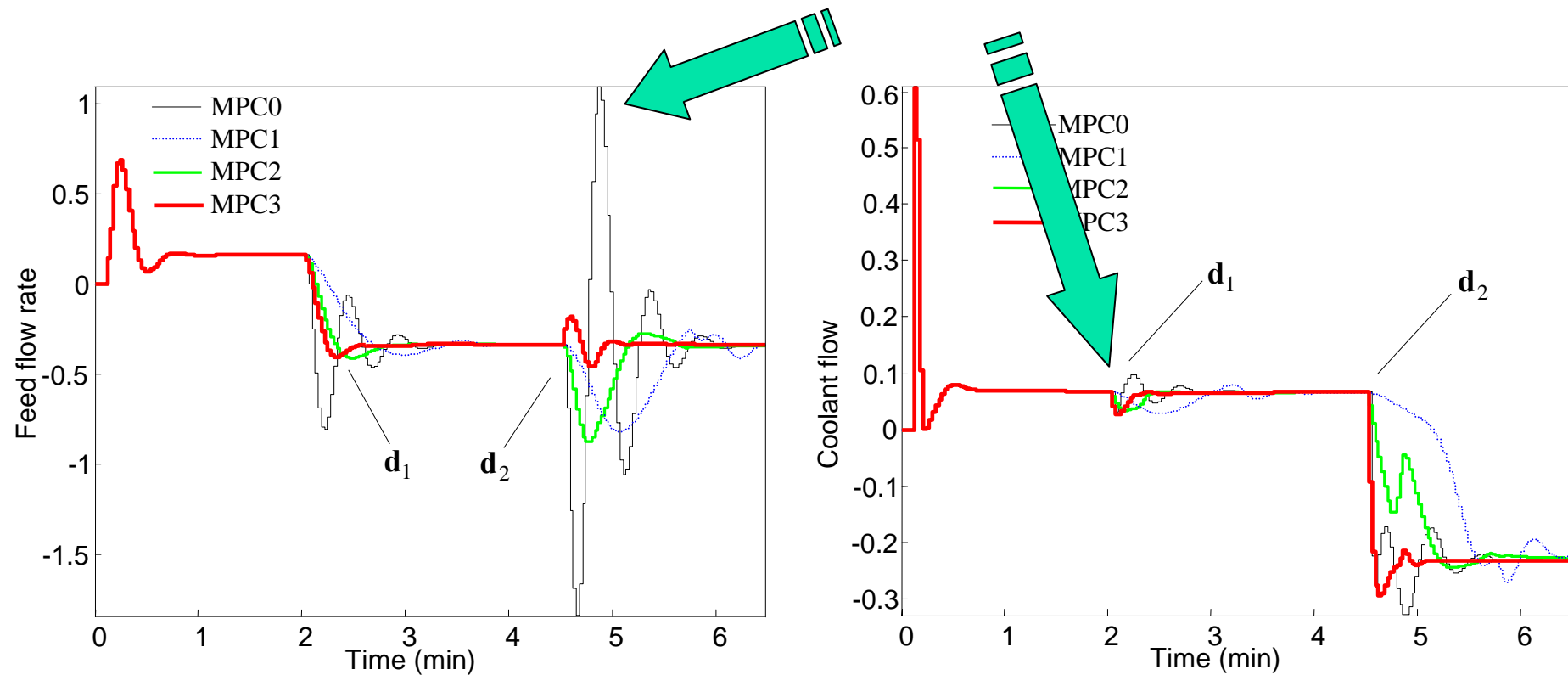
Coupling influence



# Application to a stirred tank reactor

## ■ Control signals

Coupling influence

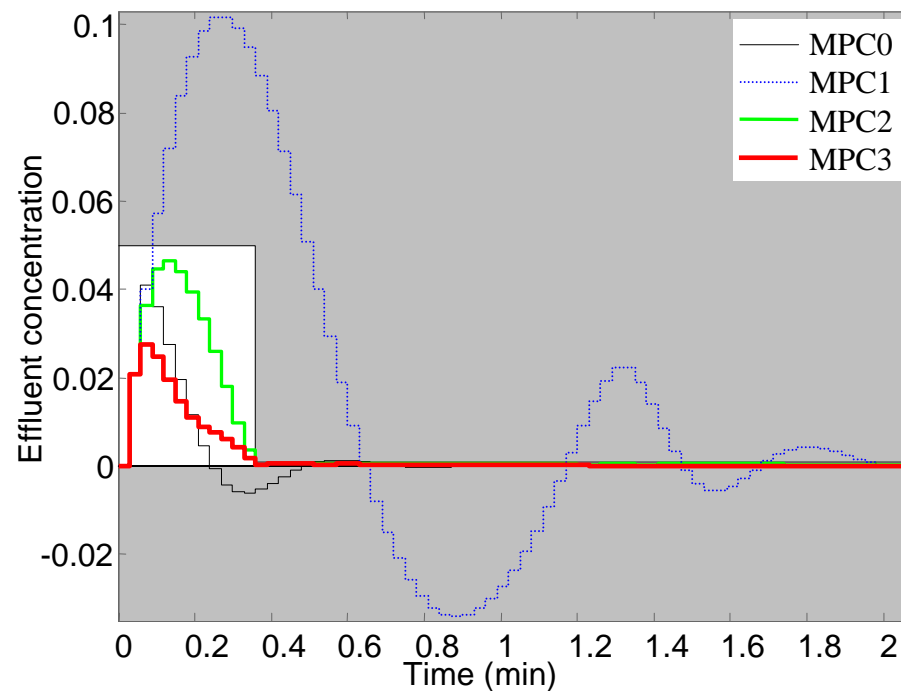




