Fault tolerant control command based on set separation

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Project and collaborations

- PhD student: F. Stoican (C3S)
- project related with CDSC center (Complex Dynamic Systems and Control, The University of Newcastle, Australia): J.A. De Dona and M.M. Seron
- practical approach, a fault tolerant control device for a position control plant: in collaboration with Blegrade University (S. Marinkovic et M. Nesic)
- periodic discussions: VIDAMES group

Outline

Outline

Fault tolerant control

Goals

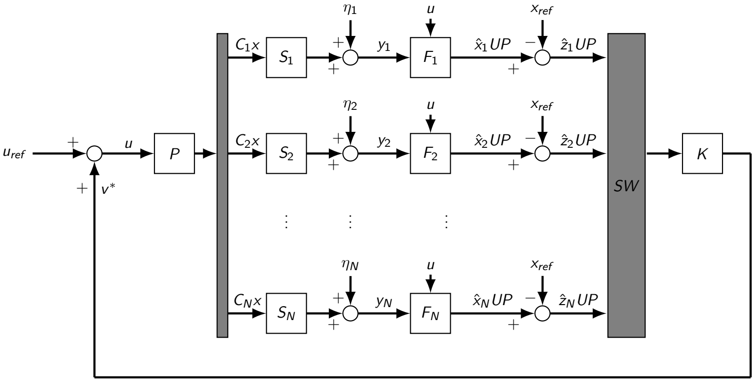
- fault detection and isolation
- command and optimisation
  - performance optimisation
  - constraint verification

Methodology

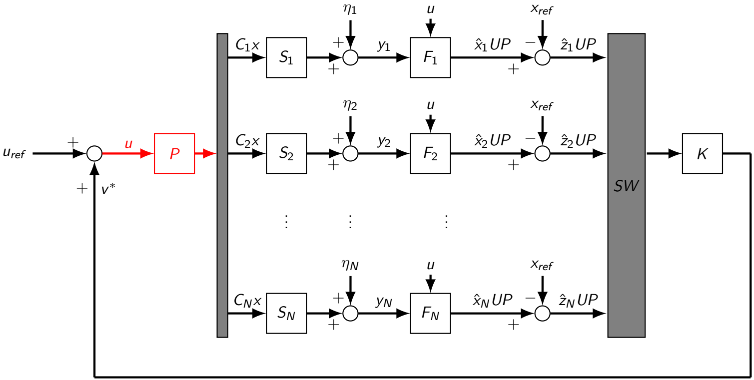
- characterisation of invariant sets in healthy/faulty cases for residual signals
- fault detection using set membership techniques

Outline

Multisensor scheme with common feedback gain

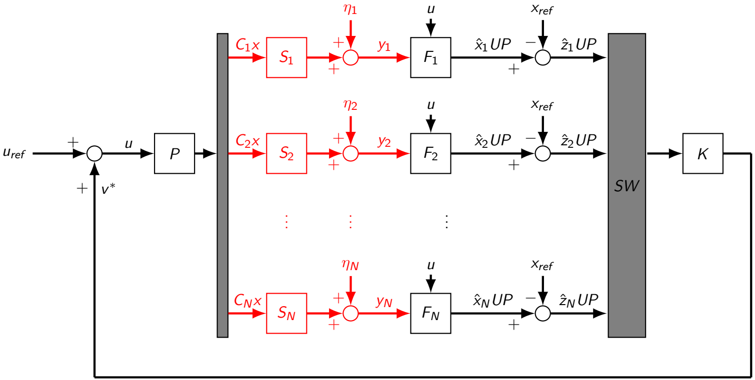


Model description – plant



$$x^+ = Ax + Bu + Ew$$

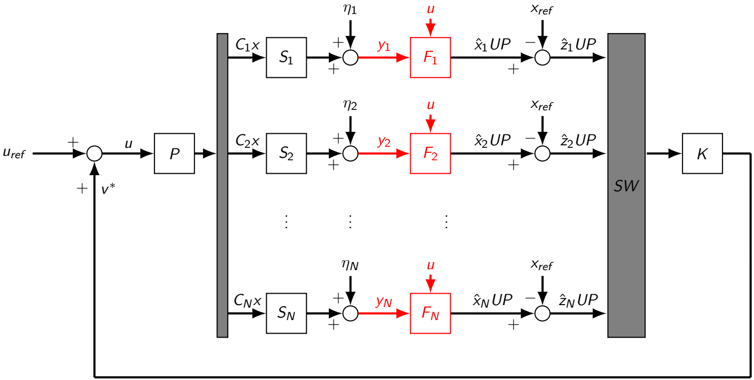
Model description – sensors



healthy behavior: $y_i = C_i x + \eta_i$

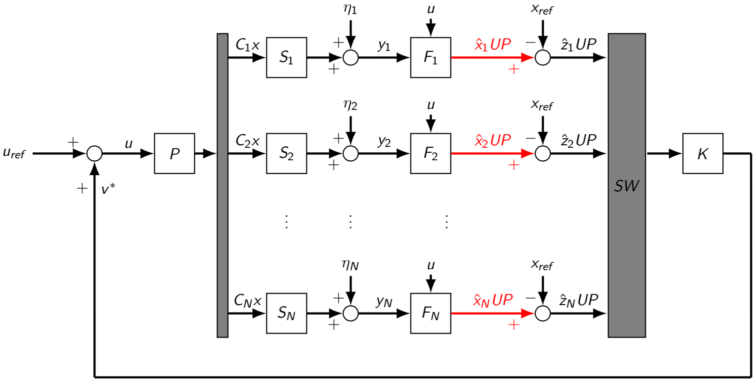
faulty behavior: $y_i^F = \eta_i^F$

Model description – estimators



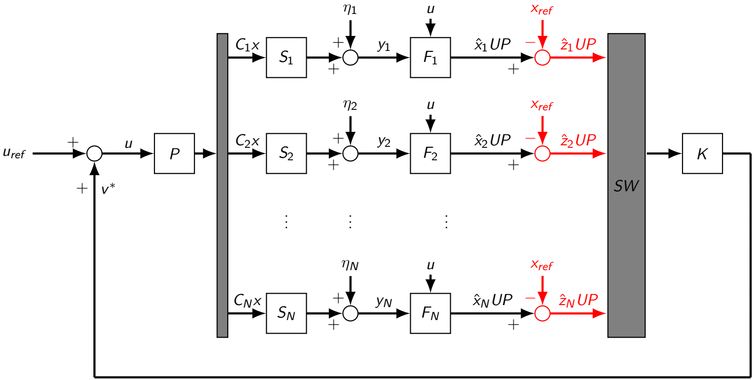
$$\hat{x}_i^+ = A\hat{x}_i + Bu + L_i\left(y_i - C_i\hat{x}_i\right)$$