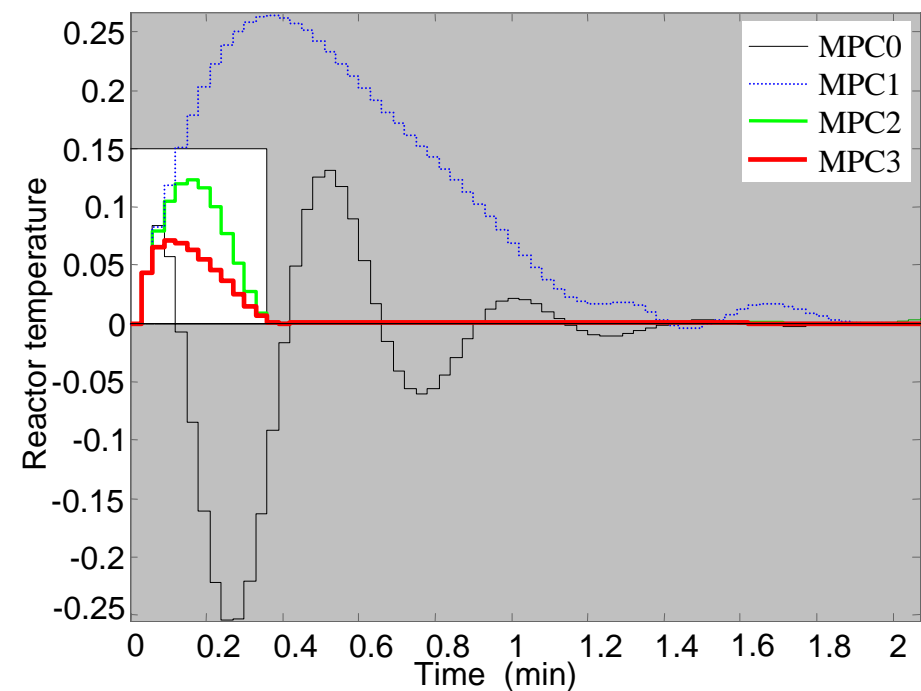
# Application to a stirred tank reactor

- Time-domain templates for disturbances rejection

$d_1 \rightarrow y_1$



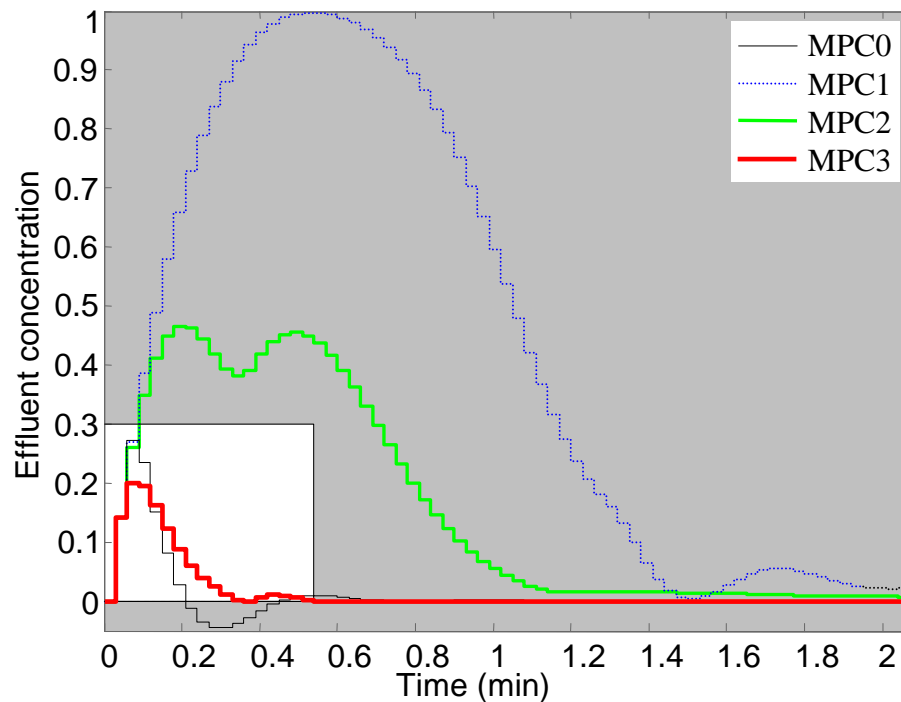
$d_2 \rightarrow y_2$



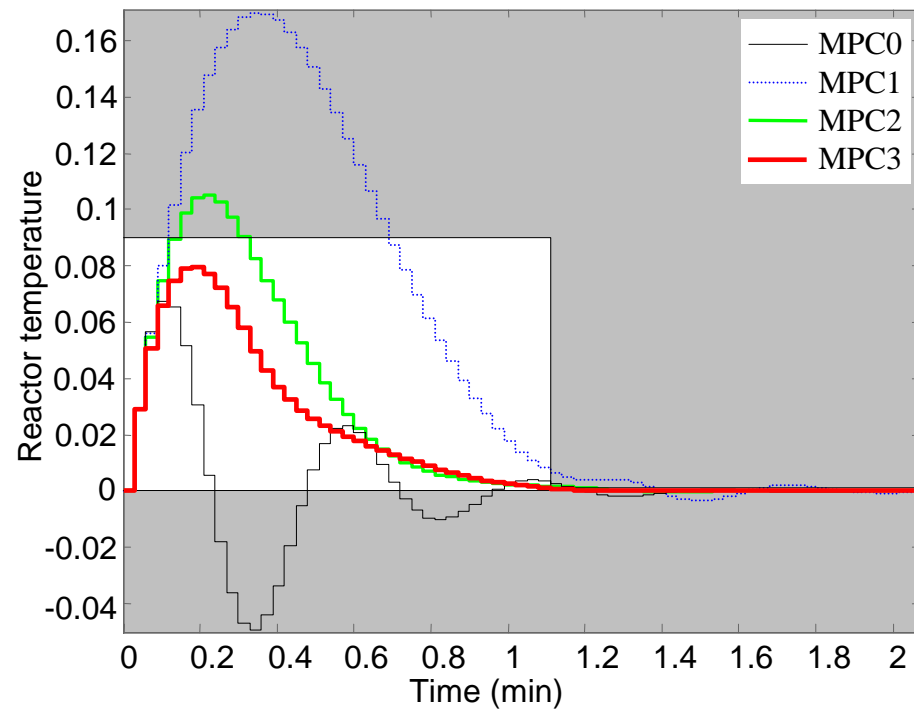
# Application to a stirred tank reactor

- Time-domain templates for disturbances rejection

$d_2 \rightarrow y_1$

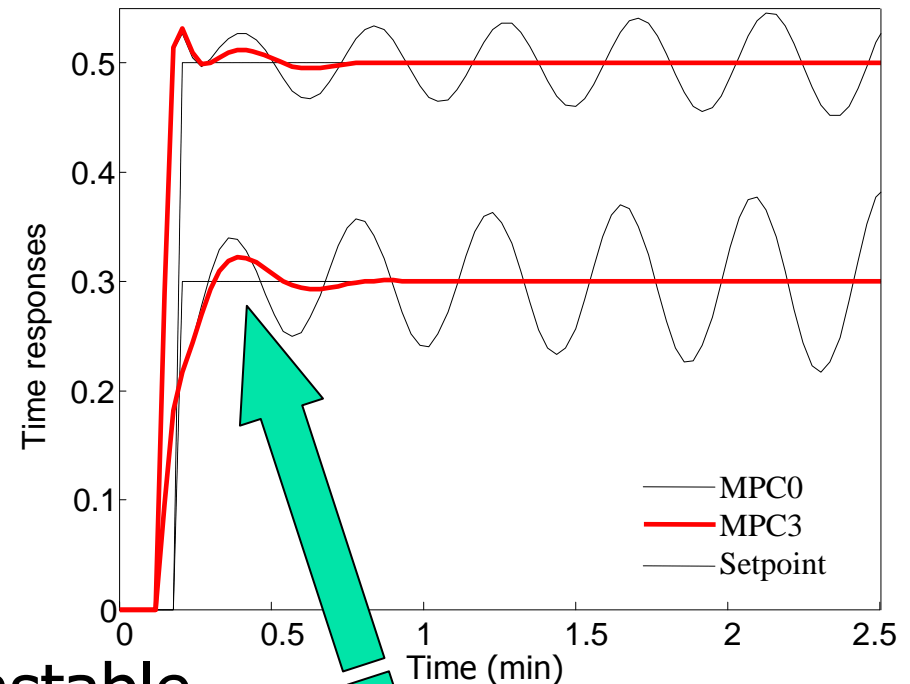
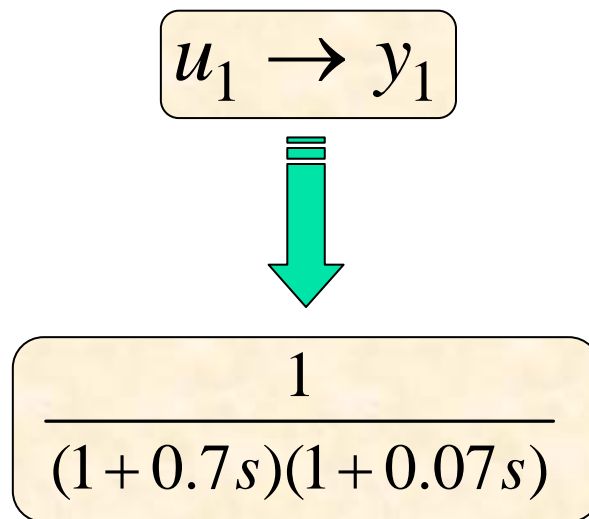


$d_1 \rightarrow y_2$



# Application to a stirred tank reactor

- Validation of robust stability under a neglected dynamics of  $u_1$  corresponding to the transfer