Model description – updates



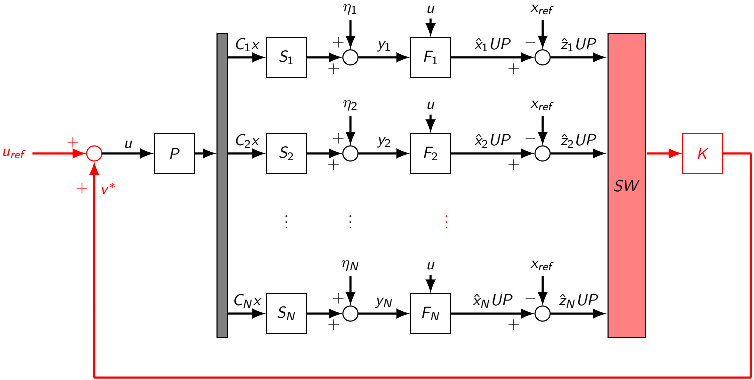
$$\hat{x}_i^{UP} = \hat{x}_i + M_i\left(y_i - C_i\hat{x}_i\right)$$

Model description – tracking error



$$\hat{z}_i^{UP} = \hat{x}_i^{UP} - x_{ref}$$

Model description – controller



$$J(\hat{z}, v) = \hat{z}' Q \hat{z} + v' R v + (A\hat{z} + Bv)' P (A\hat{z} + Bv)$$

Assumptions

- A is stabilisable and pair (A, B) is controllable
- pairs (A, C_i) are detectable for $i = 1, \ldots, N$
- adittive disturbances and the measurements perturbations are considered to be delimited by bounded polyhedral sets

Modelling equations

- plant dynamics

$$x^+ = Ax + Bu + Ew$$

- reference signal

$$x^+_{ref} = Ax_{ref} + Bu_{ref}$$

- plant tracking error

$$z^+ = x - x_{ref} = Az + B\underbrace{\left(u - u_{ref}\right)}_{v} + Ew$$

- estimations and updates of the state

$$\begin{aligned} \hat{x}_i^+ &= \left(A - L_i C_i\right)\hat{x}_i + Bu + L_i\left(y_i - C_i\hat{x}_i\right) \\ \hat{x}_i^{UP} &= \hat{x}_i + M_i\left(y_i - C_i\hat{x}_i\right) \end{aligned}$$

- estimations of the tracking error

$$\hat{z}_i^{UP} = \hat{x}_i + M_i\left(y_i - C_i\hat{x}_i\right) - x_{ref}$$

Switching criteria

At every step a pair sensor-estimator is selected to compute the command action s.t. the following cost function is minimised

$$J\left(\hat{z}^{UP}, v\right) = \left(\hat{z}^{UP}\right)' Q\hat{z}^{UP} + \left(A\hat{z}^{UP} + Bv\right)' P\left(A\hat{z}^{UP} + Bv\right)$$

for $\hat{z}^{UP} \in \hat{Z}^{UP} = \left\{\hat{z}_1^{UP}, \ldots, \hat{z}_N^{UP}\right\}$ the tracking error estimation

The command action is then defined as

$$u^* = u_{ref} - K\hat{z}^*$$

with

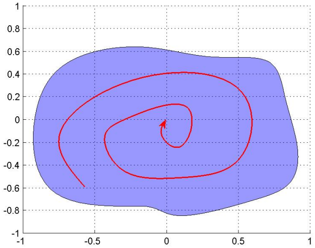
$$\begin{aligned} \hat{z}^* &= \underset{\hat{z}^{UP}}{argmin} \left\{\hat{z}^{UP,T} P\hat{z}^{UP}\ \hat{z}^{UP} \in \hat{z}^{UP}\right\} \\ &= \underset{\hat{z}^{UP}}{argmin} \left\{J\left(\hat{z}^{UP}, v\right)\ \hat{z}^{UP} \in \hat{z}^{UP}, \mathbb{R}^m\right\} \end{aligned}$$

Stability under normal functioning

Error tracking signal

$$z^+ = (A - BK)z$$

- autonomous system
- bounded perturbations



All trajectories converge assymptotically to origin

Stability under normal functioning

Error tracking signal

$$z^{+} = (A - BK)z + \begin{bmatrix} E & BK(I - M_I C_I) & -BKM_I \end{bmatrix} \begin{bmatrix} w \\ \tilde{x}_I \\ \eta_I \end{bmatrix}$$

- autonomous system
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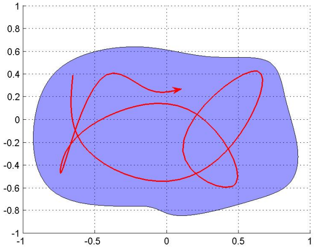
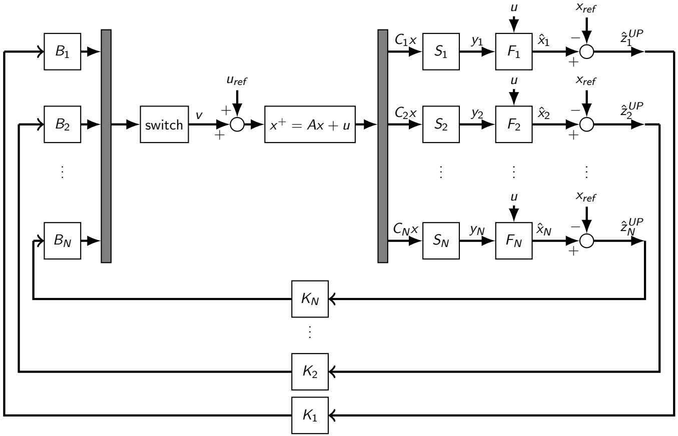