- Without robustification – unstable
- After robustification – stable

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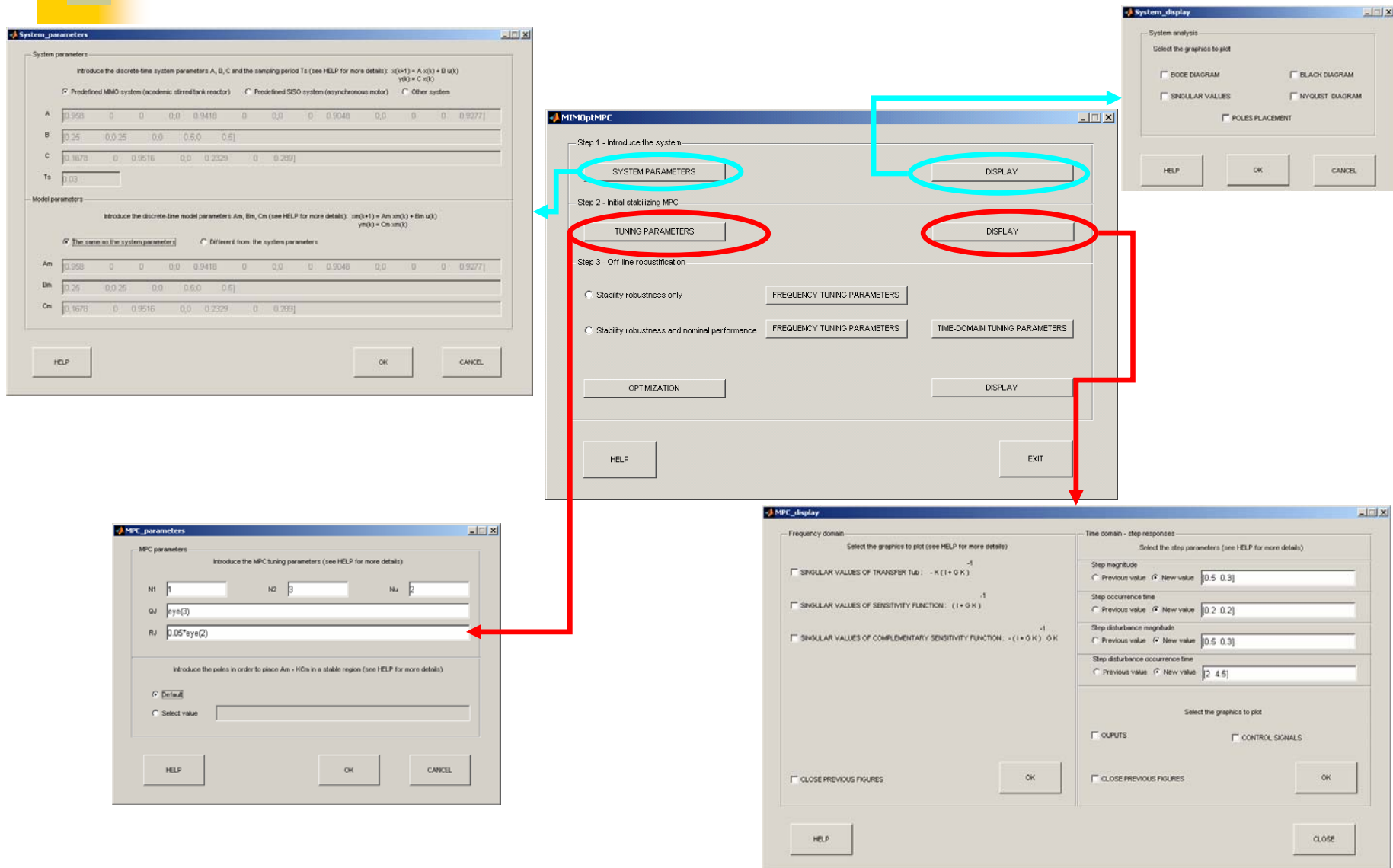
# Conclusions & current work

- Complete off-line methodology which enables robustification of an initial MIMO MPC
- Input/output behaviour remains unchanged
- Improved robustness towards unstructured uncertainties through a convex optimization problem (LMI techniques)
- Use of four time-domain templates to manage the coupling effect in an efficient way
- Compromise between robust stability and nominal performance specifications
- Reduced computational effort due to state-space representation