All trajectories converge assymptotically to a bounded region

Multisensor scheme with multiple feedback gains



The switch mechanism choses at each instant of time a sensor-feedback-actuator loop

Assumptions

- A is stabilisable and pairs (A, B_i) are controllable for $i = 1, \ldots, N$
- pairs (A, C_i) are detectable for $i = 1, \ldots, N$
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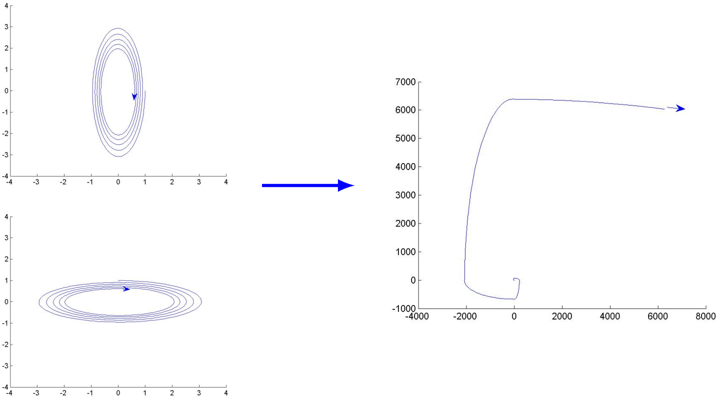
Switched systems stability

Note (Branicky (1994)): A switched system may not be stable even if all subsystems are stable



Dwell time stability

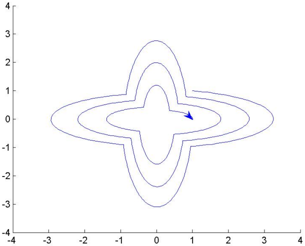
Let there be the switched autonomous system

$$x^+ = A_i x$$

Theorem (Geromel and Colaneri (2006))

Assume there exist P_i s.t.

$$\begin{cases} P_i > 0 \\ A_i' P_i A_i + P_i \leq 0 \\ {A_i'}^T P_j {A_i}^T < P_i \quad \forall j \neq i \end{cases}$$



then, the system is globally stable for any switch occuring at moments greater or equal with T

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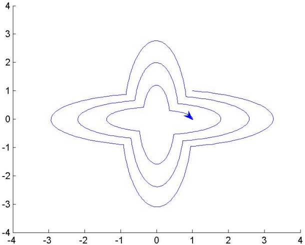

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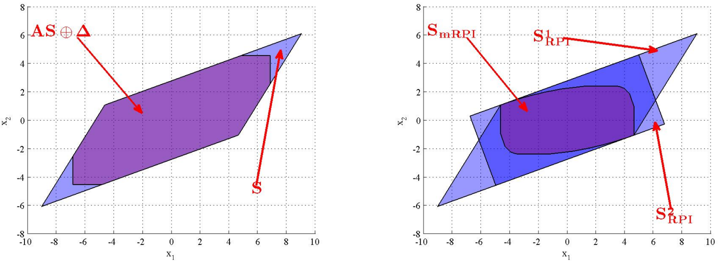
Outline

Definitions

Let there be a dynamic system defined by

$$x^+ = Ax + \delta, \quad \delta \in \Delta$$



Definition (RPI)

A set Ω is robust positively invariant (RPI) if and only if

$$x \in \Omega \rightarrow x^+ \in \Omega$$

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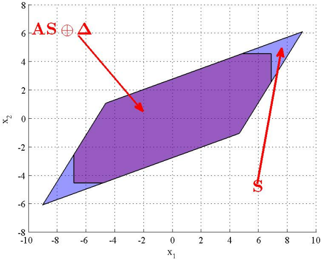

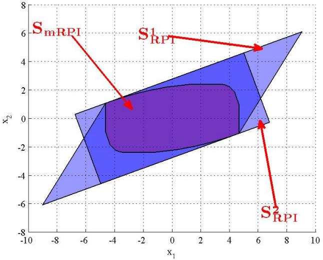

Definition (mRPI)

A set Ω is minimal robust positively invariant (mRPI) if it is contained in all RPI sets.

Definitions

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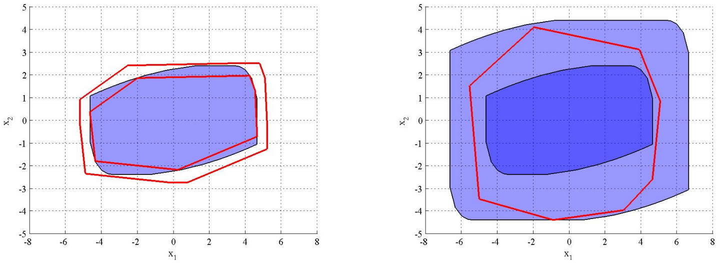

Definition (mRPI ϵ-approximations)

- *ϵ-inner approximations:* $\Phi \subseteq \Omega \subseteq \Phi \oplus \mathbb{B}^n_\infty(\epsilon)$
- *ϵ-outer approximations:* $\Omega \subseteq \Phi \subseteq \Omega \oplus \mathbb{B}^n_\infty(\epsilon)$

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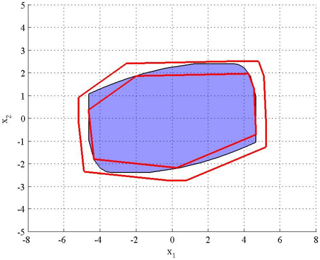

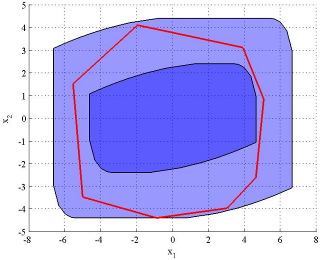

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Ultimate bounds

Theorem (Ultimate bounds – discrete case)

Consider the stable system $x^+ = Ax + Bu$. Let there be the Jordan decomposition $A = V\Lambda V^{-1}$ and assume that $\left|u(k)\right| \leq \bar{u}, \forall k \geq 0$. Then there exists $l(\epsilon)$ such that for all $k \geq l$:

$$\begin{aligned} |V^{-1}x(k)| &\leq (I - |\Lambda|)^{-1}|V^{-1}B|\bar{u} + \epsilon \\ |x(k)| &\leq |V|(I - |\Lambda|)^{-1}|V^{-1}B|\bar{u} + |V|\epsilon \end{aligned}$$

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Proof (**?**):
We can write

$$\begin{aligned} x^+ &= Ax + Bu = V\Lambda V^{-1}x + Bu \\ V^{-1}x^+ &= \Lambda V^{-1}x + V^{-1}Bu \\ |z^+| &\leq |\Lambda z + V^{-1}Bu| \leq |\Lambda|z + |V^{-1}B|\bar{u} \end{aligned}$$

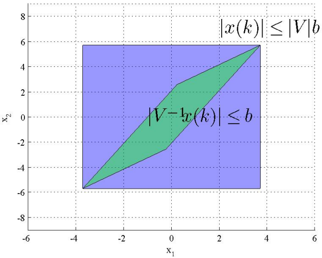
and, then:

$$|V^{-1}x| \leq (I - |\Lambda|)^{-1}|V^{-1}B|\bar{u} + \epsilon$$

Exemplification for a $\mathbb{R}^2$ case

$$x(k+1) = Ax(k) + Bu(k)$$

where $\left|u(k)\right| \leq 1$



$$A = \begin{bmatrix} 0.0241 & 0.4184 \\ -0.7869 & 1.2759 \end{bmatrix} \qquad b = \begin{bmatrix} 0.90 \\ 1.75 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.8462 \\ 0.5252 \end{bmatrix} \qquad |V|b = \begin{bmatrix} 3.70 \\ 5.68 \end{bmatrix}$$

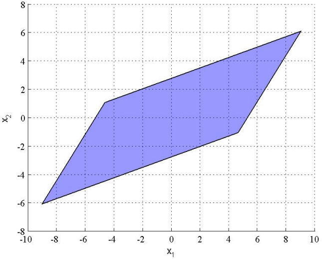
mRPI outer approximations

Note: An alternative formulation of a mRPI set can be given

$$\Omega = \bigoplus_{i=0}^{i=\infty} A^i \Delta$$

This permits the computation of a sequence of RPI outer approximations of the mRPI set

$$\Phi_{k+1} = A\Phi_k \oplus \Delta$$



Theorem (Olaru et al. (2008))

For any $\epsilon \geq 0$ exists $s \in \mathbb{N}^+$ such that the following relation is true

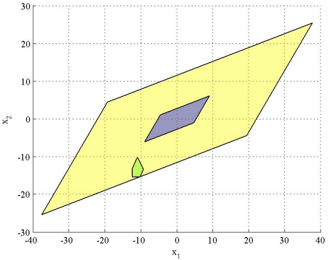
$$\Omega \subset \Phi_s \subset \Omega \oplus \mathbb{B}_p^n(\epsilon)$$

mRPI with region inclusion

Theorem (Olaru et al. (2008))

The RPI construction Φ_0 can be scaled to include a predefined region P such that the invariance is preserved

$$\begin{aligned} \mu^* &= \mu^*(\Phi_0, P) \\ &= \min_{\substack{\mu \geq 1 \\ P \subset \mu\Phi_0}} \mu \\ \Psi_0 &= \mu^* \Phi_0 \end{aligned}$$



An iterative sequence can be constructed for further enhancements

$$\Psi_{k+1} = ConvHull\{P, A\Psi_k \oplus \Delta\}$$

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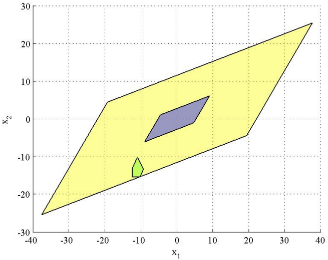

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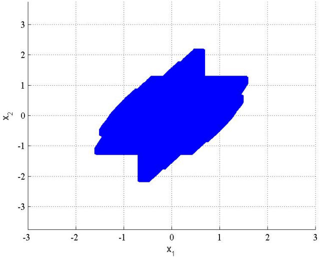
Nonconvex cases

Let there be a system switched in perturbations:

$$\begin{aligned} x^+ &= Ax + \delta_i \\ \delta_i &\in \Delta_i,\ i \in \{1, \dots, N\} \end{aligned}$$

$$kern\left(\bigcup_{i=1}^{N} \Delta_i\right) = \bigcap_{i=1}^{N} \Delta_i \neq \emptyset$$



The mRPI set is star-shaped (Rubinov and Yagubov (1986)) since the intersection of star shaped sets is also a star shaped as long as their kernels intersect.

Outline

Fault scenarios

- total output outages

$$y_i = C_i x + \eta_i \quad \xrightarrow{FAULT} \quad y_i = 0 \cdot x + \eta_i^F$$
$$y_i = C_i x + \eta_i \quad \xleftarrow{RECOVERY} \quad y_i = 0 \cdot x + \eta_i^F$$

- more complex fault scenarios (a signature matrix for each type of fault)

$$y_i = \Pi_i \left[C_i x + \eta_i \right] + \left[I - \Pi_i \right] \eta_i^F$$

Fault detection and isolation strategy

Fault detection and isolation (FDI) strategies can employ

- probability approach (sensor fusion)
- robust approach (set membership)

The robust detection requires

- sets that define the healthy/faulty functioning of the sensors
- a method of acknowledging the recovery of previously fallen sensors

Requirements

- at least one operational sensor must be acknowledged as healthy at each instant of time

Residual signals

The residual signal (Blanke et al. (2006)), composed from all the measurable quantities associated to the i^{th} sensor can be defined:

$$r_i = \hat{z}_i^{UP} - (I - M_i C_i)\, \hat{z}_i$$

Reminder:

- $z = x - x_{ref}$
- $\hat{x}_i^{Up} = \hat{x}_i + M_i(y_i - C_i\hat{x}_i)$
- $\hat{z}_i^{Up} = \hat{x}_i^{Up} - x_{ref}$

Residual signals values for a sensor

- healthy case:

$$r_i^H = M_i C_i z + M_i \eta_i$$

- faulty case:

$$r_i^F = -M_i C_i x_{ref} + M_i \eta_i^F$$

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Residual signals values for a sensor

- healthy case:

$$R_i^H = M_i C_i S_z \oplus M_i \Pi_i$$

- faulty case:

$$R_i^F = \{-M_i C_i X_{ref}\} \oplus M_i \Pi_i^F$$

Aditional sets

- Π_i, Π_i^F, W – bounding boxes for sensor and plant noises
- X_{ref} – set for the reference signal
- $\tilde{S}_i$ – invariant set for the state estimation error
- S_z – invariant set for the plant tracking error

State estimation error:

$$\tilde{x}_i^+ = x^+ - \hat{x}_i^+ = (A - L_i C_i)\,\tilde{x}_i + \begin{bmatrix} E & -L_i \end{bmatrix} \begin{bmatrix} w \\ \eta_i \end{bmatrix}$$

Plant tracking error:

$$z^+ = (A - BK)\,z + \begin{bmatrix} E & BK\,(I - M_I C_I) & BKM_I \end{bmatrix} \begin{bmatrix} w \\ \tilde{x}_I \\ \eta_I \end{bmatrix}$$

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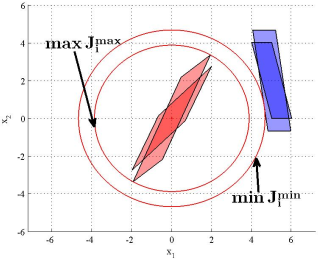
Implicit separation

Theorem (Seron et al. (2008))

The closed loop dynamics remain stable in the presence of faults if

$$\max_i J_i^{max} \leq \min_i J_i^{min}$$



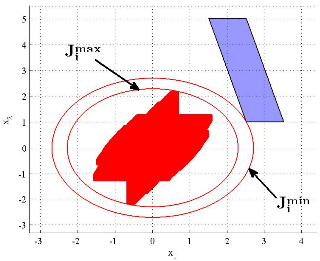
$$J_i^{min} = min\left\{\left(r_i^H\right)' r_i^H,\ r_i^H \in R_i^H\right\}$$

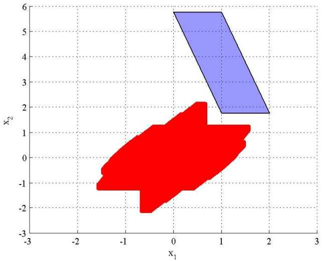
$$J_i^{max} = max\left\{\left(r_i^F\right)' r_i^F,\ r_i^F \in R_i^F\right\}$$

Implicit separation – nonconvex case

Separation through barrier using gauge functions

$$J_{gauge}\left(\hat{z}_i^{UP}\right) = \bar{J}_{LQ} \cdot \left\{\lceil \rho_H\left(\hat{z}_i^{UP}\right)\rceil - 1\right\} + J_{LQ} \cdot \lceil \rho_H\left(\hat{z}_i^{UP}\right)\rceil$$





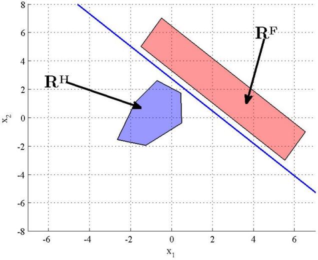
with the gauge function $\rho(x) = inf\,\{\lambda :\ x \in \lambda S\}$

Explicit separation

Each sensor is tested separately

$$R_i^H \cap R_i^F = \emptyset, \quad i = \{1, \cdots, N\}$$

$$\begin{aligned} R_i^H &= M_i(S_z \oplus C_i N_i) \\ R_i^F &= M_i(-X_{ref} \oplus C_i N_i^F) \end{aligned}$$



Theorem (Gritzmann and Klee (1994))

Any two non intersecting convex sets can be separated by a hyperplane

Outline

Recovery problem

Conditions for recovery acknowledgement

- $\hat{z}_j \in \hat{S}_H^j$ – tracking error
- $\hat{z}_j^{UP} \in \hat{S}_H^{UP,j}$ – updated tracking error
- $\tilde{x}_j \in \tilde{S}_j$ – estimation error

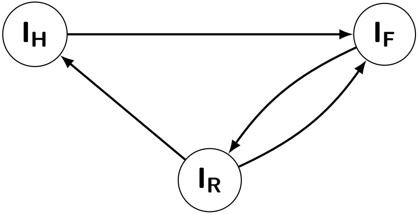
$\tilde{x}_j = x - \hat{x}_j$ is not measurable

Strategies

- necessary conditions
- sufficient conditions

Sensor partitioning

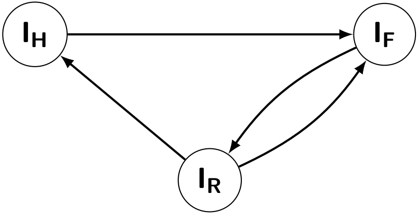
$$I = I_H \cup I_F \cup I_R$$



- $I_H = \left\{ i \in I : \tilde{x}_i \in \tilde{S}_i, \hat{z}_i \in \hat{S}_H^i, \hat{z}_i^{UP} \in \hat{S}_H^{UP,i} \right\}$
- $I_F = \left\{ i \in I : \left(\hat{z}_i \notin \hat{S}_H^i \right) \vee \left(\hat{z}_i^{UP} \notin \hat{S}_H^{UP,i} \right) \right\}$
- $I_R = \left\{ i \in I : \tilde{x}_i \notin \tilde{S}_i, \hat{z}_i \in \hat{S}_H^i, \hat{z}_i^{UP} \in \hat{S}_H^{UP,i} \right\}$