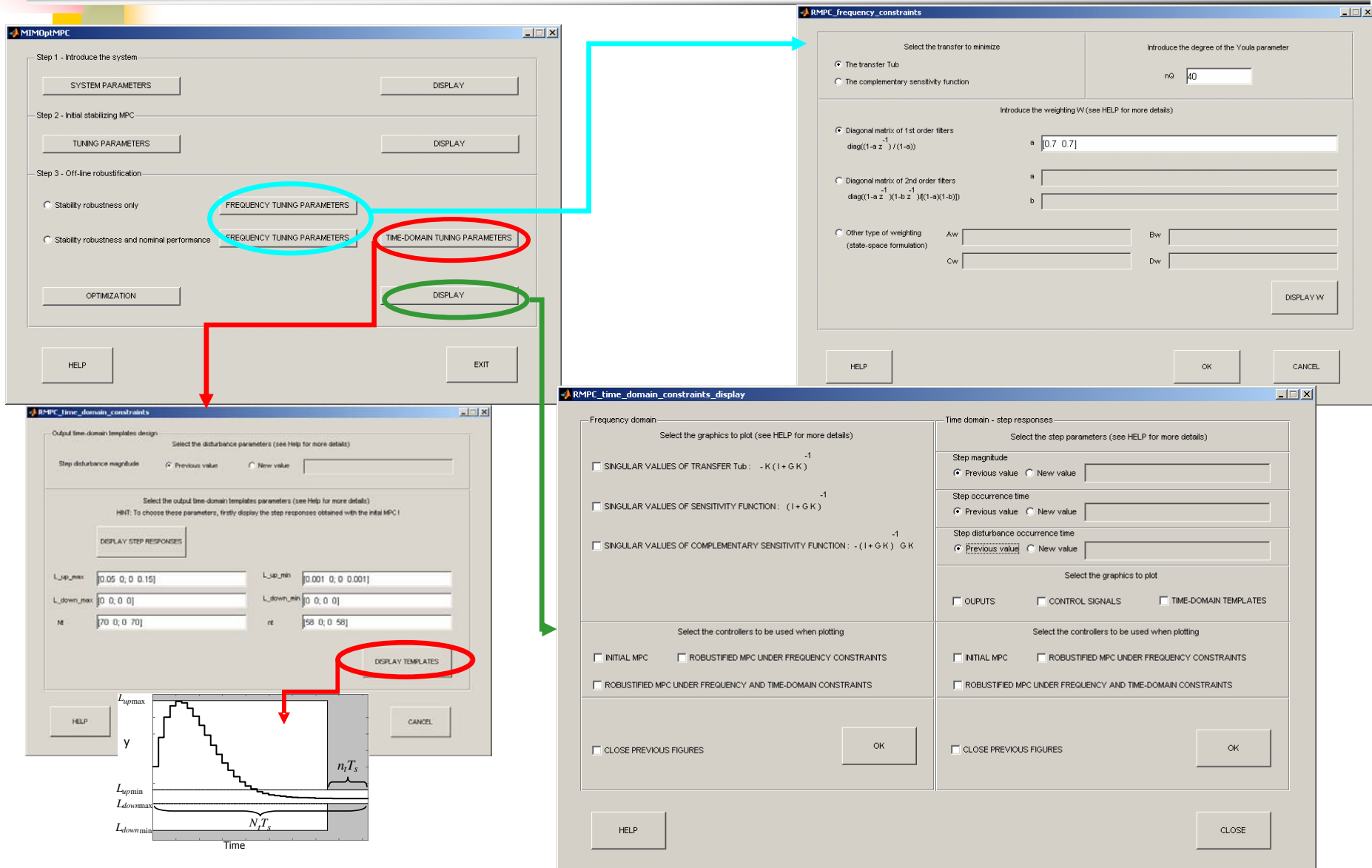
# Conclusions & current work

- Developing a new MATLAB toolbox (MIMOptMPC) based on the theoretical feature of this off-line robustification procedure
  
- An user friendly and easily extensible toolbox
  - Choice of tuning parameters and robustification options
  - Visualization tools enabling performances evaluation
  
- A helpful solution for non-specialist users as well as researchers working in the field of robust MPC

# Conclusions & current work



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The image displays three windows from the RMPC software interface:

- RMPC\_frequency\_constraints**: This window allows for selecting the transfer to minimize (The transfer Tub or The complementary sensitivity function) and introducing the degree of the Youla parameter (ng = 40). It also includes options for diagonal matrices of 1st and 2nd order filters, and other weighting options (Aw, Bw, Cw, Dw).
- RMPC\_time\_domain\_constraints\_display**: This window shows the frequency domain and time domain - step responses. It includes options to select the graphics to plot (SINGULAR VALUES OF TRANSFER Tub, SINGULAR VALUES OF SENSITIVITY FUNCTION, SINGULAR VALUES OF COMPLEMENTARY SENSITIVITY FUNCTION) and the controllers to be used when plotting (INITIAL MPC, ROBUSTIFIED MPC UNDER FREQUENCY CONSTRAINTS, ROBUSTIFIED MPC UNDER FREQUENCY AND TIME-DOMAIN CONSTRAINTS).
- RMPC\_line\_domain\_constraints**: This window is used for output time-domain templates design. It includes fields for step disturbance magnitude, output time-domain templates parameters (L<sub>up\_max</sub>, L<sub>up\_min</sub>, L<sub>down\_max</sub>, L<sub>down\_min</sub>, nt, nr), and a plot of the step response. The plot shows the output y over time, with parameters L<sub>up\_max</sub>, L<sub>up\_min</sub>, L<sub>down\_max</sub>, L<sub>down\_min</sub>, and N<sub>r</sub>T<sub>s</sub> indicated.

Arrows indicate the flow of data between these windows:

- A red arrow points from the **FREQUENCY TUNING PARAMETERS** button in the main window to the **RMPC\_frequency\_constraints** window.
- A green arrow points from the **TIME-DOMAIN TUNING PARAMETERS** button in the main window to the **RMPC\_time\_domain\_constraints\_display** window.
- A red arrow points from the **DISPLAY** button in the main window to the **RMPC\_line\_domain\_constraints** window.