Sensor transitions

$$I = I_H \cup I_F \cup I_R$$



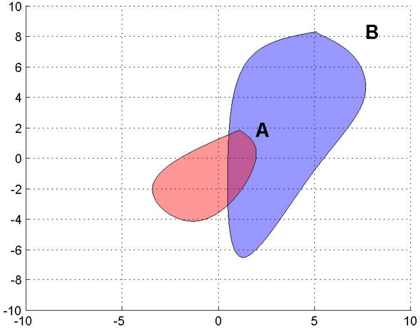
$I_H \rightarrow I_F$ If $\{i \in I_H\} \wedge \{\hat{z}_i^{UP} \in \hat{S}_{HF}^i\}$ then
$I_H = I_H \setminus \{i\};\ I_F = I_F \cup \{i\}$

$I_F \rightarrow I_R$ If $\{i \in I_F\} \wedge \{\hat{z}_i^{UP} \in \hat{S}_H^i\}$ then
$I_F = I_F \setminus \{i\};\ I_R = I_R \cup \{i\}$

$I_R \rightarrow I_F$ If $\{i \in I_R\} \wedge \{\hat{z}_i^{UP} \notin \hat{S}_H^i\}$ then
$I_R = I_R \setminus \{i\};\ I_F = I_F \cup \{i\}$

$I_R \rightarrow I_H$ If $\{i \in I_R\} \wedge \{\hat{z}_i^{UP} \in \hat{S}_H^i\} \wedge \{\tilde{x}_i \in \tilde{S}_i\}$
then $I_R = I_R \setminus \{i\};\ I_H = I_H \cup \{i\}$

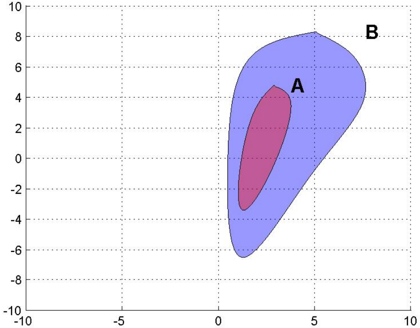
Necessary and sufficient conditions



Let $\mathcal{A}$ and $\mathcal{B}$ be twho sets, then

- $\alpha \in \mathcal{A}$, a necessary condition for $\alpha \in \mathcal{B}$ is $\mathcal{A} \cap \mathcal{B} \neq \emptyset$
- $\alpha \in \mathcal{A}$, a sufficient condition for $\alpha \in \mathcal{B}$ is $\mathcal{A} \subseteq \mathcal{B}$

Necessary and sufficient conditions

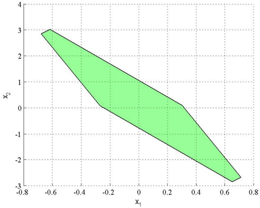


Let $\mathcal{A}$ and $\mathcal{B}$ be twho sets, then

- $\alpha \in \mathcal{A}$, a necessary condition for $\alpha \in \mathcal{B}$ is $\mathcal{A} \cap \mathcal{B} \neq \emptyset$
- $\alpha \in \mathcal{A}$, a sufficient condition for $\alpha \in \mathcal{B}$ is $\mathcal{A} \subseteq \mathcal{B}$

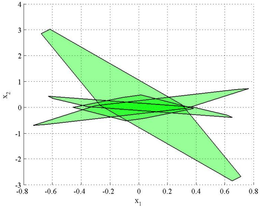
Sensor recovery – I

$$z = \underbrace{\hat{z}_I^{UP}}_{\text{measured value}} + \underbrace{\left(I - M_I C_I\right)\tilde{x}_I - M_I \eta_I}_{\text{uncertainities}}$$



Sensor recovery – I

$$z = \underbrace{\hat{z}_l^{UP}}_{\text{measured value}} + \underbrace{(I - M_l C_l)\,\tilde{x}_l - M_l \eta_l}_{\text{uncertainities}}$$

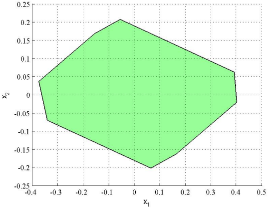


$$z \in \underbrace{\bigcap_{l \in I_H} \left[\left\{ \hat{z}_l^{UP} \right\} \oplus (I - M_l C_l)\,\tilde{S}_l \oplus (-M_l)\,N_l \right]}_{I_{I_H}}$$

$$\hat{z}_j^{UP} + (I - M_j C_j)\,\tilde{x}_j - M_j \eta_j \in I_{I_H}$$

Sensor recovery – I

$$z = \underbrace{\hat{z}_l^{UP}}_{\text{measured value}} + \underbrace{\left(I - M_l C_l\right) \tilde{x}_l - M_l \eta_l}_{\text{uncertainities}}$$



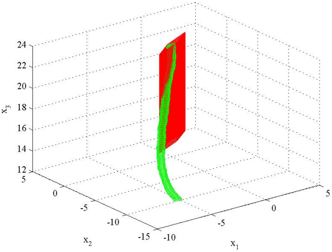
$$z \in \underbrace{\bigcap_{l \in I_H} \left[\left\{ \hat{z}_l^{UP} \right\} \oplus \left(I - M_l C_l\right) \tilde{S}_l \oplus \left(-M_l\right) N_l \right]}_{I_{I_H}}$$

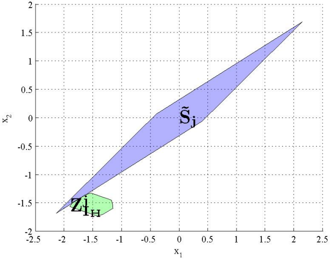
$$\tilde{x}_j \in \underbrace{\left(I - M_j C_j\right)^{-1} \left[\left\{ -\hat{z}_j^{UP} \right\} \oplus M_j N_j \oplus I_{I_H} \right]}_{Z^j_{I_H}}$$

Sensor recovery – II

Necessary condition: $\tilde{S}_j \cap Z^j_{I_H} \neq \emptyset$

Sufficient condition: $\tilde{S}_j \supseteq Z^j_{I_H}$

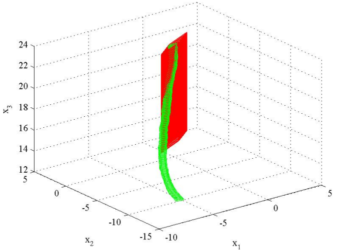


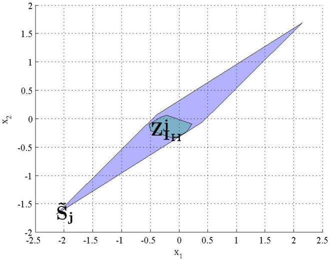


Sensor recovery – II

Necessary condition: $\tilde{S}_j \cap Z^j_{I_H} \neq \emptyset$

Sufficient condition: $\tilde{S}_j \supseteq Z^j_{I_H}$



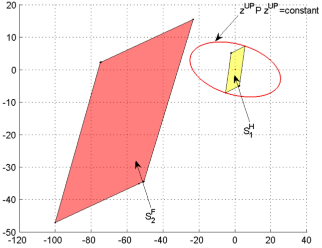


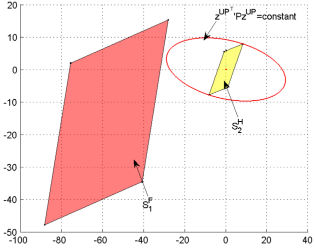
Outline

Example – Ultimate bounds vs. mRPI

For ellipsoidal separation with ultimate bounds sets the reference is

$$\begin{bmatrix} -21 \\ -5.04 \end{bmatrix} \leq x_{ref} - x^0_{ref} \leq \begin{bmatrix} 21 \\ 5.04 \end{bmatrix} \qquad x^0_{ref} = \begin{bmatrix} 54 \\ -4.96 \end{bmatrix}$$

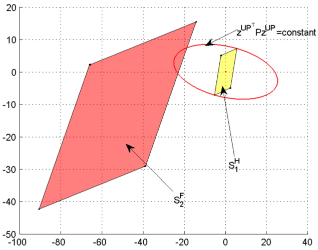


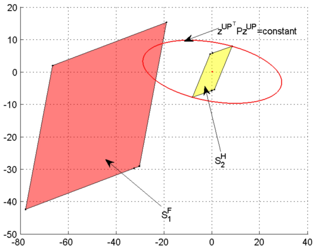


Example – Ultimate bounds vs. mRPI

For ellipsoidal separation with ultimate bounds sets the reference is

$$\begin{bmatrix} -21 \\ -5.04 \end{bmatrix} \leq x_{ref} - x_{ref}^0 \leq \begin{bmatrix} 21 \\ 5.04 \end{bmatrix} \qquad x_{ref}^0 = \begin{bmatrix} 45 \\ -4.96 \end{bmatrix}$$

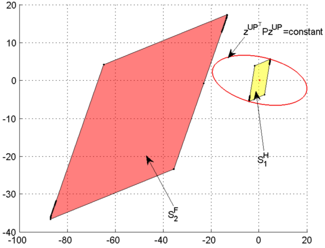


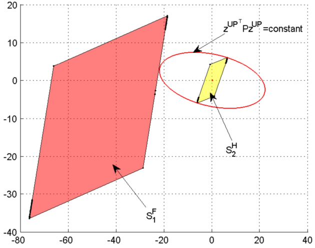


Example – Ultimate bounds vs. mRPI

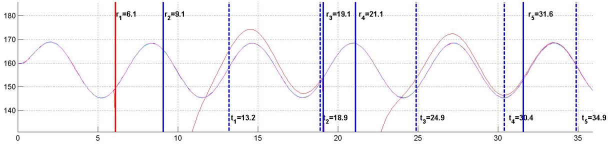
For ellipsoidal separation with mRPI approximations the reference is

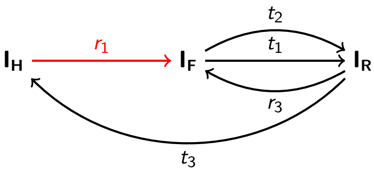
$$\begin{bmatrix} -21 \\ -5.04 \end{bmatrix} \leq x_{ref} - x^0_{ref} \leq \begin{bmatrix} 21 \\ 5.04 \end{bmatrix} \qquad x^0_{ref} = \begin{bmatrix} 45 \\ -4.96 \end{bmatrix}$$

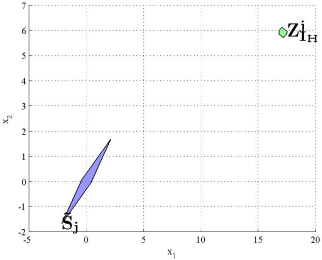




Example – Sensor recovery







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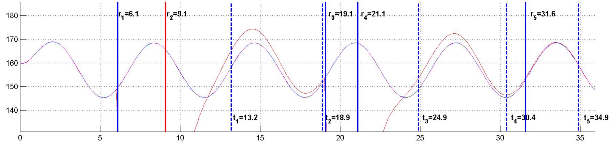

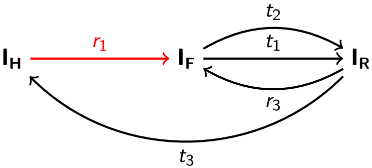

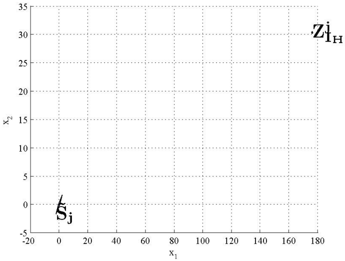

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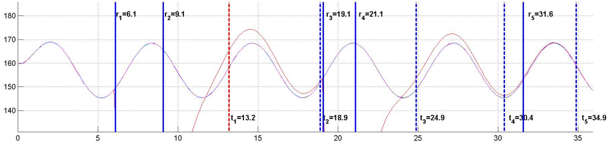
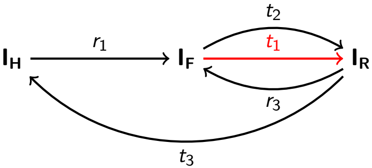
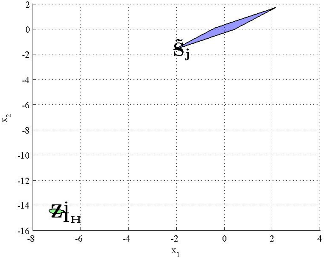
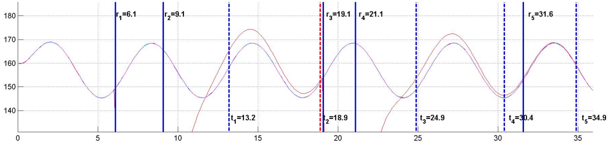
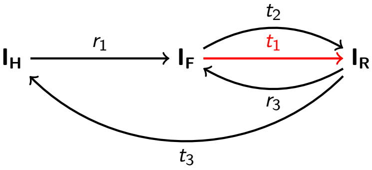


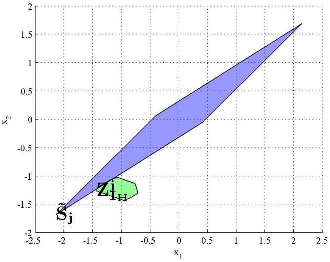




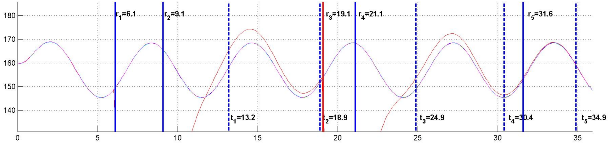
Example – Sensor recovery

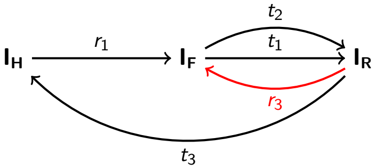


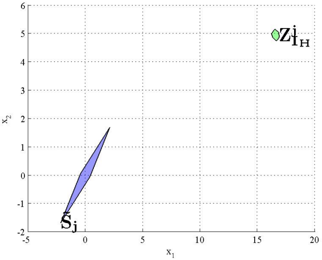




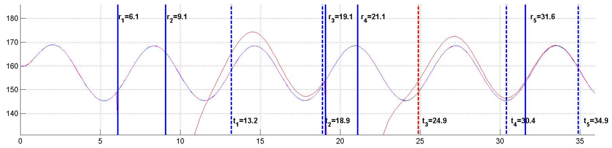
Example – Sensor recovery

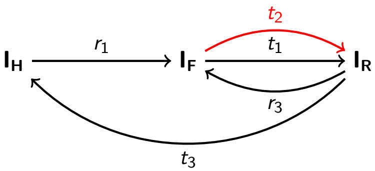


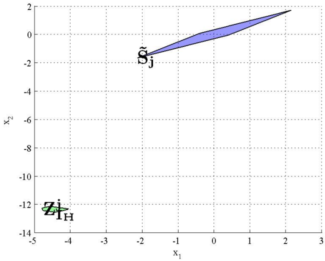




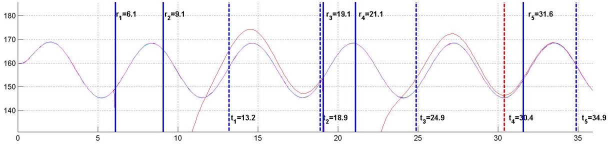
Example – Sensor recovery

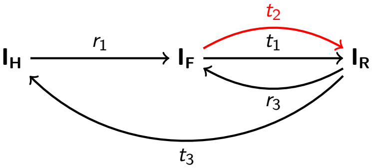


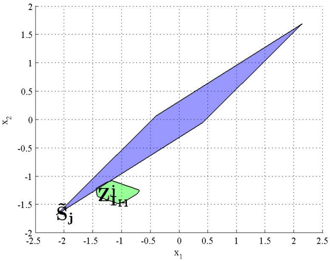




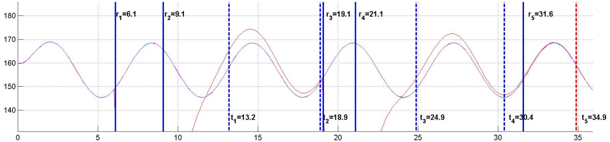
Example – Sensor recovery

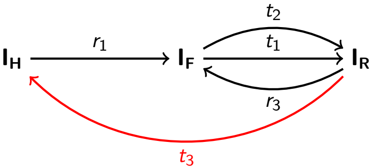


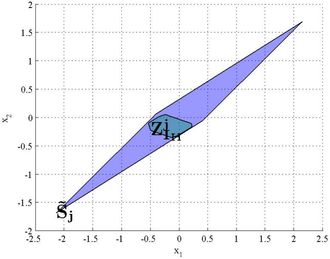




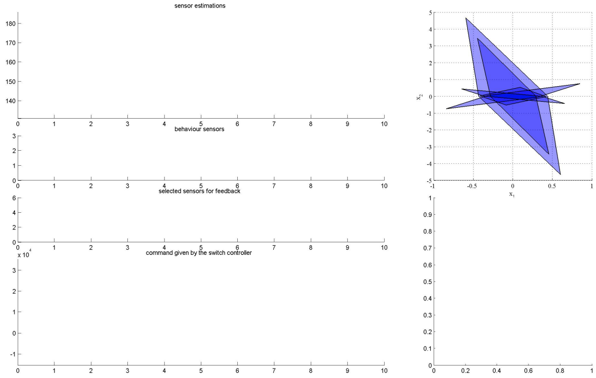
Example – Sensor recovery







Example – FTC simulation



Conclusions

Advantages

- good balance between computational effort and precision
- robust fault detection
- comparable with classical sensor fusion schemes in terms of performance

Disadvantages

- only abrupt faults discussed

Future developments

- nonconvex perturbations and nonlinear systems
- tighter approximations of the mRPI set

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Thank you!

Questions ?